Power and Bandwidth Allocation in Cooperative Dirty Paper Coding

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***Abstract*— The cooperative dirty paper coding (DPC) rate region is investigated in a two-transmitter two-receiver network with full channel state information available at all terminals. The transmitters cooperate by first exchanging messages over an orthogonal cooperation channel, then they mimic a broadcast channel (BC) and jointly perform DPC to send to the two independent receivers. The allocation of network power and bandwidth between the data and the cooperation channel is studied to characterize the cooperative DPC rate region. First, the optimal sum power allocation for a multiple access channel (MAC) is presented. Then through an application of MAC-BC capacity duality, the cooperative DPC rate region is evaluated under different bandwidth allocation assumptions. Cooperative DPC outperforms non-cooperative time-division (TD) only when the cooperation channel is strong, since the joint-encoding capacity gain is negated by the overhead of message exchanges in a weak cooperation channel. Moreover, the cooperative capacity advantage over TD is more pronounced at sum rate than when the rate vector is skewed toward one of the users.**

I. Introduction

In a wireless *ad hoc* network, neighboring nodes may cooperate by way of joint encoding or processing to increase system performance. However, such cooperation may entail overheads such as the power and bandwidth needed in the exchange of cooperation messages. Resource allocation is therefore crucial in cooperative networks in characterizing the trade-off between the costs of cooperation and its potential performance benefits. In this paper, we study the power and bandwidth allocation when a pair of cooperative transmitters first exchange messages over an orthogonal cooperation channel, then jointly perform dirty paper coding (DPC) to send to two independent receivers.

The benefits of cooperation have been studied under different performance metrics. Achievable rate regions of a channel with two cooperative transmitters and a single receiver are presented in [1]–[3]. Cooperative diversity is studied in [4], where the transmitters forward parity bits of the detected symbols to one another. Under fading channels, [5] shows that orthogonal cooperative protocols can achieve full spatial diversity order. Upper bounds and achievable multiplexing gains of cooperative networks are presented in [6]. Information-theoretic achievable rate regions and bounds are given in [7]–[11] for channels with transmitter and/or receiver cooperation.

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This paper extends the system model in [12], [13], which studies the cooperative rates of a two-transmitter two-receiver *ad hoc* network with orthogonal cooperation channels. In [12], [13], the sum rates are characterized for the DPC transmitter cooperation scheme and the Wyner–Ziv compress-and-forward receiver cooperation scheme in a network that has a symmetric topology. It is shown that the DPC scheme offers most of the capacity gain when the cooperating nodes are close together. In this paper, we focus on the DPC transmitter cooperation scheme, and consider the power and bandwidth allocation between the data channel and cooperation channel to characterize, besides the sum rate, the entire cooperative DPC rate region. The power and bandwidth allocation procedure considered in this paper, moreover, is applicable for any arbitrary network topology.

The remainder of the paper is organized as follows. Section II presents the system model and the cooperative DPC transmission scheme. Section III derives the optimal sum power allocation in a multiple access channel (MAC), the results of which are applied in Section IV when cooperative DPC power and bandwidth allocation is considered. Numerical results of the cooperative DPC rate regions under Rayleigh fading are presented in Section V. Section VI concludes the paper.

II. System Model

A. Channel Model

Consider an *ad hoc* network with two clustered transmitters and two independent receivers as shown in Fig. 1. We assume the transmitters within the cluster are close together, but the distance between the transmitter cluster and receivers is large. The channel gains are denoted by $h_1, \dots, h_4 \in \mathbb{C}$. We assume a slow fading environment where the nodes can track the channel conditions accurately. In particular, we assume all nodes have perfect channel state information (CSI), and the transmitters are able to adapt to the channel realizations $h_1, \dots, h_4$. In a fast fading environment where accurate channel tracking is difficult, the perfect CSI case provides an upper bound to the system performance.

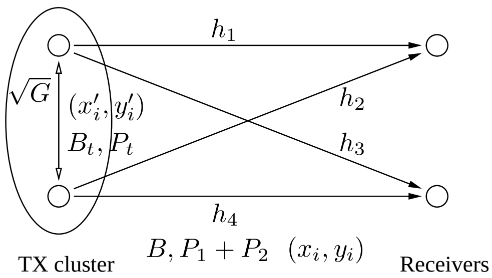

Fig. 1. System model of the cooperative transmitter cluster.

