Compound Multiple Access Channels with Conferencing Decoders

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***Abstract*— A two-user discrete memoryless compound multiple access channel with a common message and conferencing decoders is considered. The capacity region is characterized in the special cases of physically degraded channels and unidirectional cooperation, while achievable rate regions are provided for the general case. The results are then extended to the corresponding Gaussian model. In the Gaussian setup, the provided achievable rates are shown to lie within a fraction of one bit from the boundary of the capacity region in several special cases. Numerical results are also provided to obtain insights about the potential gains of decoder cooperation in the underlying model.**

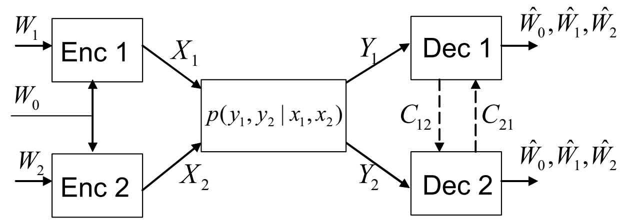

Fig. 1. A discrete-memoryless compound MAC with conferencing decoders and common information (for short, CM).

I. Introduction

Consider a communication system in which finite-capacity directed links exist between either the encoders or the decoders. This framework is widely studied in the information-theoretic literature to obtain insight into the potential advantages of cooperative transmission or reception strategies. Moreover, this system accurately models scenarios, typical in wireless communications, in which the encoders or the decoders have multiple radio interfaces providing orthogonal signal paths between nearby terminals. This information-theoretic framework is usually referred to as "conferencing", emphasizing the possibly interactive nature of the communication over such links. Conferencing encoders in a two-user multiple access channel (MAC) have been investigated in [1], [3][1] and for a two-user interference channel in [4]. These works show that conferencing encoders can create dependence between the transmitted signals by coordinating the transmission via the out-of-band links, thus mimicking multi-antenna transmitters. Conferencing decoders have instead been studied [6] - [9] for a broadcast channel and in [10] for a relay channel. Such decoders can use the out-of-band links to exchange side information about the received signals so as to mimic a multiantenna receiver (see also [11]).

This work extends the state of the art described above by considering the compound MAC with conferencing decoders and a common message (see Fig. 1). This model generalizes the setup of a single-message broadcast (multicast) channel with conferencing decoders studied in [6][2] - [9] in that, here we have two transmitters interested in broadcasting their messages to the conferencing receivers. The model also generalizes the compound MAC with common message studied in [4], by allowing conferencing among the decoders. The main contributions are the following: (*i*) The capacity region is derived for the two-user discrete-memoryless compound MAC with a common message and conferencing decoders for the special cases of physically degraded channels and unidirectional cooperation (Sec. IV); (*ii*) Achievable rate regions are given for the general model of Fig. 1 (Sec. V); (*iii*) Extension to the Gaussian case is provided, establishing the capacity region with unidirectional cooperation and deriving general achievable rates. Such achievable rates are also shown to be within a fraction of one bit of the capacity region in several special cases (Sec. VI). Finally, numerical results are also reported. In this paper, some results are stated without proof. A full treatment can be found in [17], which also considers a scenario with both conferencing encoders and decoders.

The rest of the paper is organized as follows. The system model and the definitions are introduced in Section II. Some preliminary results and an outer bound on the capacity region is given in Section III. In Section IV, the capacity region is characterized in the special cases of physically degraded channels and unidirectional cooperation. Section V is devoted to characterization the of achievable rate regions for one- and two-round conferencing. The Gaussian compound MAC with conferencing decoders is explored in Section VI,

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[1]It is noted that a MAC with conferencing encoders can be seen as a special case of a MAC with generalized feedback.

[2][8] also considers a broadcast channel with private messages to the two users.

in which it is shown that the schemes of Section V achieve all the rates within a fraction of one bit of the capacity region. Numerical results for the Gaussian setting are also presented.

II. System Model and Main Definitions

We consider the model illustrated in Fig. 1, which is a discrete-memoryless compound MAC with conferencing decoders and common information (here, for short, we will refer to this channel as CM) and is denoted by $(\mathcal{X}_1, \mathcal{X}_2, p^*(y_1, y_2|x_1, x_2), \mathcal{Y}_1, \mathcal{Y}_2)$ with input alphabets $\mathcal{X}_1, \mathcal{X}_2$ and output alphabets $\mathcal{Y}_1, \mathcal{Y}_2$. Each i-th encoder, $i = 1, 2$, is interested in sending a private message $W_i \in \mathcal{W}_i = \{1, 2, ..., 2^{nR_i}\}$ of rate R_i [bit/channel use] to both receivers, and, in addition, there is a common message $W_0 \in \mathcal{W}_0 = \{1, 2, ..., 2^{nR_0}\}$ of rate R_0 to be delivered by both encoders to both decoders. It is noted that the channel is memoryless and time-invariant in that the conditional distribution of the output symbols at any time $j = 1, 2, ..., n$ satisfies

$$p(y_{1,j}, y_{2,j}|x_1^n, x_2^n, y_1^{j-1}, y_2^{j-1}, \bar{w}) = p^*(y_{1,j}, y_{2,j}|x_{1,j}, x_{2,j})$$

with $\bar{w} = [w_0, w_1, w_2] \in \mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2$ being a given triplet of messages. Notation-wise, we employ standard conventions (see, e.g., [12]), where the probability distributions are defined by the arguments, upper-case letters represent random variables and the corresponding lower-case letters represent their realizations, and superscripts identify the number of samples to be included in a given vector, e.g., $y_1^{j-1} = [y_{1,1} \cdots y_{1,j-1}]$. It is finally noted that the channel defines to conditional marginals $p(y_1|x_1, x_2) = \sum_{y_2 \in \mathcal{Y}_2} p^*(y_1, y_2|x_1, x_2)$ and similarly for $p(y_2|x_1, x_2)$. Further definitions are in order.

