Information-Theoretic Relaying for Multicast in Wireless Networks

Ivana Marić
Stanford University
ivanam@wsl.stanford.edu

Andrea Goldsmith
Stanford University
andrea@wsl.stanford.edu

Muriel Médard
MIT
medard@mit.edu

***Abstract*— A two-sender, two-receiver channel model with one relay node is considered for the case of multicast traffic. We examine two different cooperative approaches: relaying, in which the relay time-shares between helping two senders and employs traditional routing on the network layer, and more general encoding schemes in which the relay simultaneously forwards two data streams. Outer bounds are presented and then specialized to bound the performance of any relaying scheme. Two simple encoding schemes that allow for more general relaying are considered and compared to the relaying outer bound. We also consider the performance of an *opportunistic network coding* scheme that exploits interference at the receivers by interpreting it as a form of a network code.**

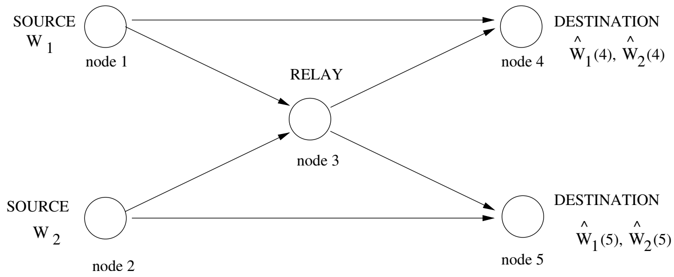

Fig. 1. Multicast relay channel.

I. Introduction and Related Work

For the networks on graphs, success of network coding has been well established and understood. Combining bits received from different senders at the network layer improves the efficiency and, in the case of multicast, achievable rates hit the cut-set bounds thus achieving capacity.

Network coding in wireless networks has received less attention, and hence the potential gains in this context are not well-understood. Wireless settings allow for any encoding scheme that satisfies node cost constraints. We will refer to such strategies as *information-theoretic relaying* in order to distinguish them from forwarding/routing at the relay. Determining optimum communication schemes in such a general setting seems a distant goal. A relay node receiving two data streams can, for example, time-share between forwarding two data streams - an approach commonly used in networks today. Alternatively, it can simultaneously transmit information about both data streams. Both approaches can be done in number of ways. For the *two-way relay channel* [1] in which two nodes exchange messages with help from a relay node, strategies that extend decode-and-forward, compress-and-forward and amplify-and-forward to relay messages from multiple sources, were proposed and investigated in [2]. Decode-and-forward was proposed for the Gaussian *multiaccess relay channel* or *multiaccess channel with dedicated relay (MAC-DR)* in [3] and further extended in [4], [5]. It is apparent that evaluating performance of various encoding schemes even for such small networks is in general complex. And yet, we seek to design and operate large networks in which even simple encoding strategies that allow for a more general approach than routing still need to be evaluated, compared to the conventional routing approach and employed.

With this goal at hand, this paper considers the smallest building block of wireless networks that have multiple sources multicasting their messages. The network has two senders, two receivers and a relay. In information-theoretic terminology this channel can be referred to as an *interference channel with a relay* in the case of unicast, and a *compound multiaccess channel with dedicated relay (CMAC-DR)* in the case of multicast. The special case of both is the relay channel [6]. We wish to understand the information-theoretic relaying approach for these channel models, as a step towards employing it in larger networks. We consider two simple relaying strategies: amplify-and-forward (AF), and the decode-and-forward (DF) strategy of [3, Sect.IV]. As in both cases the relay signal depends on transmissions from two sources, these strategies achieve simultaneous forwarding of two streams. The AF relay forwards a noisy sum of symbols from simultaneous senders which unavoidably combine in the wireless channel. It thus performs, in effect, a form of network coding referred to as *analog network coding* and implemented in [7]. Unlike the common network coding approach in which the node symbols are combined at the network layer, the analog network coding has the symbols combine at the PHY layer. We also consider the performance of an opportunistic network coding scheme that exploits interference at the receivers by interpreting it as a form of a network code.

To characterize the performance of these strategies, in Section III we first present an outer bound to the CMAC-

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[2]1-4244-1513-06/07/25.00 (c)2007 IEEE

DR capacity region that extends the MAC-DR outer bound of [3, Thm 1.]. We then specialize this bound to the relaying case to obtain a tighter outer bound on relaying performance. In Section IV the two encoding schemes are described and evaluated for CMAC-DR. Comparison between the relaying outer bound and performance of the encoding schemes is presented in Section V, exhibiting superior performance of DF. Both strategies are shown to outperform more traditional strategies such as orthogonal transmissions at the nodes. In Section VI we extend this approach to a half-duplex scenario. We conclude and propose future work in Section VII.