There are two orthogonal communication channels: the data channel between the transmitter cluster and the receivers, and the cooperation channel between the transmitters. In the data channel, Transmitter 1 wishes to send to Receiver 1 at rate R_1, and likewise Transmitter 2 to Receiver 2 at rate R_2. Let $\mathbf{x} \triangleq [x_1\ x_2]^T \in \mathbb{C}^2$ denote the transmit signals, and $\mathbf{y} \triangleq [y_1\ y_2]^T \in \mathbb{C}^2$ denote the corresponding received signals. In matrix form, the data channel can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \qquad \mathbf{H} \triangleq \begin{bmatrix} h_1 & h_2 \\ h_3 & h_4 \end{bmatrix}, \tag{1}$$

where $n_1, n_2 \sim \mathcal{CN}(0,1)$ are iid zero-mean circularly symmetric complex Gaussian (ZMCSCG) white noise with unit variance. Let B denote the bandwidth of the data channel, and $P_1 \triangleq \mathrm{E}[|x_1|^2]$, $P_2 \triangleq \mathrm{E}[|x_2|^2]$ denote the transmission power of Transmitter 1, Transmitter 2, respectively; the expectation is taken over repeated channel uses.

There is also a static, additive white Gaussian noise (AWGN) cooperation channel between the two transmitters with channel gain $\sqrt{G}$. As we assume the cooperating nodes are close together, the case of interest is when G is large. We assume the two transmitters can simultaneously transmit and receive on this full-duplex cooperation channel. Let $x_1', x_2' \in \mathbb{C}$ be the transmit signals, and $y_1', y_2' \in \mathbb{C}$ the received signals, then the cooperation channel is described by

$$y_1' = \sqrt{G}x_2' + n_3, \qquad y_2' = \sqrt{G}x_1' + n_4, \tag{2}$$

where $n_3, n_4 \sim \mathcal{CN}(0,1)$ are iid unit-variance ZMCSCG noise. Let B_t denote the transmitter cooperation channel bandwidth, and $P_t \triangleq \mathrm{E}\big[|x_1'|^2 + |x_2'|^2\big]$ denote the cooperation transmission power.

To capture the system-wide cost of cooperation, we consider a total network power constraint P on the data and cooperation transmissions:

$$P_1 + P_2 + P_t \le P. \tag{3}$$

We assume a short-term power constraint which is observed under each instance of the channel realizations; power allocation across fading states is not considered in this paper. Moreover, we consider two scenarios on the allocation of bandwidth between the data channel and the cooperation channel: Under bandwidth assumption 1), we assume dedicated orthogonal channels for cooperation, and each channel has a bandwidth of 1 Hz (i.e., $B = B_t = 1$). Under bandwidth assumption 2), however, there is a single 1 Hz channel to be divided into two different bands to implement the cooperative schemes. We thus allocate B and B_t such that $B_t + B = 1$. Bandwidth assumption 1) is applicable when the short-range cooperation communications take place in separate bands which may be spatially reused across all cooperating nodes in the system. Whereas bandwidth assumption 2) is applicable when spatial reuse is not considered.

B. Cooperative Dirty Paper Coding

In the cooperative dirty paper coding transmission scheme, the transmitters first fully exchange their intended messages over the orthogonal cooperation channel, after which the network becomes equivalent to a multi-antenna broadcast channel (BC) with a two-antenna transmitter:

$$y_1 = \mathbf{f}_1\mathbf{x} + n_1, \qquad y_2 = \mathbf{f}_2\mathbf{x} + n_2, \tag{4}$$

where $\mathbf{f}_1, \mathbf{f}_2$ are the rows of $\mathbf{H}$:

$$\mathbf{f}_1 \triangleq \big[h_1\ h_2\big], \qquad \mathbf{f}_2 \triangleq \big[h_3\ h_4\big]. \tag{5}$$

The transmitters then *jointly* encode both messages using dirty paper coding (DPC) [14], which is capacity-achieving for the multi-antenna Gaussian BC [15]. Causality is not violated since we can offset the transmitter cooperation and DPC communication by one block. We assume the transmitters are synchronized, which may be achieved through an external clock or exchanging timing reference signals.

In this paper, we investigate the power and bandwidth allocation between the data and the cooperation channel, and characterize the rate region of the cooperative DPC transmission scheme. We characterize the convex hull of the cooperative DPC rates in terms of the maximization:

$$d^*(\mu) \triangleq \max\ \mu R_1 + (1-\mu)R_2, \tag{6}$$

where the maximization is over the set of rates (R_1, R_2) achievable by the cooperative DPC scheme. As μ ranges from 0 to 1, the cooperative DPC rate region is given by the intersection of the halfspaces:

$$\mathcal{C}_{\text{coop-DPC}} = \bigcap_{0\le\mu\le1} \big\{(R_1,R_2) \mid \mu R_1 + (1-\mu)R_2 \le d^*(\mu)\big\}. \tag{7}$$

III. Optimal MAC Sum Power Allocation

In the cooperative DPC scheme, after the transmitters exchange messages, the cooperative network becomes a BC with two independent receivers. As the capacity region of the BC equals that of its dual MAC under a sum power constraint [16], [17], in this section we derive the optimal allocation of sum power between the users in a MAC, and hence via duality also the BC capacity region. In the following sections we focus on the instantaneous rates for a given fading state (i.e., conditioned on $\mathbf{H}$). Numerical results for Rayleigh fading channels are presented in Section V, where the ergodic rate regions are computed by averaging over the fading states.