Definition 1: A $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n, K)$ code for the CM channel consists of two encoding functions ($i = 1, 2$)

$$f_i\colon \mathcal{W}_0 \times \mathcal{W}_i \rightarrow \mathcal{X}_i^n, \tag{1}$$

a set of $2K$ "conferencing" functions and corresponding output alphabets $\mathcal{V}_{i,k}$ ($k = 1, 2, ..., K$):

$$g_{1,k}\colon \mathcal{Y}_1^n \times \mathcal{V}_{2,1} \times \cdots \times \mathcal{V}_{2,k-1} \rightarrow \mathcal{V}_{1,k} \tag{2a}$$
$$g_{2,k}\colon \mathcal{Y}_2^n \times \mathcal{V}_{1,1} \times \cdots \times \mathcal{V}_{1,k-1} \rightarrow \mathcal{V}_{2,k}, \tag{2b}$$

and decoding functions:

$$h_1\colon \mathcal{Y}_1^n \times \mathcal{V}_{2,1} \times \cdots \times \mathcal{V}_{2,K} \rightarrow \mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2 \tag{3a}$$
$$h_2\colon \mathcal{Y}_2^n \times \mathcal{V}_{1,1} \times \cdots \times \mathcal{V}_{1,K} \rightarrow \mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2. \tag{3b}$$

Notice that the conferencing functions (2) prescribe K conferencing rounds between the decoders that start as soon as the two decoders receive the entire block of n output symbols y_1^n and y_2^n. Each conference round, say the kth, corresponds to a simultaneous and bidirectional exchange of messages between the two decoders taken from the alphabets $\mathcal{V}_{1,k}$ and $\mathcal{V}_{2,k}$, similarly to [1] and [14]. It is noted that other works have used slightly different definitions of conferencing rounds [8], [6], [16]. After K conferencing rounds, the receivers decode with functions in (3) by capitalizing on the exchanged conferencing messages. Due to the orthogonality between the main channel and the conferencing links, the two phases of transmission on one hand and conferencing/ decoding on the other can take place simultaneously in a pipelined fashion.

Definition 2: A rate triplet (R_0, R_1, R_2) is said to be achievable for the CM channel with conferencing links with capacities (C_{12}, C_{21}) (see Fig. 1), if for any $\varepsilon > 0$ there exists for all n sufficiently large an $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n, K)$ code with any $K \geq 0$ such that the probability of error at the two receivers satisfies

$$P_e = \frac{1}{2^{n(R_0+R_1+R_2)}} \sum_{\bar{w}} \Pr\left[\begin{array}{c} \{h_1(Y_1^n, V_2^k) \neq \bar{w}\} \cup \\ \{h_2(Y_2^n, V_1^k) \neq \bar{w}\} | \bar{w} \text{ sent} \end{array}\right] \leq \varepsilon,$$

and the conferencing alphabets are such that

$$\sum_{k=1}^{K} |\mathcal{V}_{1,k}| \leq nC_{12} \text{ and } \sum_{k=1}^{K} |\mathcal{V}_{2,k}| \leq nC_{21}. \tag{4}$$

The capacity region $\mathcal{C}_{CM}(C_{12}, C_{21})$ is the closure of the set of all achievable rates (R_0, R_1, R_2) in the presence of conferencing links with capacities (C_{12}, C_{21}).

III. Preliminaries and Outer bound

Similarly to [4], it is useful to define the rate region $\mathcal{R}_{MAC,i}(p(u), p(x_1|u), p(x_2|u))$ for the MAC seen at the i-th receiver ($i = 1, 2$) as the set of rates

$$\begin{aligned} \mathcal{R}_{MAC,i}(p(u), p(x_1|u), p(x_2|u)) = \\ \{(R_0, R_1, R_2)\colon R_j \geq 0,\ j = 0, 1, 2 \\ R_1 \leq I(X_1; Y_i|X_2 U), \\ R_2 \leq I(X_2; Y_i|X_1 U), \\ R_1 + R_2 \leq I(X_1 X_2; Y_i|U), \\ R_0 + R_1 + R_2 \leq I(X_1 X_2; Y_i)\}, \end{aligned}$$

where the joint distribution of the involved variables is given by $p(u)p(x_1|u)p(x_2|u)p(y_i|x_1, x_2)$.

If $C_{12} = C_{21} = 0$, the capacity region $\mathcal{C}_{CM}(0, 0)$ of the CM is given by [4]:

$$\begin{aligned} \mathcal{C}_{CM}(0, 0) &= \bigcup \left\{ \bigcap_{i=1,2} \mathcal{R}_{MAC,i}(p(u), p(x_1|u), p(x_2|u)) \right\} \\ &= \bigcup \big\{ (R_0, R_1, R_2)\colon\ R_j \geq 0,\ j = 0, 1, 2, \\ R_1 &\leq \min\{I(X_1; Y_1|X_2 U),\ I(X_1; Y_2|X_2 U)\} \\ R_2 &\leq \min\{I(X_2; Y_1|X_1 U),\ I(X_2; Y_2|X_1 U)\} \\ R_1 + R_2 &\leq \min\{I(X_1 X_2; Y_1|U),\ I(X_1 X_2; Y_2|U)\} \\ R_0 + R_1 + R_2 &\leq \min\{I(X_1 X_2; Y_1),\ I(X_1 X_2; Y_2)\} \big\}, \end{aligned}$$

where the union is taken over all joint distributions that factorize as $p(u)p(x_1|u)p(x_2|u)p^*(y_1, y_2|x_1, x_2)$.

It is remarked that no convex hull operation is necessary in evaluating $\mathcal{C}_{CM}(0, 0)$ as the region is convex [4] (see also [2], Appendix A).

We now derive an outer bound to the capacity region $\mathcal{C}_{CM}(C_{12}, C_{21})$ with conferencing at the decoders. To this end, it is useful to define the rate region achievable when the two receivers are allowed to fully cooperate (FC), thus equivalently forming a two-antenna receiver. In this case, we have:

$$\begin{aligned}\mathcal{R}_{MAC,FC}&(p(u),p(x_1|u),p(x_2|u)) = \\ &\big\{(R_0,R_1,R_2){:}R_j\geq 0,\ j=0,1,2,\\ &R_1\leq I(X_1;Y_1,Y_2|X_2,U),\\ &R_2\leq I(X_2;Y_1,Y_2|X_1,U),\\ &R_1+R_2\leq I(X_1,X_2;Y_1,Y_2|U),\\ &R_0+R_1+R_2\leq I(X_1,X_2;Y_1,Y_2)\big\}\end{aligned}$$

where the joint distribution is of the form

$$p(u)p(x_1|u)p(x_2|u)p^*(y_1,y_2|x_1,x_2). \tag{5}$$

Proposition 1: We have $\mathcal{C}_{CM}(C_{12},C_{21}) \subseteq \mathcal{C}_{CM-out}(C_{12},C_{21})$ where (dropping the dependence on $p(u),p(x_1|u),p(x_2|u)$ to simplify the notation)

$$\begin{aligned}\mathcal{C}_{CM-out}(C_{12},C_{21}) = \bigcup\{&(\mathcal{R}_{MAC,1}+C_{12})\cap\\ &(\mathcal{R}_{MAC,2}+C_{21})\cap(\mathcal{R}_{MAC,FC})\},\end{aligned}$$

where union is taken over joint distributions of the form (5).

