On the Capacity of Interference Channels with a Cognitive Transmitter

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***Abstract*— Outer and inner bounds are established on the capacity region of two-sender, two-receiver interference channels where one transmitter knows both messages. The transmitter with extra knowledge is referred to as being cognitive. One of the outer bounds is based on the Nair-El Gamal outer bound for broadcast channels. The inner bound is based on strategies that generalize prior work to include rate-splitting, dirty-paper coding, and carbon-copying. The bounds are demonstrated for Gaussian channels.**

I. Introduction

Cognitive radio [1] technology is aimed at developing smart radios that are both aware of and adaptive to the environment. Such radios can efficiently sense the spectrum, decode information from detected signals and use that knowledge to improve the overall system performance. This technology motivates new information-theoretic models that try to capture the cognitive radio characteristics. In this paper, we consider channel models with two senders and two receivers in which one of the senders is cognitive in the sense that it knows the message of the other encoder. Without such information, we have the interference channel [2], [3] in which senders are unaware of each other's messages. The extra information allows the cognitive user to cooperate by forming channel inputs based on both users' messages, thus improving its own rate and the rate of the other user. We refer to this channel as an *interference channel with unidirectional cooperation (ICUC)*. This channel was dubbed the *cognitive radio channel* in [4], [5] where achievable rates were presented. For the Gaussian case of weak interference, the capacity region of this channel was determined in [6] and [7].

We present an outer bound on the ICUC capacity which is based on the broadcast outer bound of Nair and El Gamal, [8], [9]. We then present an achievable rate region which improves on the regions in [6], [7], [10]. In addition to using rate-splitting [3] to allow the receivers to decode part of the interference, the region is based on ideas of [11] and [12] that extend the Gel'fand-Pinsker [13] and Costa [14] approaches.

The assumption that the full message of one sender is available to the cognitive user may be an over-idealization. The capacity for this model constitutes an outer bound on the performance of more realistic models. In our ongoing work, we are considering more general models where only part of the message is known to the cognitive user [15].

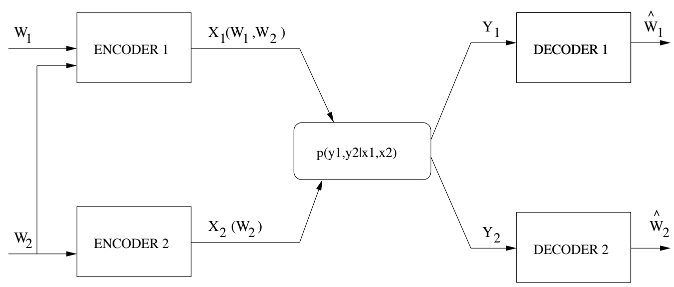

Fig. 1. Interference channel with unidirectional cooperation.

II. The Discrete Memoryless Channel

Consider a channel with finite input alphabets $\mathcal{X}_1, \mathcal{X}_2$, finite output alphabets $\mathcal{Y}_1, \mathcal{Y}_2$, and a conditional probability distribution $p(y_1, y_2|x_1, x_2)$, where $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$ are channel inputs and $(y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2$ are channel outputs. Each encoder t, $t = 1, 2$, wishes to send a message $W_t \in \{1, \ldots, M_t\}$ to decoder t in N channel uses. Message W_2 is also known at encoder 1, thus allowing for unidirectional cooperation. The channel is memoryless and time-invariant in the sense that

$$\begin{aligned} p(y_{1,n}, y_{2,n}|\mathbf{x}_1^n, \mathbf{x}_2^n, \mathbf{y}_1^{n-1}, \mathbf{y}_2^{n-1}, \bar{w}) \\ = p_{Y_1,Y_2|X_1,X_2}(y_{1,n}, y_{2,n}|x_{1,n}, x_{2,n}) \end{aligned} \quad (1)$$

for all n, where X_1, X_2 and Y_1, Y_2 are random variables representing the respective inputs and outputs, $\bar{w} = [w_1, w_2]$ denotes the messages to be sent, and $\mathbf{x}_t^n = \left[x_{t,1}, \quad \ldots, \quad x_{t,n}\right]$. We will follow the convention of dropping subscripts of probability distributions if the arguments of the distributions are lower case versions of the corresponding random variables. To simplify notation, we also drop superscripts when $n = N$.