Consider a MAC with two single-antenna transmitters and a two-antenna receiver. Let the channel be described by

$$\mathbf{y} = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \mathbf{n}, \tag{8}$$

where $\mathbf{y} \in \mathbb{C}^2$ is the received signal vector; $x_1, x_2 \in \mathbb{C}$ are the signals sent by Tx 1 and Tx 2, respectively; $\mathbf{h}_1 \triangleq [h_{11}\ h_{12}]^T \in \mathbb{C}^2$, $\mathbf{h}_2 \triangleq [h_{21}\ h_{22}]^T \in \mathbb{C}^2$ are the channel vectors; and $\mathbf{n} \in \mathbb{C}^2$ is unit-variance ZMCSCG white noise with $\mathrm{E}[\mathbf{n}\mathbf{n}^H] = \mathbf{I}$. We assume perfect CSI, and thus $\mathbf{h}_1, \mathbf{h}_2$ are known to the transmitters and the receiver. Suppose Tx 1 has power P_1, and Tx 2 has power P_2. Since the inputs x_1, x_2 are scalars (each transmitter has only one antenna), the MAC capacity region is given by the pentagon [18]:

$$R_1 \le \log|\mathbf{I} + \mathbf{h}_1 P_1 \mathbf{h}_1^H| \tag{9}$$

$$R_2 \le \log|\mathbf{I} + \mathbf{h}_2 P_2 \mathbf{h}_2^H| \tag{10}$$

$$R_1 + R_2 \le \log|\mathbf{I} + \mathbf{h}_1 P_1 \mathbf{h}_1^H + \mathbf{h}_2 P_2 \mathbf{h}_2^H|, \tag{11}$$

where R_1 is the transmission rate of Tx 1, and R_2 is that of Tx 2.

Suppose we write $P_1 = \alpha P_s$, $P_2 = (1-\alpha)P_s$, where P_s can be interpreted as the sum power constraint, and the power allocation parameter $\alpha \in [0, 1]$ is to be optimized. To numerically calculate the optimal sum power allocation at the sum capacity, an iterative algorithm was given in [19], which was based on the iterative waterfilling algorithm for MIMO MACs with individual power constraints [20]. However, we wish to find the optimal sum power allocation for all points on the capacity region boundary (i.e., including non-sum-capacity rate vectors). Hence we solve the optimal sum power allocation analytically using the Lagrange method, and its derivation is given in Appendix I.

In the next section we will again refer to the optimal MAC sum power allocation when we consider the power and bandwidth allocation in the cooperative DPC scheme. As noted in Appendix I, the BC capacity region boundary comprises three segments: the rate vectors achieved by decode order (1) (the receiver decodes in the order: Tx 2, Tx 1), the ones by decode order (2) (the receiver decodes in the order: Tx 1, Tx 2), and the ones on the straight line segment achieved via time-sharing between the two decode orders. When we examine the power and bandwidth allocation in cooperative DPC in Section IV, all three segments of the BC region boundary need to be considered.

IV. Cooperative DPC Power and Bandwidth Allocation

In cooperative DPC, the transmitters can perform joint encoding only when they know the codewords of each other. The target transmission rate vector (R_1, R_2), therefore, must be supported by *both* the cooperation channel as well as the resulting BC after the transmitters have exchanged messages. Since capacity is non-decreasing in transmit power, there is no surplus in power under optimal allocation: we suppose the transmitters use power P_t to exchange their messages through the cooperation channel, and then perform DPC over the data channel with the remaining power $P_1 + P_2 = P - P_t$. The cooperative DPC rate region can then be characterized by the intersection of the cooperation channel and the BC capacity regions:

$$\max_{P_t, B_t, B} \mu R_1 + (1-\mu) R_2 \tag{12}$$

$$\text{such that: } (R_1, R_2) \in \mathcal{C}_{\mathrm{BC}}(P - P_t, B) \tag{13}$$