Similarly to $\mathcal{C}_{CM}(0,0)$, region $\mathcal{C}_{CM-out}(C_{12},C_{21})$ can be proved to be convex following [2], Appendix A.

Proof: Follows from cut-set arguments. ∎

IV. Capacity Region with Physically Degraded Channels and Unidirectional Cooperation

The next proposition establishes the capacity region $\mathcal{C}_{CMD}(C_{12},C_{21})$ for the CM channel with degraded outputs.

Proposition 2: If the CM channel is physically degraded as $(X_1,X_2)-Y_1-Y_2$, then the capacity region is obtained as

$$\begin{aligned}\mathcal{C}_{CMD}(C_{12},C_{21}) &= \mathcal{C}_{CM-out}(C_{12},0)\\ =\bigcup\Big\{&(R_0,R_1,R_2){:}\ R_j\geq 0,\\ R_1&\leq\min\{I(X_1;Y_1|X_2,U),(X_1;Y_2|X_2,U)+C_{12}\},\\ R_2&\leq\min\{I(X_2;Y_1|X_1,U),I(X_2;Y_2|X_1,U)+C_{12}\},\\ R_1+R_2&\leq\min\{I(X_1,X_2;Y_1|U),I(X_1,X_2;Y_2|U)+C_{12}\},\\ R_0+R_1+R_2&\leq\min\{I(X_1,X_2;Y_1),I(X_1,X_2;Y_2)+C_{12}\}\Big\},\end{aligned}$$

Notice that here $p^*(y_1,y_2|x_1,x_2) = p(y_1|x_1,x_2)p(y_2|y_1)$ due to degradedness.

Proof: See Appendix. ∎

Establishment of the capacity region is also possible in the special case where only unidirectional cooperation is allowed, that is $C_{12}=0$ or $C_{21}=0$. This result is akin to [9] where a broadcast channel with two receivers and unidirectional cooperation was considered.

Proposition 3: In the case of unidirectional cooperation ($C_{12}=0$ or $C_{21}=0$), the capacity region is given by, respectively,

$$\mathcal{C}_{CM}(0,C_{21}) = \mathcal{C}_{CM-out}(0,C_{21}) \tag{7}$$
$$\mathcal{C}_{CM}(C_{12},0) = \mathcal{C}_{CM-out}(C_{12},0). \tag{8}$$

Proof: Achievability follows from the scheme used in the proof of Proposition 2. The converse is immediate. ∎

V. General Achievable Rates

Achievable rates can be derived for the general CM channel, extending the analysis of [8] from the broadcast setting with one transmitter to the CM channel. Notice that [8] uses a different definition for the operation over the conferencing channels but this turns out to be immaterial for the achievable rates discussed below.

Proposition 4: The following rate region is achievable with one-round conferencing, i.e., $K=1$:

$$\begin{aligned}\mathcal{R}_{OR}(C_{12},C_{21}) = \bigcup\{&(R_0,R_1,R_2){:}\ R_j\geq 0,\ j=0,1,2,\\ R_1&\leq\min\{I(X_1;Y_1,\hat{Y}_2|X_2,U),I(X_1;Y_2,\hat{Y}_1|X_2,U)\},\\ R_2&\leq\min\{I(X_2;Y_1,\hat{Y}_2|X_1,U),I(X_2;Y_2,\hat{Y}_1|X_1,U)\},\\ R_1+R_2&\leq\min\{I(X_1,X_2;Y_1\hat{Y}_2|U),I(X_1,X_2;Y_2,\hat{Y}_1|U)\}\\ R_0+R_1+R_2&\leq\min\{I(X_1,X_2;Y_1,\hat{Y}_2),I(X_1,X_2;Y_2,\hat{Y}_1)\}\}\end{aligned}$$

subject to

$$C_{12}\geq I(Y_1;\hat{Y}_1|Y_2) \tag{10a}$$
$$C_{21}\geq I(Y_2;\hat{Y}_2|Y_1) \tag{10b}$$

with $|\widehat{\mathcal{Y}}_i|\leq|\mathcal{Y}_i|+1$, and the union is taken over all joint distributions that factorize as

$$p(u)p(x_1|u)p(x_2|u)p^*(y_1,y_2|x_1,x_2)p(\hat{y}_1|y_1)p(\hat{y}_2|y_2).$$

Proof: (Sketch): The proof follows similarly to Theorem 3 in [8] and is thus only sketched here. A one-step conference ($K=1$) is used. Encoding and transmission at the transmitters are performed as for a MAC channel with common information (see proof of Proposition 2). Each receiver compresses its received signal by Wyner-Ziv quantization exploiting the fact that the other receiver has its own correlated observation given by the corresponding output sequence. The compression indices are exchanged during the single conferencing round via symbols $\mathcal{V}_{1,1}$ and $\mathcal{V}_{2,1}$. Decoding is then carried out at each receiver using joint typicality: For instance, receiver 1 looks for jointly typical sequences $(u^n(w_0),x_1^n(w_0,w_1),x_2^n(w_0,w_2),y_1^n,\hat{y}_2^n)$ with $w_i\in\mathcal{W}_i$, where $\hat{y}_2^n$ is the compressed version of the channel output received by the second decoder. ∎

Remark 1: The one-round strategy of Proposition 4 does not subsume the scheme used in Proposition 2 and Proposition 3 to achieve capacity in the presence of physically degraded channels and unidirectional cooperation. Since this latter scheme, as detailed in the Appendix, is also based on a one-round strategy (where only one of the conferencing links is used to convey partial information about the decision of the sending decoder), the rate region obtained as the convex hull of the union $\mathcal{R}_{OR}(C_{12},C_{21})\cup\mathcal{C}_{CM}(0,C_{21})\cup\mathcal{C}_{CM}(C_{12},0)$ is also achievable with $K=1$ and generally includes $\mathcal{R}_{OR}(C_{12},C_{21})$.

Remark 2: In the one-round schemes of Proposition 2 and Proposition 3, one of the receivers decodes both messages only from its received signal from the transmitters, and forwards the bin indices over the conferencing link. We can consider an alternative one-round scheme in which each receiver only decodes one of the messages from its received signal and forwards the bin index for the decoded message over the conferencing link. Then each receiver decodes the remaining message from both the received signal and the bin index. Ignoring the common message ($R_0 = 0$), convex hull of the following rate region can be achieved by this one-round scheme, e.g. $K = 1$.

$$\mathcal{R}_{dfTR}(C_{12}, C_{21}) = \bigcup \{(R_1, R_2)\colon R_j \geq 0,\ j = 0, 1, 2,$$
$$R_1 \leq \min\{I(X_1; Y_1), I(X_1; Y_2|X_2) + C_{12}\},$$
$$R_2 \leq \min\{I(X_2; Y_2), I(X_2; Y_1|X_1) + C_{21}\},$$

where the union is taken over all joint distributions that factorize as $p(x_1)p(x_2)p^*(y_1, y_2|x_1, x_2)$.