II. System Model

Consider a channel model with two senders, two receivers, and a relay, as shown in Fig. 1. The nodes are full-duplex. Encoder t, $t = 1, 2$, wishes to send a message $W_t \in \{1, \ldots, M_t\}$ in N channel uses to both decoders at the multicast rate $R_t = \log_2 M_t/N$. Encoders 1 and 2 transmit independent channel inputs X_1 and X_2, respectively, and the relay transmits X_3 based on previously received signals [8]

$$X_{3,n} = f_n(Y_{3,n-1}, \ldots, Y_{3,1}). \tag{1}$$

The encoding strategy of (1) captures all possible relaying strategies such as routing, DF and AF and can be in that sense thought of as *universal relaying*. Each transmitting node t has a block power constraint

$$\frac{1}{N}\sum_{n=1}^{N} E[X_{tn}^2] \leq P_t. \tag{2}$$

The received signal at the relay is given by

$$Y_3 = h_{13}X_1 + h_{23}X_2 + Z_3 \tag{3}$$

and signals at the two destination nodes

$$Y_4 = h_{14}X_1 + h_{24}X_2 + h_{34}X_3 + Z_4 \tag{4}$$

$$Y_5 = h_{15}X_1 + h_{25}X_2 + h_{35}X_3 + Z_5 \tag{5}$$

where the Z_t are independent, zero-mean, unit-variance Gaussian random variables.

The capacity region of the compound MAC-DR is the closure of the set of all rate pairs (R_1, R_2) at which two destination nodes can, for sufficiently large N, decode the messages (W_1, W_2) with arbitrarily small positive error probability.

III. Outer Bound

Similarly to [3, Thm. 1], a cut-set outer bound on the performance of any encoding scheme in CMAC-DR can be developed starting with Fano's inequality [9].

Lemma 1: The set of nonnegative rate pairs (R_1, R_2) that for $t = 1, 2$ satisfy

$$\sum_{t\in S} R_t \leq \min\{I(X_S, X_3; Y_d|X_{S^C}), I(X_S; Y_3, Y_d|X_{S^C}, X_3)\} \tag{6}$$

where S is any subset of $\{1, 2\}$ and S^C its complement, $d = 4, 5$, and probability distribution $p(x_1, x_2, x_3)$ factors as

$$p(x_1)p(x_2)p(x_3|x_1, x_2) \tag{7}$$

is an outer bound to the CMAC-DR capacity region.

Proof: The proof follows the same steps as in [8, Thm. 4] and is therefore omitted. ■

Since channel inputs at two sources are always independent, the input distribution factors as in (7). When compared to [3, Thm. 1], the number of bounds doubles in order for the reliable communications to be achieved at both receivers. The outer bound in Lemma 1 holds for any encoding scheme in CMAC-DR. Note that the channel input at the relay (1) in general depends on both (W_1, W_2).

As a specific encoding scheme, the relay can decide to time-share in relaying information of two sources during the transmission of a block of N bits. We have

$$X_3[n] = f_{3,n}(W_1) \qquad \text{for } 1 \leq n \leq k \tag{8}$$

$$X_3[n] = f_{3,n}(W_2) \qquad \text{for } k < n \leq N. \tag{9}$$

As we wish to compare general relaying that also allows for the network coding approach to pure relaying given by (8)-(9), we next develop an outer bound that applies to this specific encoding scheme (8)-(9). An outer bound on individual rates R_1 and R_2 for this case was developed in [10]. We strengthen this routing outer bound by bounding the sum rate $R_1 + R_2$.