$$(R_1, R_2) \in \mathcal{C}_{\mathrm{co}}(P_t, B_t), \tag{14}$$

where $\mathcal{C}_{\mathrm{BC}}(P - P_t, B)$ is the BC capacity region with power $P - P_t$ and bandwidth B, and similarly $\mathcal{C}_{\mathrm{co}}(P_t, B_t)$ is the transmitter cooperation channel capacity region with power P_t and bandwidth B_t. By duality, the BC capacity region equals that of the dual MAC under the same sum power constraint, i.e., $\mathcal{C}_{\mathrm{BC}}(P - P_t, B) = \mathcal{C}_{\mathrm{MAC}}(P - P_t, B)$. The cooperation channel is full-duplex with channel gain $\sqrt{G}$; hence $(R_1, R_2) \in \mathcal{C}_{\mathrm{co}}(P_t, B_t)$ iff

$$(2^{R_1/B_t} - 1)B_t/G + (2^{R_2/B_t} - 1)B_t/G \le P_t, \tag{15}$$

which follows from the capacity of an AWGN channel. Under bandwidth assumption 1), we set $B_t = B = 1$, and P_t is the only optimization variable. Under bandwidth assumption 2), we write $B = 1 - B_t$, so the optimization is over P_t, B_t.

We note that for some given P_t, B_t, the cooperative DPC achievable rate prescribed by that power/bandwidth allocation can be computed. For fixed P_t, B_t, the regions $\mathcal{C}_{\mathrm{BC}}$ and $\mathcal{C}_{\mathrm{co}}$ are convex; hence so is their intersection. If only the BC region constraint (13) is active, then the problem reduces to a sum power allocation optimization for the dual MAC, which is considered in Section III, and the solution is given in Appendix I. In this case, the maximizing BC rate vector $(R_{1,\mathrm{BC}}^*, R_{2,\mathrm{BC}}^*)$ lies on the boundary of $\mathcal{C}_{\mathrm{BC}}$ but in the interior of $\mathcal{C}_{\mathrm{co}}$. Of the P_t, B_t when this is true, we denote them as region (i) in the power/bandwidth allocation space. To test if $(R_{1,\mathrm{BC}}^*, R_{2,\mathrm{BC}}^*)$ lies in the interior of $\mathcal{C}_{\mathrm{co}}$, we stipulate its power requirement be strictly feasible for the cooperation channel:

$$(2^{R_{1,\mathrm{BC}}^*/B_t} - 1)B_t/G + (2^{R_{2,\mathrm{BC}}^*/B_t} - 1)B_t/G < P_t. \tag{16}$$

Additional consideration is needed, however, when we maximize the sum rate of the BC region. When $\mu = 0.5$ in (12), all rate vectors on the time-sharing segment of the BC region boundary are optimal and have the same sum rate. Accordingly, we select the rate vector that requires the minimum cooperation channel power when we apply (16) to determine its feasibility. The minimum cooperation channel power is derived in Appendix II-B.

Conversely, if only the cooperation channel region constraint (14) is active, the maximization becomes a sum power allocation optimization problem in the cooperation channel. This can be solved with similar steps as in MAC optimal sum power allocation, and the solution is given in Appendix II-A. In this case, the maximizing cooperation channel rate vector $(R_{1,\mathrm{co}}^*, R_{2,\mathrm{co}}^*)$ lies on the boundary of $\mathcal{C}_{\mathrm{co}}$ but in the interior of $\mathcal{C}_{\mathrm{BC}}$. We denote the corresponding P_t, B_t as region (ii). The condition for region (ii) can be verified by checking if

$(R_{1,\text{co}}^*, R_{2,\text{co}}^*)$ is strictly feasible in the dual MAC with decode order (1), (2) or time-sharing.

Lastly, if both constraints (13) and (14) are active, the maximizing rate vector lies on the intersection of the boundaries of $\mathcal{C}_{\text{BC}}$ and $\mathcal{C}_{\text{co}}$. We denote the corresponding P_t, B_t as region (iii). In this case, the rate vector is computed by equating the two capacity region boundaries. Specifically, under bandwidth assumption 1), as $B_t = B$, the intersecting rate vectors are found by solving a set of quadratic equations when the rates are achievable by MAC decode order (1) or (2). If an intersecting rate vector lies on the time-sharing segment of the BC region boundary, then the solution is numerically computed as it involves solving equations with non-integer powers. Under bandwidth assumption 2), all the intersecting rate vectors have to be numerically computed as they again involve solving equations with non-integer powers. Of all intersecting rate vectors, the one that maximizes $\mu R_1 + (1-\mu) R_2$ produces the cooperative DPC achievable rate.