Now consider a special case in which the channel from the transmitters to receivers is composed of two links of capacity C_i from transmitter i to receiver i, $i = 1, 2$. It is possible to show that the capacity region of this special setup is given by $\{(R_1, R_2){:}0 \leq R_1 \leq \min\{C_1, C_{12}\}, 0 \leq R_2 \leq \min\{C_2, C_{21}\}\}$, and is achievable by the above scheme, in which each receiver decodes the message of the transmitter it is connected to, and forwards it to the other receiver over the conferencing link. We should note that the quantization based protocols fail to achieve this capacity.

The achievable scheme of Proposition 4 has one round of conferencing. Below, we construct an examples for which this scheme fails to achieve the outer bound (6).

Example 1. Consider a symmetric scenario (i.e., $p^*(y_1, y_2|x_1, x_2) = p^*(y_2, y_1|x_1, x_2) = p^*(y_1, y_2|x_2, x_1) = p^*(y_2, y_1|x_2, x_1)$) with $R_0 = 0$ and equal rates $R_1 = R_2 = R$, and fix U to a constant without loss of generality (given the absence of a common message) and input distribution to $p(x_1)p(x_2)$. We are interested in finding the maximum achievable equal rate $R_1 = R_2 = R$. Assume that the conferencing capacities satisfy $C_{12} = H(Y_1|Y_2) = H(Y_2|Y_1)$ and $\frac{1}{2}I(X_1X_2; Y_2|Y_1) \leq C_{21} < H(Y_1|Y_2)$. In this case, it can be seen that the maximum equal rate is upper bounded as $R \leq R_{out} = \frac{1}{2}I(X_1X_2; Y_1Y_2)$ by the outer bound (6), which corresponds to the maximum equal rate of a system with full cooperation at the receiver side. This bound can be achieved if both receivers have access to both outputs Y_1 and Y_2. With the one-round strategy, since $C_{12} = H(Y_1|Y_2)$ receiver 1 can provide Y_1 to receiver 2 via Slepian-Wolf compression, but receiver 2 cannot do the same with receiver 1 since $C_{21} < H(Y_1|Y_2)$. Therefore, rate R_{out} cannot be achieved by such a strategy, which in fact attains equal rate $R_{OR} = \frac{1}{2}I(X_1X_2; Y_1\hat{Y}_2) < R_{out}$ (recall (10)).

We now consider a second strategy that generalizes the previous one and is based on two rounds of conferencing ($K = 2$). As will be shown below, this strategy is able to improve upon the one-round scheme, while still failing to achieve the outer-bound (6) in the general case.

Proposition 5: The following rate region is achievable with two rounds of conferencing, i..e., $K = 2$ [3],

$$\mathcal{R}_{TR}(C_{12}, C_{21}) = \mathrm{co} \bigcup \{\mathcal{R}_{TR,12} \cup \mathcal{R}_{TR,21}\} \tag{11}$$

[3] Notation "co" indicates the convex hull operation.

where

$$\mathcal{R}_{TR,12} = \{(R_0, R_1, R_2)\colon R_j \geq 0,\ j = 0, 1, 2,$$
$$R_1 \leq \min\{I(X_1; Y_1|X_2, U) + C_{21}, I(X_1; Y_2, \hat{Y}_1|X_2, U)\},$$
$$R_2 \leq \min\{I(X_2; Y_1|X_1, U) + C_{21}, I(X_2; Y_2, \hat{Y}_1|X_1, U)\},$$
$$R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1|U) + C_{21}, I(X_1, X_2; Y_2, \hat{Y}_1|U)\},$$

$$R_0 + R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1) + C_{21}, I(X_1, X_2; Y_2, \hat{Y}_1)\}\},$$

and $\mathcal{R}_{TR,21}$ is similarly defined

$$\mathcal{R}_{TR,21} = \{(R_0, R_1, R_2)\colon R_j \geq 0,\ j = 0, 1, 2,$$
$$R_1 \leq \min\{I(X_1; Y_1, \hat{Y}_2|X_2, U), I(X_1; Y_2|X_2, U) + C_{12}\},$$
$$R_2 \leq \min\{I(X_2; Y_1, \hat{Y}_2|X_1, U), I(X_1; Y_2|X_2, U) + C_{12}\},$$
$$R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1, \hat{Y}_2|U), I(X_1; Y_2|X_2, U) + C_{12}\},$$
$$R_0 + R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1, \hat{Y}_2), I(X_1; Y_2|X_2) + C_{12}\}\},$$

subject to

$$C_{12} \geq I(Y_1; \hat{Y}_1|Y_2) \tag{13a}$$
$$C_{21} \geq I(Y_2; \hat{Y}_2|Y_1) \tag{13b}$$

with $|\widehat{\mathcal{Y}}_i| \leq |\mathcal{Y}_i| + 1$, and the union is taken over all joint distributions that factorize as $p(u)p(x_1|u)p(x_2|u)p^*(y_1, y_2|x_1, x_2)p(\hat{y}_1|y_1)p(\hat{y}_2|y_2)$.

Proof: (Sketch): The proof is quite similar to Theorem 4 in [8] so that here we only sketch the main points. Conferencing takes place via $K = 2$ rounds. Moreover, two possible strategies are considered, hence the convex hull operation in (11), achieved by time-sharing. The achievable rate region $\mathcal{R}_{TR,12}$ can be obtained as follows. Receiver 2 randomly partitions the message sets $\mathcal{W}_0$, $\mathcal{W}_1$ and $\mathcal{W}_2$ into $2^{n\alpha_0 C_{12}}$, $2^{n\alpha_1 C_{12}}$ and $2^{n\alpha_2 C_{12}}$ subsets, respectively, for a given $0 \leq \alpha_i \leq 1$ and $\sum_{i=0}^{2} \alpha_i = 1$, as in the proof of Proposition 2. Encoding and transmission are performed as for the MAC channel with common information. Receiver 1 compresses its received signal using Wyner-Ziv quantization as for the scheme discussed in the proof of Proposition 4. This index is sent in the first conferencing round (notice that $|\mathcal{V}_{1,1}| = nC_{12}$ and $|\mathcal{V}_{2,1}| = 0$). Upon reception of the compression index $\mathcal{V}_{1,1}$, receiver 2 proceeds to decoding via joint typicality and then sends the subset indices (see proof of Proposition 2) to receiver 1 via $\mathcal{V}_{2,2}$ (now, $|\mathcal{V}_{1,2}| = 0$ and $|\mathcal{V}_{2,2}| = nC_{21}$). The latter decoder performs joint-typicality decoding on the subsets of messages left undecided by its received channel output. The rate region $\mathcal{R}_{TR,21}$ is obtained similarly by simply swapping the role of decoder 1 and decoder 2. ■