One can observe from (8)-(9) that channel inputs in the pure relaying scheme have different probability distributions depending on whether $n \leq k$. Taking that into account and evaluating rates (6) for Gaussian CMAC-DR with Gaussian inputs, yields:

Lemma 2:

$$R_1 \leq \frac{k}{N}\min\{C((h_{1t}^2P_1)^E), C(P_1(1-\rho^2)(h_{13}^2 + h_{1t}^2))\} + \frac{N-k}{N}C(h_{1t}^2P_1) \tag{10}$$

$$R_2 \leq \frac{k}{N}C(h_{2t}^2P_2) + \frac{N-k}{N}C((h_{2t}^2P_2)^E) \tag{11}$$

$$R_1 + R_2 \leq \frac{k}{N}C((h_{1t}^2P_1)^E + h_{2t}^2P_2) + \frac{N-k}{N}C(h_{1t}^2P_1 + (h_{2t}^2P_2)^E) \tag{12}$$

where $0 \leq \rho \leq 1$, and

$$(h_{1t}^2P_1)^E = h_{1t}^2P_1 + h_{3t}^2P_3 + 2\rho h_{1t}h_{3t}\sqrt{P_1P_3} \tag{13}$$

$$(h_{2t}^2P_2)^E = h_{2t}^2P_2 + h_{3t}^2P_3 + 2h_{2t}h_{3t}\sqrt{P_2P_3} \tag{14}$$

and

$$C(x) = \frac{1}{2}\log_2(1+x). \tag{15}$$

Each of the bounds in Lemma 2 has a simple interpretation. Consider the bound on R_1. During time $n \leq k$, the relay helps in transmission of the message W_1. Thus the relay channel upper bounds [8, Thm. 4] apply, yielding the first term in (10). The second term is the rate achieved with direct transmission of W_1 from node 1 to the receiver t for $n > k$. Since we are interested in an outer bound, in both cases we can assume that W_2 (and thus X_2) is known to the receivers, creating no interference at nodes 4 and 5. Similar reasoning leads to a

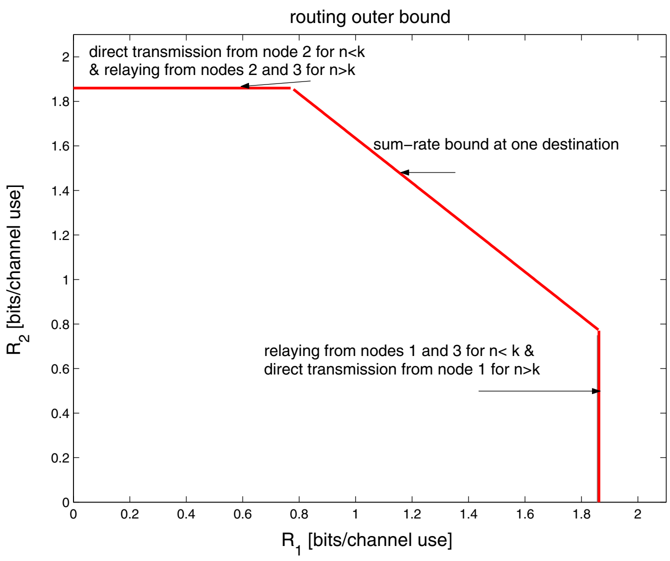

Fig. 2. Routing outer bound.

bound on R_2 in (11). The difference is in that the relay was receiving information about W_2 through X_2 for $n \leq k$ prior to relaying it. In fact, we assume that the relay was able to decode W_2 during that period. The relay upper bound is then given by the second term in (11). Finally, the two terms in the sum-rate bound (12) have the form as in the MAC where one of the users has additional power, which occurs due to the relay help. The bound at one of the destinations is illustrated in Fig. 2.

IV. Encoding Strategies

We consider two simple encoding schemes that allow the relay to simultaneously transmit information about both messages W_1, W_2. Two schemes respectively extend amplify-and-forward and decode-and-forward. We are interested in their performance for CMAC-DR.

A. Amplify-and-Forward

Consider a simple scheme as in [10] given by

$$X_3[n] = \alpha Y_3[n-1] \tag{16}$$

where α is chosen to satisfy the relay power constraint

$$\alpha \leq \sqrt{\frac{P_3}{h_{13}^2 P_1 + h_{23}^2 P_2 + 1}}. \tag{17}$$

Since Y_3 is the sum of two inputs X_1 and X_2, the above strategy combines bits from different nodes, i.e. performs a form of network coding at the relay, referred to as analog network coding in [10]. We remark that this combining occurs at the PHY layer in contrast to the network coding approach that combines bits on the network layer. From (4), (5) and (16), the received signal at the destination $t, t = 4, 5$ at time n is

$$\begin{aligned} Y_t[n] = {} & h_{1t}X_1[n] + \alpha h_{13}h_{3t}X_1[n-1] + h_{2t}X_2[n] \\ & + \alpha h_{23}h_{3t}X_2[n-1] + \alpha h_{3t}Z_3[n-1] + Z_t[n]. \end{aligned} \tag{18}$$