In regions (i) and (ii), the weighted sum of rates $\mu R_1 + (1-\mu) R_2$ is concave in P_t, B_t. In fact, as channel capacity increases with available transmit power and bandwidth, the rate in region (i) is monotonically increasing in $P - P_t, 1 - B_t$, and the rate in region (ii) is monotonically increasing in P_t, B_t. Consequently, if we apply one of the standard one-dimensional or multi-dimensional numerical optimization algorithms (e.g., see [21]), we arrive at a maximum in region (iii), and the suboptimal rates in regions (i) and (ii) are rejected due to their monotonicity. Numerically we have observed that region (iii) appears to be concave in P_t, B_t; if that is the case, it would imply a local maximum in the region is indeed a global maximum. However, as the rates in this region are numerically computed, its concavity cannot be readily verified.

V. Numerical Results

In this section, we illustrate the rate regions of the cooperative DPC transmission scheme. We consider the network in Fig. 1, and assume that the channels $h_1, \ldots, h_4$ are under independent Rayleigh fading with unit power. The numerical results are generated via Monte Carlo simulation with 1000 instances of channel realizations. For each channel realization, we evaluate the power and bandwidth allocation in cooperative DPC as described in Section IV. We then compute the ergodic rate regions by averaging over the channel realizations.

The cooperative DPC rate regions are plotted in Fig. 2 and Fig. 3, for $G = 0$ dB and $G = 10$ dB, respectively. We assume a network power constraint of $P = 10$ dB. For comparison, in the plots we show also the BC ergodic capacity region that corresponds to the case when the two transmitters are colocated ($G = \infty$). In addition, we compare the cooperative DPC rates against a non-cooperative transmission scheme where each transmitter sends to its respective receiver by time-division (TD). Note that under each channel realization, the TD capacity region boundary is given by a straight line segment. The TD ergodic capacity regions shown in the plots are averaged over the channel realizations.

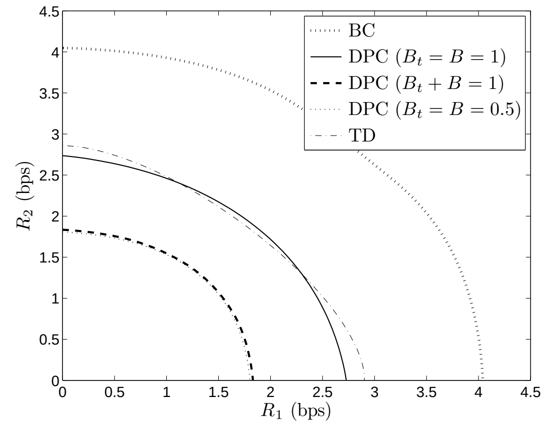

Fig. 2. Cooperative DPC rate regions ($G = 0$ dB).

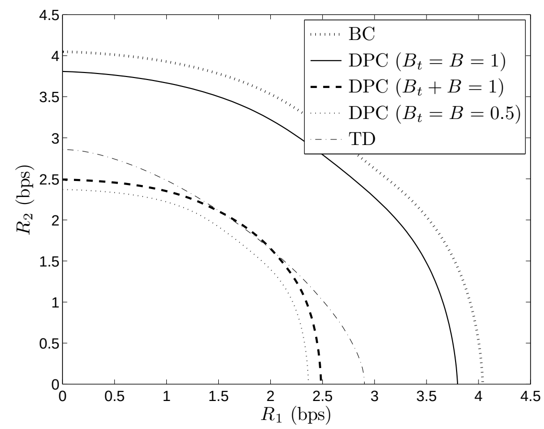

Fig. 3. Cooperative DPC rate regions ($G = 10$ dB).

We observe that when the cooperation channel is weak (i.e., when G is small), the transmitters need to spend much power and bandwidth exchanging messages: the cooperative schemes fail to surpass the non-cooperative TD capacity, especially when bandwidth needs to be allocated between the data and the cooperation channel. When G is large, however, cooperative DPC begins to outperform the non-cooperative TD transmission scheme. For a given G, the cooperative capacity gain over TD is more pronounced at sum rate (i.e., $\mu = 0.5$) than when the rate vector is skewed toward one of the users (i.e., $\mu \approx 0$ or $\mu \approx 1$). We also show the DPC regions under equal bandwidth allocation ($B_t = B = 0.5$) for comparison. Equal bandwidth allocation is close to optimal bandwidth allocation ($B_t + B = 1$) at $G = 0$ dB, but the performance gap widens as G increases.