Remark 3: An alternative two-round strategy to the one in Proposition 5 may prescribe the use of Wyner-Ziv compression in both conferencing rounds. Specifically, after the first conferencing round, each decoder, to elaborate say the first, can compress its received sequence Y_1^n based, not only on Y_2^n, but also conditionally on the knowledge of the sequence $\hat{Y}_2^n$ (clearly also known by the second decoder) received in the first round. The achievable rate region of such scheme can be easily obtained following similar considerations to those leading to Proposition 5 and is not explicitly given here. It is generally not obvious whether such a strategy or the one of Proposition 5 should be preferred, so that the convex hull of the union of the two regions is generally achievable and (possibly) includes both regions. Another possibility would be to share the conferencing capacity link in the second round between binning of the messages (as in Proposition 5) and conditional Wyner-Ziv compression (as discussed in this Remark). This is not further elaborated upon here.

Example 1 (cont'd): To see the impact of the two-round scheme, here we reconsider Example 1 discussed above for which the one-round scheme does not achieve the outer bound. However, it can be seen that the two-round does indeed achieve the outer bound. In fact, receiver 1 can provide Y_1 to receiver 2 via Slepian-Wolf compression as for the one round case, while receiver 2 does not send anything in the first conferencing round ($\hat{Y}_2$ is a constant). Now, receiver 2 decodes and sends the bin index of the decoded messages to receiver 1 in the second conferencing round according to the two-round strategy discussed above (receiver 1 is silent in the second round). Since $C_{21} \geq \frac{1}{2}I(X_1X_2;Y_2|Y_1)$ by assumption, it can be seen from Proposition 5 that the maximum equal rate achieved by the two round scheme is $R_{TR} = R_{out}$.

We finally remark that it is possible in principle to extend the achievable rate regions derived above to more than two conferencing rounds, following [6], [5]. While conceptually not difficult, description of the achievable rate region would require cumbersome notation and is thus omitted here.

VI. Gaussian Compound MAC

Here we consider the Gaussian version of the CM channel

$$Y_1 = \gamma_{11}X_1 + \gamma_{21}X_2 + Z_1 \tag{14a}$$

$$Y_2 = \gamma_{22}X_2 + \gamma_{12}X_1 + Z_2, \tag{14b}$$

with channel gains $\gamma_{ij} \geq 0$, white zero-mean unit-power Gaussian noise $\{Z_i\}_{i=1}^n$ and per-symbol power constraints $E[X_i^2] \leq P_i$, $i = 1, 2$. Notice that channel (14) is not physically degraded.

The outer bound of Proposition 1 can be extended to (14) by using standard arguments. In particular, the capacity region over the Gaussian CM $\mathcal{C}_{CM}^{\mathcal{G}}(C_{12}, C_{21})$ satisfies the following.

Proposition 6: We have $\mathcal{C}_{CM}^{\mathcal{G}}(C_{12}, C_{21}) \subseteq \mathcal{C}_{CM-out}^{\mathcal{G}}(C_{12}, C_{21})$ where $\mathcal{C}_{CM-out}^{\mathcal{G}}(C_{12}, C_{21})$ is the rate region of Proposition 1 as evaluated with a Gaussian joint distribution $p(u)p(x_1|u)p(x_2|u)$ characterized by

$$X_i = \sqrt{P - P_i'}U + \sqrt{P_i'}V_i, \tag{15}$$

with $0 \leq P_i' \leq P_i$, $i = 1, 2$, where is U, V_1 and V_2 are independent Gaussian zero-mean unit-power random variables.

The full expressions of the rate bounds can be derived from Proposition 1 and the discussion above, and are found in [17]. Here we note that P_i' can be interpreted as the power that the i-th transmitter invests in transmitting its own private message.

Proof: The proof is based on showing that a joint Gaussian distribution on U, X_1 and X_2 exhausts the outer bound region of Proposition 1 when evaluated with input power constraints $E[X_i^2] \leq P_i$. This can be done following the steps of [3], where the proof is given for a single MAC channel with common information (see also [15]). ■

The achievable rates in Proposition 4 (for $K = 1$) and Proposition 5 (for $K = 2$) can also be extended to the Gaussian CM. In so doing, here we focus on jointly Gaussian random variables (15) for the input $p(u)p(x_1|u)p(x_2|u)$ and Gaussian test channels $p(\hat{y}_1|y_1)$ and $p(\hat{y}_2|y_2)$ for Wyner-Ziv compression. Specifically, for the latter, we select the variables at hand as $\hat{Y}_i = Y_i + Z_{q,i}$ where the compression noise $Z_{q,i}$ is zero-mean Gaussian with variance σ_i^2, and is independent of Y_i. Due to the constraints in (10), the compression noise variances σ_i^2 should satisfy:

$$\sigma_1^2 \geq \frac{1 + (\gamma_{11}^2 + \gamma_{12}^2)P_1 + (\gamma_{21}^2 + \gamma_{22}^2)P_2 + \mathcal{K}}{(2^{2C_{12}} - 1)(1 + \gamma_{12}^2 P_1 + \gamma_{22}^2 P_2)} \tag{16a}$$

$$\sigma_2^2 \geq \frac{1 + (\gamma_{11}^2 + \gamma_{12}^2)P_1 + (\gamma_{21}^2 + \gamma_{22}^2)P_2 + \mathcal{K}}{(2^{2C_{21}} - 1)(1 + \gamma_{11}^2 P_1 + \gamma_{21}^2 P_2)}. \tag{16b}$$

with

$$\mathcal{K} \triangleq (\gamma_{12}\gamma_{21} - \gamma_{11}\gamma_{22})^2 P_1 P_2. \tag{17}$$

The rate regions for $K = 1$ and $K = 2$ in the Gaussian CM then follow easily from Proposition 4 and Proposition 5, respectively, with the given choices for the random variables at hand. Full expressions of the rate bounds can be found in [17]. Examples for special cases of interest are provided below.