Denote the effective noise as

$$W_t[n] = \alpha h_{3t} Z_3[n-1] + Z_t[n]. \tag{19}$$

The received signal (18) becomes

$$\begin{aligned} Y_t[n] = {} & h_{1t}X_1[n] + \alpha h_{13}h_{3t}X_1[n-1] + h_{2t}X_2[n] \\ & + \alpha h_{23}h_{3t}X_2[n-1] + W_t[n]. \end{aligned} \tag{20}$$

The above equation describes a multiaccess (MAC) channel with a unit memory. This observation was also used in [4], [5] to evaluate achievable rates in MAC-DR. The capacity region of the MAC channel with output t is [11, Corollary 1]:

$$\begin{aligned} \mathcal{C}_{\text{MAC,t}} = & \bigcup_{S_1(f), S_2(f)} \{(R_1, R_2) : R_1 \geq 0, R_{2t} \geq 0, \\ R_1 \leq & \int_0^{1/2} \log\left(1 + S_1(f)T_{1t}(f)\right) df \\ R_2 \leq & \int_0^{1/2} \log\left(1 + S_2(f)T_{2t}(f)\right) df \\ R_1 + R_2 \leq & \int_0^{1/2} \log\left(1 + S_1(f)T_{1t}(f) + S_2(f)T_{2t}(f)\right) df\} \end{aligned} \tag{21}$$

where

$$T_{1t}(f) = \frac{h_{1t}^2 + \alpha^2 h_{13}^2 h_{3t}^2 + 2\alpha h_{1t}h_{13}h_{3t}\cos 2\pi f}{\alpha^2 h_{3t}^2 + 1} \tag{22}$$

$$T_{2t}(f) = \frac{h_{2t}^2 + \alpha^2 h_{23}^2 h_{3t}^2 + 2\alpha h_{2t}h_{23}h_{3t}\cos 2\pi f}{\alpha^2 h_{3t}^2 + 1} \tag{23}$$

and $S_t(f)$ are the input power spectral densities. The choice of rates (R_1, R_2) that belong to the region (21) for $t = 1, 2$ will guarantee that W_1, W_2 are multicast to both nodes 4 and 5, [12, Sect. II]. Under the assumption that there is no delay at the relay, the outputs (20) describe two MAC channels for which the capacity regions are easily calculated to be

$$\begin{aligned} \mathcal{C} = & \{(R_1, R_2) : R_1 \geq 0, R_2 \geq 0, \\ R_1 \leq & \frac{1}{2}\log\left(1 + \frac{P_1(h_{1t} + \alpha h_{13}h_{3t})^2}{(\alpha h_{3t})^2 + 1}\right) \\ R_2 \leq & \frac{1}{2}\log\left(1 + \frac{P_2(h_{2t} + \alpha h_{23}h_{3t})^2}{(\alpha h_{3t})^2 + 1}\right) \\ R_1 + R_2 \leq & \frac{1}{2}\log \\ & \left(1 + \frac{P_1(h_{1t} + \alpha h_{13}h_{3t})^2 + P_2(h_{2t} + \alpha h_{23}h_{3t})^2}{(\alpha h_{3t})^2 + 1}\right)\}. \end{aligned} \tag{24}$$

B. Decode-and-Forward

The decode-and-forward strategy of [8, Sec. IV] was extended for Gaussian MAC-DR channel by Kramer and Wijngaarden in [3]. We adopt this encoding scheme for the CMAC-DR. In the two-user case, the channel inputs are

$$X_t = \sqrt{P_t}(\sqrt{\alpha_t}V_t + \sqrt{\bar{\alpha}_t}U_t) \tag{25}$$

$$X_3 = \sqrt{P_3}(\sqrt{\beta}V_1 + \sqrt{\bar{\beta}}V_2) \tag{26}$$

where V_t, U_t are the independent, zero-mean, unit-variance, Gaussian random variables, and $0 \leq \alpha_1, \alpha_2, \beta \leq 1$. Observe from (25)-(26) that the relay signal depends on both messages W_1 and W_2. For Gaussian MAC-DR with output Y_t, achievable rates are [3, Sect. IV]:

$$\mathcal{R}_{\text{MAC-DR},t}(p(x_1), p(x_2)) = \Big\{(R_1, R_2) : R_1 \geq 0, R_2 \geq 0,$$
$$R_1 \leq \min\{I(X_1; Y_3|X_2, V_1, V_2), I(X_1, V_1; Y_t|X_2, V_2)\}$$
$$R_2 \leq \min\{I(X_2; Y_3|X_1, V_1, V_2), I(X_2, V_2; Y_t|X_1, V_1)\}$$
$$R_1 + R_2 \leq \min\{I(X_1, X_2; Y_3|V_1, V_2), I(X_1, V_1, X_2, V_2; Y_t)\}\Big\} \tag{27}$$

which with Gaussian inputs evaluate to

$$\mathcal{R}_{\text{MAC-DR},t}(\alpha_1, \alpha_2, \beta) = \Big\{(R_1, R_2) : R_1 \geq 0, R_2 \geq 0,$$
$$R_1 \leq \min\{C(h_{13}^2\bar{\alpha}_1 P_1), C(h_{1t}^2 P_1 + \beta h_{3t}^2 P_3 + 2h_{1t}h_{3t}\sqrt{\alpha_1 P_1 \beta P_3})\}$$
$$R_2 \leq \min\{C(h_{23}^2\bar{\alpha}_2 P_2), C(h_{2t}^2 P_2 + \bar{\beta} h_{3t}^2 P_3 + 2h_{2t}h_{3t}\sqrt{\alpha_2 P_2 \bar{\beta} P_3})\}$$
$$R_1 + R_2 \leq \min\{C(h_{13}^2\bar{\alpha}_1 P_1 + h_{23}^2\bar{\alpha}_2 P_2), C(h_{1t}^2 P_1 + h_{2t}^2 P_2 + h_{3t}^2 P_3 + 2h_{1t}h_{3t}\sqrt{\alpha_1 P_1 \beta P_3} + 2h_{2t}h_{3t}\sqrt{\alpha_2 P_2 \bar{\beta} P_3}\}\Big\}. \tag{28}$$

The first term in bounds (27) and (28) guarantees that the relay can reliably decode. The contribution of the relay to the data rates is reflected in the second term in each bound.

We adopt the encoding scheme given by (25)-(26) for the CMAC-DR. Again, it is straightforward to observe (see [12, Sect. II] for more details) that rates that satisfy

$$\mathcal{R}_{\text{CMAC-DR}} = \bigcup_{\alpha_1, \alpha_2, \beta} \Big\{\mathcal{R}_{\text{MAC-DR},1} \bigcap \mathcal{R}_{\text{MAC-DR},2}\Big\} \tag{29}$$

are achievable in the CMAC-DR.

C. MAC to the Relay and Broadcast to Destinations

We wish to compare out two proposed strategies to an encoding scheme that resembles more what is done in networks today. Therefore, we also consider a MAC/BC scheme in which the sources first transmit data to the relay and destinations do not observe these inputs. The relay decodes both messages and broadcasts it to destinations. From (3), the relay can decode (W_1, W_2) if rates satisfy MAC bounds:

$$R_1 \leq C\left(h_{13}^2 P_1\right)$$
$$R_2 \leq C\left(h_{23}^2 P_2\right)$$
$$R_1 + R_2 \leq C\left(h_{13}^2 P_1 + h_{23}^2 P_2\right). \tag{30}$$

Note that (30) upper bounds the performance of a scheme in which sources time-share in transmitting to the relay [9].

Because indexes (W_1, W_2) are to be decoded at both nodes 4 and 5, they have a role of a common message in the broadcast channel (BC) from the relay to nodes 4 and 5. Their sum-rate is then bounded by the worse channel

$$R_1 + R_2 \leq \min\{C(h_{34}^2 P_3), C(h_{35}^2 P_3)\}. \tag{31}$$

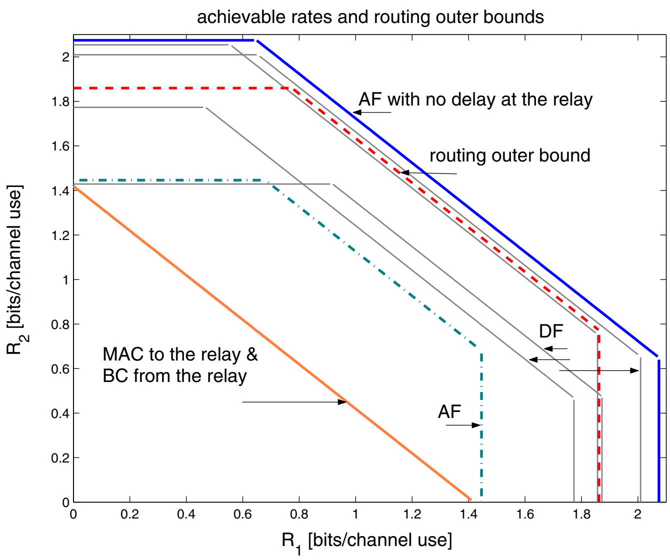

Fig. 3. Outer bounds on routing and achievable rates with more general relaying.