VI. Conclusions

In this paper we have studied the power and bandwidth allocation in an *ad hoc* network with a pair of cooperating transmitters. The transmitters cooperate by first exchanging their messages over an orthogonal cooperation channel, then they perform DPC to transmit the messages jointly to the independent receivers. We derive the optimal sum power allocation for a MAC, which we then use in an application of MAC-BC capacity duality to compute the cooperative DPC rate regions under different bandwidth allocation assumptions. When the cooperation channel is weak, cooperative DPC performs worse than non-cooperative TD, since the overhead of exchanging messages between the transmitters outweighs the capacity gain from joint-encoding. When the cooperation channel is strong, on the other hand, cooperative DPC outperforms non-cooperative TD transmission. We have considered the case where the transmitters cooperate but the receivers decode independently in this paper. A future work is to extend the analysis to investigate the rate region under receiver cooperation, as well as that under transmitter-and-receiver cooperation.

APPENDIX I
MAC OPTIMAL SUM POWER ALLOCATION

Consider the multiple-antenna MAC described in (8): each of the two transmitters has a single antenna, the receiver has two antennas, and the channel vectors are given by $\mathbf{h}_1, \mathbf{h}_2$. We write Tx 1's power constraint as $P_1 = \alpha P_s$, and Tx 2's power constraint as $P_2 = (1-\alpha)P_s$, where P_s is the sum power constraint. We consider the optimization of the power allocation parameter $\alpha \in [0,1]$ in this section.

Let B be the bandwidth of the MAC, then in terms of the sum power the MAC capacity region (9), (10), and (11) can be written as

$$R_1 \le B \log\big|\mathbf{I} + \mathbf{h}_1(\alpha P_s/B)\mathbf{h}_1^H\big| \tag{17}$$

$$R_2 \le B \log\big|\mathbf{I} + \mathbf{h}_2((1-\alpha) P_s/B)\mathbf{h}_2^H\big| \tag{18}$$

$$R_1 + R_2 \le B \log\big|\mathbf{I} + \mathbf{h}_1(\alpha P_s/B)\mathbf{h}_1^H + \mathbf{h}_2((1-\alpha) P_s/B)\mathbf{h}_2^H\big|. \tag{19}$$

For notational convenience, we define

$$K_1 \triangleq \|\mathbf{h}_1\|_F^2 P_s/B, \quad K_2 \triangleq \|\mathbf{h}_2\|_F^2 P_s/B, \quad K_0 \triangleq h_0 P_s^2/B^2,$$

where $\|\cdot\|_F$ denotes the Frobenius norm, and

$$h_0 \triangleq |h_{11}|^2|h_{22}|^2 + |h_{21}|^2|h_{12}|^2 - 2\Re\{h_{11}h_{12}^* h_{21}^* h_{22}\}.$$

Now the capacity region can be written more compactly as

$$R_1 \le B \log\big(1 + K_1\alpha\big) \tag{20}$$

$$R_2 \le B \log\big(1 + K_2(1-\alpha)\big) \tag{21}$$

$$R_1 + R_2 \le B \log\big(1 + K_1\alpha + K_2(1-\alpha) + K_0\alpha(1-\alpha)\big). \tag{22}$$

The capacity region of a MAC under a sum power constraint is closed and convex [16]; under each given fading state (i.e., conditioned on $\mathbf{h}_1, \mathbf{h}_2$), it can be characterized by the following convex optimization:

$$\max_{(R_1,R_2)\in\mathcal{C}_{\text{MAC}}} \mu R_1 + (1-\mu)R_2, \tag{23}$$

where $\mu \in [0,1]$ is given, and $\mathcal{C}_{\text{MAC}}$ is the MAC capacity region described in (20), (21) and (22). As μ ranges from 0 to 1, (23) traces the boundary of the capacity region.

When $\mu > 0.5$ (i.e., R_1 is weighted more favorably than R_2), it is optimal to decode Tx 1's signal last, after decoding and canceling Tx 2's signal [18]. We call this decode order (1), and the following rates are achieved:

$$R_1^{(1)} = B \log\big(1 + K_1\alpha\big) \tag{24}$$

$$R_2^{(1)} = B \log\Big(1 + \frac{(1-\alpha)(K_2 + K_0\alpha)}{1 + K_1\alpha}\Big). \tag{25}$$

Under decode order (1), the maximization in (23) becomes

$$\max_{0\le\alpha\le1} \mu R_1^{(1)} + (1-\mu)R_2^{(1)} \tag{26}$$

$$\Rightarrow \max_{0\le\alpha\le1} (2\mu-1)B\log\big(1+K_1\alpha\big) + (1-\mu)B\log\big(1 + K_1\alpha + K_2(1-\alpha) + K_0\alpha(1-\alpha)\big). \tag{27}$$