A. Discussion

Here we draw some conclusions on the optimality of the one and two-round schemes discussed above for the Gaussian CM. We start with the one-round scheme and notice that, by comparison with the outer bound of Proposition 6, it can be easily seen that the scheme at hand is optimal in the asymptotic regime of large conferencing capacities $C_{12} \to \infty$ and $C_{21} \to \infty$. In fact, in such regime the quantization noise variances in (16) tend to zero, so that the performance approaches that of a system with full cooperation at the decoder side, which coincide with the outer bound of Proposition 6. Further conclusions on the gap between the outer bound and the performance achievable with one round of conferencing at the decoders can be drawn in two special cases. Consider first the case of a broadcast channel with conferencing encoders, which is obtained by setting $R_0 = R_2 = 0$ and thus $P_2 = 0$ without loss of generality (a symmetric statement can be straightforwardly obtained for $R_0 = R_1 = 0$). In this case, we will show below that the one-round scheme achieves the outer bound of Proposition 6 to within half a bit, irrespective of the channel gains of the broadcast channel and the capacities of the conferencing links. To elaborate, we notice that the outer bound of Proposition 6 for the special case at hand becomes

$$R_1 \le R_{1,out} = \min\{\mathcal{C}(\gamma_{11}^2 P_1) + C_{21}, \mathcal{C}(\gamma_{12}^2 P_1) + C_{12}, \mathcal{C}((\gamma_{11}^2 + \gamma_{12}^2)P_1)\}, \tag{18}$$

where we have defined $\mathcal{C}(x) = 0.5\log(1+x)$[4], whereas the rate achievable with one-round conferencing is

$$R_{1,OR} = \min\left\{\mathcal{C}\left(\gamma_{11}^2 P_1 + \frac{\gamma_{12}^2 P_1}{1+\sigma_2^2}\right), \mathcal{C}\left(\gamma_{12}^2 P_1 + \frac{\gamma_{11}^2 P_1}{1+\sigma_1^2}\right)\right\}$$

Using these two expressions, we can prove the following proposition (see [17] for a full proof).

Proposition 7: We have $R_{1,OR} \ge R_{1,out} - \frac{1}{2}$. Moreover, for the symmetric channel case, i.e., $\gamma_{11}^2 = \gamma_{12}^2$, we have $R_{1,OR} \ge R_{1,out} - \frac{\log 3 - 1}{2}$.

Next we consider the symmetric Gaussian CM, that is, we let $R_0 = 0$, $\gamma_{ij} = 1$ for $i, j \in \{1, 2\}$, and $P_1 = P_2 \triangleq P$. We also assume symmetric conferencing link capacities $C_{12} = C_{21} \triangleq C$. In such a case, the outer bound and the achievable rate region with one-round conferencing are given by, respectively:

$$\begin{aligned} \mathcal{C}_{CM-out}^{\mathcal{G}}(C) = \{(R_1, R_2) : R_1 \ge 0,\ R_2 \ge 0, \\ R_1 \le \min\{\mathcal{C}(P) + C,\ \mathcal{C}(2P)\}, \\ R_2 \le \min\{\mathcal{C}(P) + C,\ \mathcal{C}(2P)\}, \\ R_1 + R_2 \le \min\{\mathcal{C}(2P) + C,\ \mathcal{C}(4P)\}\}, \end{aligned}$$

and

$$\begin{aligned} \mathcal{R}_{OR}^{\mathcal{G}}(C) = \Bigg\{(R_1, R_2) : R_1 \ge 0,\ R_2 \ge 0, \\ R_1 \le \mathcal{C}\left(\left(1 + \frac{1}{1+\sigma^2}\right)P\right), \\ R_2 \le \mathcal{C}\left(\left(1 + \frac{1}{1+\sigma^2}\right)P\right), \\ R_1 + R_2 \le \mathcal{C}\left(\left(1 + \frac{1}{1+\sigma^2}\right)2P\right)\Bigg\}. \end{aligned}$$

with

$$\sigma^2 \triangleq \frac{1+4P}{(1+2P)(2^{2C}-1)}.$$

The following result can be proved (see [17]).

Proposition 8: $\mathcal{R}_{OR}^{\mathcal{G}} \supseteq \{(R_1, R_2) : R_1 \ge 0, R_2 \ge 0, (R_1 + \delta, R_2 + (\Delta - \delta)) \in \mathcal{C}_{CM-out}^{\mathcal{G}}(C)$ for all $\delta \in [0, \Delta]\}$ with $\Delta = \frac{\log 3 - 1}{2}$.

The proposition above is equivalent to saying that the total rate loss of using one round of conferencing is less than $\left(\frac{\log 3 - 1}{2}\right) \approx 0.293$ bits. It should be pointed out that one round of conferencing at the encoders is also optimal in all the cases where the capacity region is known [1], [4].

[4]All logarithms are base 2.

Let us now consider the two-round scheme. Since the rate region achievable by this scheme subsumes that attainable with $K = 1$ ($\mathcal{R}_{TR}^{\mathcal{G}}(C_{12}, C_{21}) \supseteq \mathcal{R}_{OR}^{\mathcal{G}}(C_{12}, C_{21})$), all the conclusions above on the one-round scheme also apply to the two-round strategy. Moreover, it should be noted that the two-round approach was defined as single-session in [16] and shown therein to be optimal among several classes of multi-session protocols for a broadcast channel with cooperating decoders. Finally, we can prove the following.

Proposition 9: The two-round scheme is optimal for *uni-directional cooperation*: $\mathcal{R}_{TR}^{\mathcal{G}}(0, C_{21}) = \mathcal{C}_{CM-out}^{\mathcal{G}}(0, C_{21})$ and $\mathcal{R}_{TR}^{\mathcal{G}}(C_{12}, 0) = \mathcal{C}_{CM-out}^{\mathcal{G}}(C_{12}, 0)$, thus establishing the capacity of the Gaussian CM for this special case.

Finally, we would like to comment on the sum-rate multiplexing gain of the Gaussian CM. Consider a symmetric system with $P_1 = P_2 \triangleq P$, $\gamma_{11} = \gamma_{22}$, $\gamma_{12} = \gamma_{21}$, and $C_{12} = C_{21} \triangleq C$. We are interested in the conditions on the conferencing capacity C under which we can achieve the maximum multiplexing gain on the sum-rate, $\lim_{P\to\infty} \sup_{(0,R_1,R_2)\in\mathcal{C}_{CM}^{\mathcal{G}}(C,C)} (R_1 + R_2)/(0.5\log P) = 2$, corresponding to full cooperation. From the outer bound of Proposition 6, it can be seen that C should scale at least as $0.5 \log P$ since the sum rate is limited by $I(X_1X_2; Y_2|U) + C_{12} = \mathcal{C}(P(\gamma_{11}^2 + \gamma_{21}^2)) + C$. By considering the achievable regions with one or two conferencing rounds, we can conclude that, if C scales as $(1+\epsilon)\log P$ with any $\epsilon > 0$, then the optimal multiplexing gain is indeed achievable. This is because, with $C = 0.5(1+\epsilon)\log P$ the quantization noise variances in (16) are proportional to $P^{-\epsilon}$ and thus tend to zero for large P. This result would hold even if the decoders ignored the side information at the other decoder. In this case we would have $\sigma_i^2 = \frac{\gamma_{11}^2 P + \gamma_{21}^2 P + 1}{2^{2C} - 1}$, which is still proportional to $P^{-\epsilon}$ for $C = 0.5(1+\epsilon)\log P$.