The relay can alternatively time-share in transmitting to destinations. We next consider a variation of this scheme that uses the idea of network coding. Unlike in the analog network coding, in this strategy the relay decodes (W_1, W_2).

D. Network Coding Approach

Suppose that in the above scheme, nodes time-share in transmitting their messages. The relay can decode (W_1, W_2) if, for $t = 1, 2$

$$R_t \leq C(h_{t3} P_t)/2 \tag{32}$$

where $1/2$ is due to the fact that each source transmits only for half the time when compared to the MAC/BC scenario. Furthermore, suppose that rates R_1 and R_2 are chosen so to allow nodes 4 and 5 to respectively decode W_1 and W_2

$$R_1 \leq C(h_{14} P_1)/2 \tag{33}$$
$$R_2 \leq C(h_{25} P_2)/2. \tag{34}$$

Therefore, at the end of the first slot, each destination decodes one of two messages and needs to decode the other one. In the next time slot, the relay can use the encoding scheme for the broadcast channel in which receivers have side information [13], (see also work by Oechtering et. al. in [14], Xie in [15] and Kramer and Shamai in [16], [17]). The relay encodes (W_1, W_2) using a single codebook. For nodes 4 and 5 to respectively be able to decode W_2 and W_1, rates have to satisfy

$$R_2 \leq C(h_{34} P_3) \tag{35}$$
$$R_1 \leq C(h_{35} P_3). \tag{36}$$

This approach can be applied, more generally, when destinations obtain parts of messages from source transmissions [17].

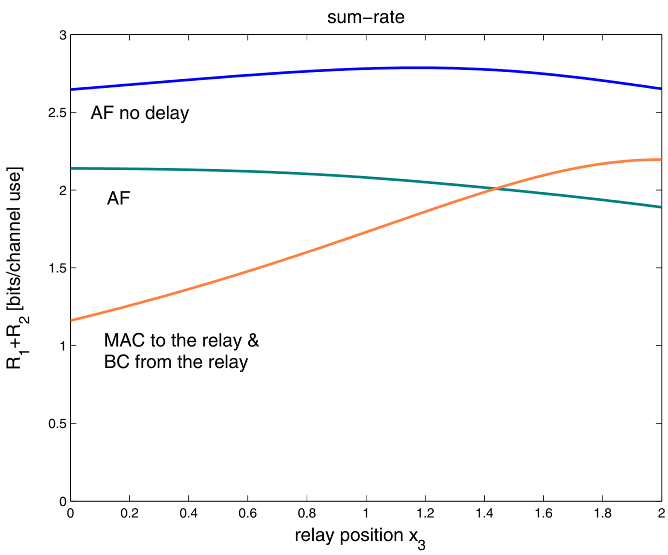

Fig. 4. Sum-rate achieved with AF and with the MAC/BC scheme.

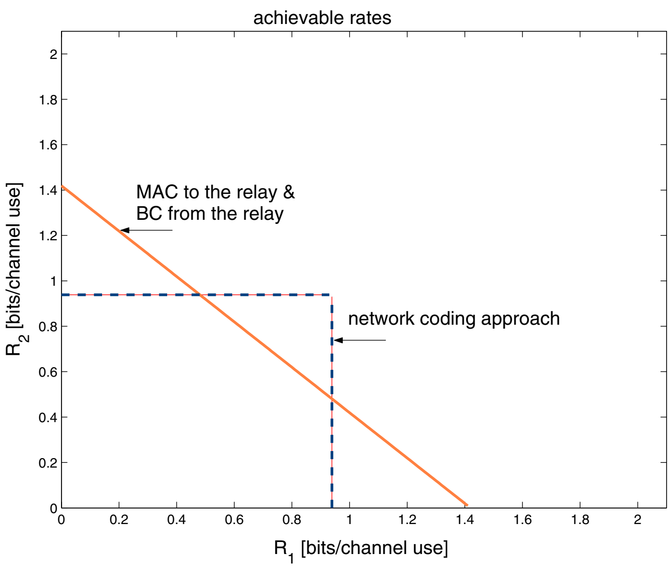

Fig. 5. Achievable rates with MAC/BC scheme and network coding.