Note that $K_1, K_2, K_0 \ge 0$ from their definitions, so (27) is concave in α. Next we form the Lagrangian:

$$L(\alpha,\lambda_1,\lambda_2) = -(2\mu-1)B\log\big(K_1\alpha+1\big) + \lambda_1(\alpha-1) - \lambda_2\alpha - (1-\mu)B\log\big(-K_0\alpha^2 + (K_1 - K_2 + K_0)\alpha + K_2 + 1\big). \tag{28}$$

Applying the Karush-Kuhn-Tucker (KKT) conditions, the gradient of the Lagrangian vanishes at the optimal sum power allocation α^*. It can be found by solving a quadratic equation with the coefficients:

$$a \triangleq K_1K_0 \tag{29}$$

$$b \triangleq (-K_1^2 + K_1K_2 - K_1K_0 - 2K_0)\mu + 2K_0 \tag{30}$$

$$c \triangleq (-2K_1K_2 - K_1 - K_2 + K_0)\mu + K_1K_2 + K_2 - K_0. \tag{31}$$

The KKT conditions state that at the optimal α^*, either one of the inequality constraints is active, or the gradient of the objective function is zero. When $a \ne 0$, the solution can be summarized as

$$\alpha^* = \begin{cases} 0 & \text{if } b^2 - 4ac < 0 \text{ or } r_1 > 1 \text{ or } r_2 < 0 \\ r_2 & \text{else if } 0 \le r_2 \le 1 \\ r_1 & \text{else if } 0 \le r_1 \le 1 \\ 1 & \text{else,} \end{cases} \tag{32}$$

where r_1, r_2 are the roots of the quadratic:

$$r_1 = (-b - \sqrt{b^2 - 4ac})/(2a) \tag{33}$$

$$r_2 = (-b + \sqrt{b^2 - 4ac})/(2a). \tag{34}$$

In the case of $a = 0$, the optimal power allocation is given by

$$\alpha^* = \begin{cases} -c/b & \text{if } b \ne 0 \text{ and } 0 \le -c/b \le 1 \\ 0 & \text{else if } c > 0 \\ 1 & \text{else.} \end{cases} \tag{35}$$

On the other hand, when $\mu < 0.5$ (i.e., R_2 is weighted more favorably than R_1), the reverse decoder order is optimal: we decode Tx 2's signal last, after decoding and canceling Tx 1's signal. We call this decode order (2), and similarly steps can be used to derive the optimal sum power allocation α^* under decode order (2). When $\mu = 0.5$, decode orders (1) and (2) result in the same power allocation α^* and the same sum rate $R_1 + R_2$. The linear combination of the rate vectors $(R_1^{(1)}, R_2^{(1)})$ and $(R_1^{(2)}, R_2^{(2)})$ can be achieved via time-sharing between the two decode orders.

Appendix II
Cooperation Channel Power Allocation

We consider power allocation in the full-duplex AWGN transmitter cooperation channel described in (2): the channel gain in each direction is $\sqrt{G}$, and we let P_t, B_t, respectively, be the transmit power constraint and the bandwidth of the channel.

A. Optimal Sum Power Allocation

Suppose Transmitter 1 uses power τP_t, and Transmitter 2 uses power $(1-\tau)P_t$, where $\tau \in [0,1]$. In this section, we consider the optimal cooperation channel sum power allocation. The capacity region of the cooperation channel can be characterized by the convex optimization:

$$\max_{0 \le \tau \le 1} \; \mu R_1 + (1-\mu) R_2, \tag{36}$$

where $\mu \in [0,1]$ is given, R_1 is the rate Transmitter 1 sends to Transmitter 2, and R_2 is the rate in the other direction. Being an AWGN channel in each direction, the channel capacities are given by

$$R_1 = B_t \log(1 + \tau G P_t / B_t) \tag{37}$$

$$R_2 = B_t \log(1 + (1-\tau) G P_t / B_t). \tag{38}$$

The maximization is concave in τ, and steps similar to those in Appendix I can be used to derive the solution. The optimal cooperation channel sum power allocation is found to be:

$$\tau^* = \min\Big\{\max\Big\{0, \frac{2\mu - 1 + \mu G P_t/B_t}{G P_t / B_t}\Big\}, 1\Big\}. \tag{39}$$

B. Minimum Power Requirement

On the BC region boundary time-sharing segment, all rate vectors have the same sum rate $R_1 + R_2$; however, they do not have the same power requirement in the cooperation channel. In this section, we find the rate vector on the BC time-sharing segment that requires the minimum cooperation channel power. We make use of this result in Section IV when we test if a rate vector on the BC region boundary lies inside of the cooperation channel capacity region.