As a final remark, extending the achievable rates defined above for the Gaussian channel (and assuming Gaussian channel and compression codebooks as done above) to more than two conferencing rounds would not lead to any further gain, as with Gaussian variables "conditional" compression or compression with side information have the same efficiency (see [6] for a discussion).

B. Numerical results

In this section, we present some numerical examples to get further insight into the impact of decoder conferencing on the Gaussian CM. Fig. 2 shows the outer bound, the rate regions achievable with one-round and two-round strategies as well as with no cooperation ($C_{12} = C_{21} = 0$) for $R_0 = 0$ (so that selecting $P_i' = P_i$ is sufficient in all the capacity regions), and a symmetric scenario with $P_1 = P_2 = 5dB$, $\gamma_{12}^2 = \gamma_{21}^2 = -3dB$, $\gamma_{11}^2 = \gamma_{22}^2 = 0dB$, $C_{21} = C_{12} = 0.3$. It can be seen that cooperation via conferencing decoders enlarges the achievable rate region in terms of both the sum-rate and the individual rates. Moreover, the two-step strategy

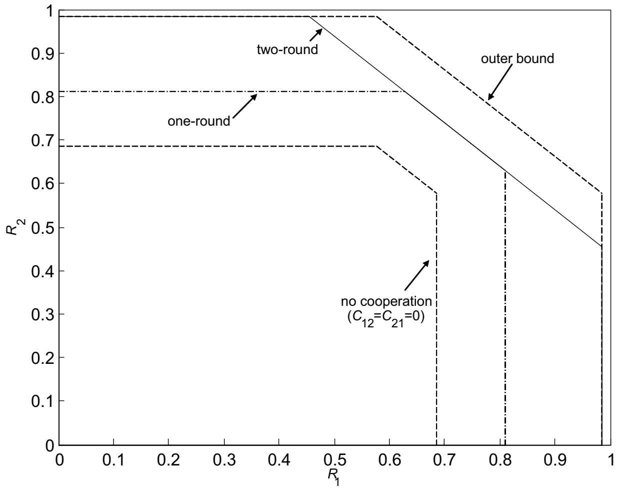

Fig. 2. Outer bound, rate region achievable with one-round and two-round strategies and with no cooperation ($C_{12} = C_{21} = 0$) for $R_0 = 0$, and a symmetric scenario with $P_1 = P_2 = 5dB$, $\gamma_{12}^2 = \gamma_{21}^2 = -3dB$, $\gamma_{11}^2 = \gamma_{22}^2 = 0dB$, $C_{21} = C_{12} = 0.5$.

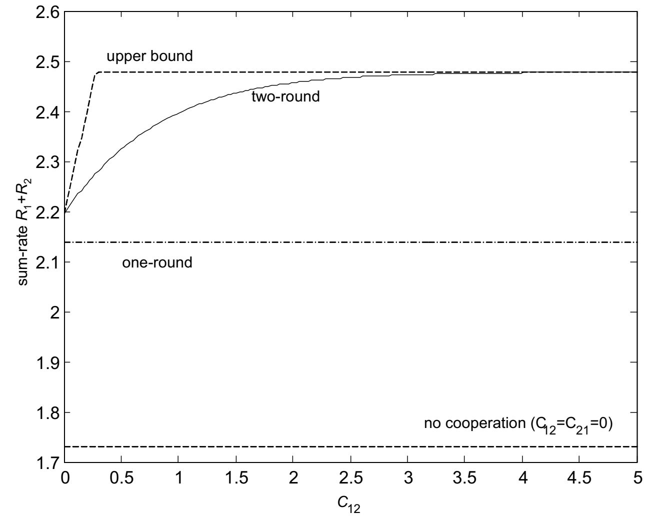

Fig. 4. Sum of the private rates $R_1 + R_2$ (with $R_0 = 0$) versus the conferencing link capacity C_{12} for the outer bound, the one-round and two-round strategies and with no cooperation ($P_1 = P_2 = 10dB$, $\gamma_{12}^2 = 0dB$, $\gamma_{22}^2 = 0dB$, $\gamma_{21}^2 = -3dB$, $\gamma_{11}^2 = -3dB$, $C_{21} = 0.8$).

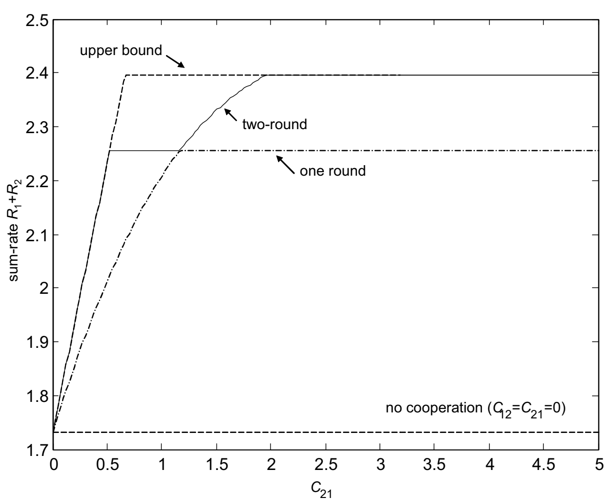

Fig. 3. Sum of the private rates $R_1 + R_2$ (with $R_0 = 0$) versus the conferencing link capacity C_{21} for the outer bound, the one-round and two-round strategies and with no cooperation ($P_1 = P_2 = 10dB$, $\gamma_{12}^2 = 0dB$, $\gamma_{22}^2 = 0dB$, $\gamma_{21}^2 = -3dB$, $\gamma_{11}^2 = -3dB$, $C_{12} = 0.2$).

provides relevant gains with respect to the one-step approach, while still falling short of the outer bound.