V. Numerical Results

We compare achievable rates with three encoding strategies to the relaying bounds for a symmetric topology of Fig. 1. The performance is shown in Fig. 3 for $P_1 = P_2 = 50$, $P_3 = 20$. Achievable rates for DF are given by the union of all pentagons (28) obtained for different values of α_1, α_2 and β. Four such pentagons are shown in Fig. 3. We observe that the AF region is contained in the DF region and thus DF outperforms AF, as in the relay channel [18]. Observe that the achievable sum-rate with DF outperforms the relaying sum-rate bound. AF outperforms the relaying bound only under the assumption of no delay at the relay, given by (24). We suspect that this is due to the looseness of relaying bound which neglects the effect of interference at the receivers due to a genie. Both AF and DF outperform the MAC/BC scheme. For the considered scenario, the performance of the latter is limited by the broadcast rate from the relay to the destinations. We also compare the AF sum-rate to the MAC/BC sum-rate for different positions of the relay in Fig. 4. We observe that MAC/BC outperforms AF only when the relay is close to destinations where the BC rate is the highest. Performance comparison between the MAC/BC scheme and the network coding scheme is shown in Fig. 5.

VI. The Half-Duplex Scenario

We next consider a more practical scenario in which nodes cannot transmit and receive at the same time. We compare the AF and DF strategies. As before, the sources $t = 1, 2$ transmit W_t over a block of length N with average power P_t and the relay listens. Received signals at nodes $t = 3, 4, 5$ are

$$Y_t[n] = h_{1t}X_1[n] + h_{2t}X_2[n] + Z_t[n]. \tag{37}$$

In the second block, the relay transmits with power P_3. The channel outputs at destination nodes $t = 4, 5$ are

$$Y_t[n] = h_{3t}X_3[n] + Z_t[n]. \tag{38}$$

For the AF strategy we have from (16) and (37)

$$X_3[n] = \alpha h_{13}X_1[n-1] + \alpha h_{23}X_2[n-1] + \alpha Z_3[n-1]. \tag{39}$$

From (37), (38) and (39), received signals at destination nodes $t = 4, 5$ over two consecutive blocks become

$$\mathbf{Y}_t = \begin{bmatrix} h_{1t} \\ \alpha h_{13}h_{3t} \end{bmatrix} X_1[n-1] + \begin{bmatrix} h_{2t} \\ \alpha h_{23}h_{3t} \end{bmatrix} X_2[n-1] + \begin{bmatrix} Z_t[n-1] \\ W_t[n] \end{bmatrix} \tag{40}$$

where equivalent noise $W_t[n]$ is given by (19). The above equation describes the MAC with one transmit and two receive antennas. The achievable rates satisfy for $t = 1, 2$

$$R_1 \leq C\left((h_{1t}^2 + \frac{\alpha^2 h_{13}^2 h_{3t}^2}{\alpha^2 h_{3t}^2 + 1})P_1\right)$$
$$R_2 \leq C\left((h_{2t}^2 + \frac{\alpha^2 h_{23}^2 h_{3t}^2}{\alpha^2 h_{3t}^2 + 1})P_2\right)$$
$$R_1 + R_2 \leq C\left((h_{1t}^2 + \frac{\alpha^2 h_{13}^2 h_{34}^2}{\alpha^2 h_{3t}^2 + 1})P_1 + (h_{2t}^2 + \frac{\alpha^2 h_{23}^2 h_{3t}^2}{\alpha^2 h_{3t}^2 + 1})P_2\right)\Big\}.$$

Consider next the DF strategy adapted for orthogonal transmissions. In the first block, source t encodes w_t with codebook $\mathbf{x}_t(w_t)$ and transmits X_t. The relay decodes and re-encodes w_t using codebooks $\mathbf{v}_t(w_t)$. In the second block, the relay splits its power to forward for both sources and transmits