Let us consider the time-sharing rates between two given rate vectors $(R_1^{(1)}, R_2^{(1)})$ and $(R_1^{(2)}, R_2^{(2)})$. Suppose time-sharing achieves the rate vector

$$R_1 = t R_1^{(1)} + (1-t) R_1^{(2)} \tag{40}$$

$$R_2 = t R_2^{(1)} + (1-t) R_2^{(2)}, \tag{41}$$

for some $t \in [0,1]$. To support rate vector (R_1, R_2), the power requirement on the cooperation channel is given by

$$P_{\text{co}}(t) = (2^{R_1/B_t} - 1) B_t / G + (2^{R_2/B_t} - 1) B_t / G. \tag{42}$$

Assuming $(R_1^{(1)}, R_2^{(1)}) \neq (R_1^{(2)}, R_2^{(2)})$, with the Lagrange method we can find the optimal time-sharing variable t^* that minimizes the cooperation channel requirement:

$$t^* = \min\{\max\{0, \tilde{t}\}, 1\}, \tag{43}$$

where

$$\tilde{t} = \frac{R_2^{(2)} - R_1^{(2)} + \log\big((R_2^{(2)} - R_2^{(1)})/(R_1^{(1)} - R_1^{(2)})\big)}{R_1^{(1)} - R_1^{(2)} + R_2^{(2)} - R_2^{(1)}}. \tag{44}$$

References

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[2] ——, "User cooperation diversity—Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, Nov. 2003.

[3] K. A. Yazdi, H. El Gamal, and P. Schniter, "On the design of cooperative transmission schemes," in *Proc. Allerton Conf. Commun., Contr., Comput.*, Monticello, IL, Oct. 2003, pp. 1576–1585.

[4] T. E. Hunter and A. Nosratinia, "Cooperation diversity through coding," in *Proc. IEEE Int. Symp. Inform. Theory*, Lausanne, Switzerland, June 2002, p. 220.

[5] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[6] A. Host-Madsen and A. Nosratinia, "The multiplexing gain of wireless networks," in *Proc. IEEE Int. Symp. Inform. Theory*, Adelaide, Australia, Sept. 2005, pp. 2065–2069.

[7] A. Host-Madsen, "Capacity bounds for cooperative diversity," *IEEE Trans. Inform. Theory*, vol. 52, no. 4, pp. 1522–1544, Apr. 2006.

[8] M. A. Khojastepour, A. Sabharwal, and B. Aazhang, "Improved achievable rates for user cooperation and relay channels," in *Proc. IEEE Int. Symp. Inform. Theory*, Chicago, IL, June 2004, p. 4.

[9] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channel," *IEEE Trans. Inform. Theory*, vol. 51, no. 6, pp. 2020–2040, June 2005.

[10] A. Host-Madsen, "On the achievable rate for receiver cooperation in ad-hoc networks," in *Proc. IEEE Int. Symp. Inform. Theory*, Chicago, IL, June 2004, p. 272.

[11] ——, "A new achievable rate for cooperative diversity based on generalized writing on dirty paper," in *Proc. IEEE Int. Symp. Inform. Theory*, Yokohama, Japan, June 2003, p. 317.

[12] N. Jindal, U. Mitra, and A. Goldsmith, "Capacity of ad-hoc networks with node cooperation," in *Proc. IEEE Int. Symp. Inform. Theory*, Chicago, IL, June 2004, p. 271.

[13] C. T. K. Ng, N. Jindal, A. J. Goldsmith, and U. Mitra, "Capacity gain from two-transmitter and two-receiver cooperation," *IEEE Trans. Inform. Theory*, accepted, 2007.

[14] M. H. M. Costa, "Writing on dirty paper," *IEEE Trans. Inform. Theory*, vol. 29, no. 3, pp. 439–441, May 1983.

[15] H. Weingarten, Y. Steinberg, and S. Shamai (Shitz), "The capacity region of the Gaussian MIMO broadcast channel," in *Proc. IEEE Int. Symp. Inform. Theory*, Chicago, IL, June 2004, p. 174.

[16] N. Jindal, S. Vishwanath, and A. Goldsmith, "On the duality of Gaussian multiple-access and broadcast channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 5, pp. 768–783, May 2004.

[17] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2658–2668, Oct. 2003.

[18] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley-Interscience, 1991.

[19] N. Jindal, W. Rhee, S. Vishwanath, S. A. Jafar, and A. Goldsmith, "Sum power iterative water-filling for multi-antenna Gaussian broadcast channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 4, pp. 1570–1580, Apr. 2005.

[20] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi, "Iterative water-filling for Gaussian vector multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 1, pp. 145–152, Jan. 2004.

[21] G. E. Forsythe, M. A. Malcolm, and C. B. Moler, *Computer Methods for Mathematical Computations*. Prentice-Hall, Inc., 1977.