Fig. 3 and Fig. 4 show the sum of the private rates $R_1 + R_2$ (with $R_0 = 0$) versus C_{21} and C_{12}, respectively, for the outer bound, the one-round and two-round strategies and with no cooperation. In both cases, we consider a case where receiver 1 has a worse signal quality than receiver 2 (stochastically degraded): $P_1 = P_2 = 10dB$, $\gamma_{12}^2 = 0dB$, $\gamma_{22}^2 = 0dB$, $\gamma_{21}^2 = -3dB$, $\gamma_{11}^2 = -3dB$. Fig. 3 shows the achievable sum-rate versus C_{21} for $C_{12} = 0.2$. It is seen that if $C_{21} = 0$ the upper bound coincides with the rate achievable with no cooperation, showing that if the link between the "good" and the degraded receivers is disabled, the performance is dominated by the worst receiver and there is no gain in having $C_{12} > 0$. Increasing C_{21} enables the rate of the worst receiver to be increased via cooperation, thus harnessing significant gains with respect to the case of no cooperation. In particular, it is seen that for C_{21} sufficiently small (here $C_{21} \lesssim 0.5$) the two-step strategy is optimal, since in this region the performance is dominated by the worst receiver whose achievable rate increases linearly with C_{21} due to cooperation via binning of the message set performed at the good receiver. The one-step protocol instead lags behind and its performance saturates at $\mathcal{C}\left(\gamma_{22}^2 P_2 + \gamma_{21}^2 P_1 + \frac{\gamma_{11}^2 P_1 + \gamma_{21}^2 P_2}{1+\sigma_1^2}\right) \simeq 2.26$. Finally, for sufficiently large C_{21}, the achievable sum-rate at the worst receiver becomes larger than 2.26 and the performance tends to the sum-rate of the best receiver, $\mathcal{C}\left(\gamma_{22}^2 P_2 + \gamma_{12}^2 P_1\right) + C_{12} \simeq 2.4$, unless C_{12} is too large.

Further insight is shown in Fig. 4 where the sum-rate is plotted versus C_{12} for $C_{21} = 0.8$. We notice that for $C_{12} = 0$ only the two-step protocol is able to achieve the upper bound, since in such a regime it is optimal for the good receiver to decode and bin its reconstruction. Moreover, similarly, increasing C_{12} enhances the gain of the two-round strategy over the one-round strategy up to the point where the perfomance is limited by the sum-rate at the worse receiver, i.e., by $\mathcal{C}\left(P_1\left(\gamma_{11}^2 + \gamma_{12}^2\right) + P_2\left(\gamma_{21}^2 + \gamma_{22}^2\right) + \mathcal{K}\right) \simeq 2.48$, which coincides with the upper bound.

VII. Conclusions

We have investigated a compound MAC with conferencing decoders. The compound MAC can be seen as a combination of two single-message broadcast (multicast) channels from the standpoint of the transmitters, or two MACs as seen by the receivers, and it is an extension of the previously studied channel models. A number of capacity results have been derived in this paper that shed light on the performance of such systems. Among the results, we have shown that, one round of conferencing at the decoders in a compound Gaussian MAC achieves the entire capacity region within a fraction of one bit/s/Hz in several special cases. As a possible extension of this work here we mention the study of an interference channel, rather than a compound MAC, with conferencing decoders.

VIII. Appendix: Proof of Proposition 2

Converse: The converse follows immediately from Proposition 1 and the data processing theorem. In fact, it is easy to see that, because of the physical degradedness, receiver 1 cannot benefit from V_2^K, which is a function of Y_2^n and Y_1^n via V_1^k. We refer to [17] for a full proof.

Achievability: *Codeword generation* at the transmitters is performed as for the MAC with common information [2] [13]:

Generate 2^{nR_0} sequences $u^n(w_0)$ of length n, with elements independent identically distributed (i.i.d.) according to the distribution $p(u)$, $w_0 \in \mathcal{W}_0$. For any sequence $u^n(w_0)$, generate 2^{nR_i} independent sequences $x_i^n(w_0, w_i)$, $w_i \in \mathcal{W}_i$, i.i.d. according to $p(x_i|u_i(w_0))$, for $i = 1, 2$.

At receiver 1, the message sets $\mathcal{W}_0, \mathcal{W}_1, \mathcal{W}_2$ are partitioned into $2^{n\alpha_0 C_{12}}$, $2^{n\alpha_1 C_{12}}$ and $2^{n\alpha_2 C_{12}}$ subsets, respectively, for some given $0 \leq \alpha_i \leq 1$ and $\sum_{i=0}^{2} \alpha_i = 1$. This is done by assigning each codeword in the message set $\mathcal{W}_i$ independently and randomly to the index set $\{1, 2, ..., 2^{n\alpha_i C_{12}}\}$.

Encoding at transmitter i is performed by sending codeword $x_i^n(w_0, w_i)$ corresponding to the common message $w_0 \in \mathcal{W}_0$ and the local message $w_i \in \mathcal{W}_i$ $(i = 1, 2)$. Encoding at decoder 1 takes place after decoding messages W_0, W_1 and W_2 (see description of decoding below). In particular, decoder 1 sends over the conferencing link the indices of the subsets where the estimated messages W_0, W_1 and W_2 lie. Notice that this requires nC_{12} bits and $K = 1$ conferencing rounds (i.e., $|\mathcal{V}_{1,1}| = nC_{12}$). We emphasize again that the conferencing link from decoder 2 to decoder 1 is not used ($|\mathcal{V}_{2,k}| = 0$).

Decoding at the first decoder is carried out by finding jointly typical sequences $(u^n(w_0), x_1^n(w_0, w_1), x_2^n(w_0, w_2), y_1^n)$ with $w_i \in \mathcal{W}_i$ [12]. As discussed above, once the first decoder has obtained the messages W_0, W_1 and W_2, it sends the corresponding subset indices to receiver 2 over the conferencing link. Decoding at receiver 2 then takes place again based on a standard MAC joint-typicality decoder with the caveat that the messages W_0, W_1 and W_2 are now known to belong to the reduced set given by the subsets mentioned above.

The *analysis of the probability of error* follows immediately from [2] [13]. In particular, as far as receiver 1 is concerned, it can be seen from [2] [13] that a sufficient condition for the probability of error go to zero as $n \to \infty$ is given by $(R_0, R_1, R_2) \in \mathcal{R}_{MAC,1}(p(u), p(x_1|u), p(x_2|u))$. Considering receiver 2, a sufficient condition for decaying error probability is that the rates belong to the following region:

$$
\begin{aligned}
\{(R_0, R_1, R_2)\colon R_j \geq 0,\ j = 0, 1, 2, \qquad\qquad & \qquad (19)\\
R_1 \leq I(X_1; Y_2|X_2, U) + \alpha_1 C_{12} &\\
R_2 \leq I(X_2; Y_2|X_1, U) + \alpha_2 C_{12} &\\
R_1 + R_2 \leq I(X_1, X_2; Y_2|U) + (\alpha_1 + \alpha_2) C_{12} &\\
R_0 + R_1 + R_2 \leq I(X_1, X_2; Y_2) + C_{12}\}, &
\end{aligned}
$$

for the given α_i. Taking the union over all allowed α_i concludes the proof.

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