$$X_3 = c_1V_1 + c_2V_2 \tag{41}$$

where c_1 and c_2 satisfy the relay power constraint

$$c_1 = \sqrt{\beta P_3}, \qquad c_2 = \sqrt{(1-\beta)P_3} \tag{42}$$

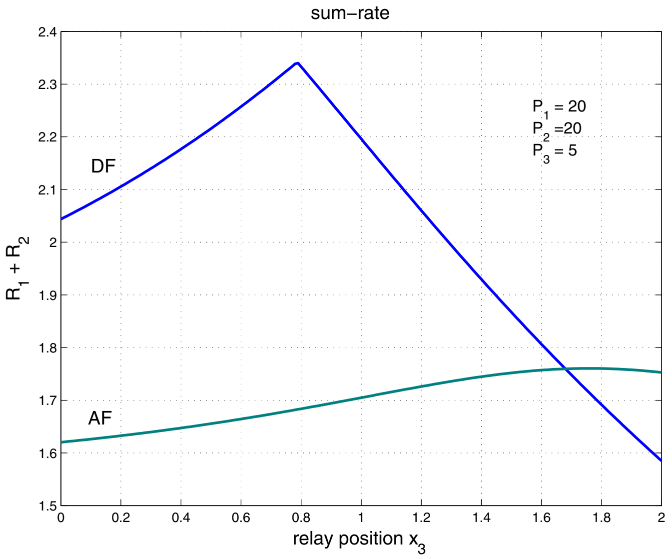

Fig. 6. Sum rate achieved in the half-duplex scenario with AF and DF for different position of the relay.

for $0 \le \beta \le 1$. Recall that V_1 and V_2 are independent zero-mean, unit-variance Gaussian random variables. Channel outputs (37)-(38) over two consecutive slots become

$$\mathbf{Y}_t = \begin{bmatrix} h_{1t} & 0 \\ 0 & c_1 h_{3t} \end{bmatrix} \begin{bmatrix} X_1[n-1] \\ V_1[n] \end{bmatrix} + \begin{bmatrix} h_{2t} & 0 \\ 0 & c_2 h_{3t} \end{bmatrix} \begin{bmatrix} X_2[n-1] \\ V_2[n] \end{bmatrix} + \begin{bmatrix} Z_t[n-1] \\ Z_t[n] \end{bmatrix}. \tag{43}$$

For the relay to be able to decode, the rate pair (R_1, R_2) has to satisfy MAC bounds (30). In addition, rates have to belong to two rate regions specified by two-transmit, two-receive antenna MACs at nodes 4 and 5, (43). We have

$$\begin{aligned} \mathcal{R}_{\text{HD-DFD},t} = \Big\{ &(R_1, R_2) : R_1 \ge 0, R_2 \ge 0, \qquad (44) \\ R_1 &\le C\left((h_{1t}^2 P_1\right) + C\left(c_1^2 h_{3t}^2 P_1\right) \\ R_2 &\le C\left((h_{2t}^2 P_2\right) + \left(c_2^2 h_{3t}^2 P_2\right) \\ R_1 + R_2 &\le C\left(h_{1t}^2 P_1 + h_{2t}^2 P_2\right) + C\left(c_1^2 h_{3t}^2 P_1 + c_2^2 h_{3t}^2 P_2\right) \Big\}. \end{aligned}$$

For a symmetric scenario, the sum-rate for both schemes is shown in Fig. 6, showing superior performance of DF for a range of values when the relay is further away from destinations.

VII. Conclusions and Future Work

This paper considers cross-layer, information-theoretic encoding strategies for the CMAC-DR with the goal of evaluating their multicast performance and comparing it to a pure relaying and routing approach. Among the variety of possible cooperative schemes, we seek simple schemes that can easily be employed in large wireless networks. We compare the performance of two schemes that extend AF and DF to allow forwarding for multiple sources. We also consider the performance of an opportunistic network coding scheme that exploits interference at the receivers by interpreting it as a form of a network code. As the capacity of relay networks is an open problem, we employ upper bounds to characterize the relaying performance. And although the achievable schemes come close to the derived relaying outer bound, they do not, in general, outperform it. We suspect that this is due to the looseness of relaying bound which neglects the effect of interference at the receivers due to a genie. We also observe that the DF scheme achieves higher gains than AF. Results in this paper point to the directions of our current and future work. We wish to tighten the outer bounds on relaying schemes which would allow for a stronger statement about the gains from the general, information-theoretic approach. We also plan to compare the strategies analyzed in this paper to other strategies that employ a network coding approach as in [14] and [19], [20]. We then wish to extend the analysis to more general network scenarios. We remark that the considered opportunistic network coding translates more easily to large networks than DF does, as each network node can exploit overheard signals in the same way, as parts of a network code. This scheme can also be employed along other techniques that allow for mixing data at the relays. Therefore, this is a promising approach for more general network scenarios and larger topologies.

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