The communication system is shown in Figure 1. An (M_1, M_2, N, P_e) code has two encoding functions

$$\mathbf{x}_1 = f_1(W_1, W_2) \quad (2)$$

$$\mathbf{x}_2 = f_2(W_2) \quad (3)$$

two decoding functions

$$\hat{W}_t = g_t(\mathbf{Y}_t) \quad t = 1, 2 \quad (4)$$

and an error probability

$$P_e = \max\{P_{e,1}, P_{e,2}\} \quad (5)$$

where, for $t = 1, 2$, we have

$$P_{e,t} = \sum_{(w_1,w_2)} \frac{1}{M_1 M_2} P\left[g_t(\mathbf{Y}_t) \neq (w_t) | (w_1, w_2) \text{ sent}\right]. \quad (6)$$

A rate pair (R_1, R_2) is achievable if, for any $\epsilon > 0$, there is an (M_1, M_2, N, P_e) code such that

$$M_t \geq 2^{NR_t}, \quad t = 1, 2, \text{ and } P_e \leq \epsilon.$$

The capacity region of the ICUC is the closure of the set of all achievable rate pairs (R_1, R_2).

A. Outer Bound

Theorem 1: The set of rate pairs (R_1, R_2) satisfying

$$R_1 \leq I(V, U_1; Y_1) \quad (7)$$
$$R_2 \leq I(V, U_2; Y_2) \quad (8)$$
$$R_1 + R_2 \leq \min\{I(V, U_1; Y_1) + I(U_2; Y_2 | U_1, V), \quad (9)$$
$$I(U_1; Y_1 | U_2, V) + I(V, U_2; Y_2)\} \quad (10)$$

for input distributions $p(v, u_1, u_2, x_1, x_2)$ that factor as

$$p(u_1)p(u_2)p(v|u_1, u_2)p(x_2|u_2)p(x_1|u_1, u_2) \quad (11)$$

is an outer bound to the ICUC capacity region.

Proof: Consider a code (M_1, M_2, N, P_e) for the ICUC. We first consider the bound (10). Fano's inequality implies that for reliable communication we require

$$N(R_1 + R_2) \quad (12)$$
$$\leq I(W_1; \mathbf{Y}_1) + I(W_2; \mathbf{Y}_2) \quad (13)$$
$$\leq I(W_1; \mathbf{Y}_1 | W_2) + I(W_2; \mathbf{Y}_2) \quad (14)$$
$$= \sum_{i=1}^{N} I(W_1; Y_1^i | W_2, Y_{2,i+1}^N) - I(W_1; Y_1^{i-1} | W_2, Y_{2,i}^N) + I(W_2; Y_{2,i} | Y_{2,i+1}^N) \quad (15)$$
$$= \sum_{i=1}^{N} I(W_1; Y_1^i | W_2, Y_{2,i+1}^N) - [I(W_1, Y_{2,i}; Y_1^{i-1} | W_2, Y_{2,i+1}^N) - I(Y_{2,i}; Y_1^{i-1} | W_2, Y_{2,i+1}^N)] + I(W_2; Y_{2,i} | Y_{2,i+1}^N) \quad (16)$$
$$= \sum_{i=1}^{N} I(W_1; Y_{1,i} | W_2, V_i) - I(Y_{2,i}; Y_1^{i-1} | W_1, W_2, Y_{2,i+1}^N) + I(W_2, Y_1^{i-1}; Y_{2,i} | Y_{2,i+1}^N) \quad (17)$$
$$\leq \sum_{i=1}^{N} I(W_1; Y_{1,i} | W_2, V_i) + I(W_2, V_i; Y_{2,i}) \quad (18)$$

where (14) follows from the independence of W_1, W_2; in (17), we let $Y_{t,i}^j = (Y_{t,i}, \ldots, Y_{t,j})$ and $V_i = [Y_1^{i-1}, Y_{2,i+1}^N]$.

We next consider the bound (8). Fano's inequality implies

$$NR_2 \leq I(W_2; \mathbf{Y}_2) \quad (19)$$
$$= \sum_{i=1}^{N} I(W_2; Y_{2,i} | Y_{2,i+1}^N) \quad (20)$$
$$\leq \sum_{i=1}^{N} I(W_2, Y_1^{i-1}, Y_{2,i+1}^N; Y_{2,i}) \quad (21)$$
$$= \sum_{i=1}^{N} I(W_2, V_i; Y_{2,i}). \quad (22)$$

Note that for (12)-(22) we have used only the independence of W_1 and W_2, and the non-negativity of mutual information. The bounds (7) and (9) thus follow by symmetry.

We introduce random variables $U_{1,i} = W_1$ and $U_{2,i} = W_2$ for all i, to get the bounds in the form (7)-(10). Observe that $U_{1,i}$ and $U_{2,i}$ are independent. Furthermore, due to unidirectional cooperation, the joint probability distribution factors as in (11). ■

We observe that the outer bound in Thm. 1 is of the same form as the outer bound for the broadcast channel in [9, Sect. 3]. The difference is the factorization of the input distribution. In fact, one can restrict attention to distributions (11) where X_2 is a function of U_2 and X_1 is a function of (U_1, U_2). The bounds (7)-(10) can thus be written as

$$R_1 \leq I(V, U_1; Y_1) \quad (23)$$
$$R_2 \leq I(V, U_2, X_2; Y_2) \quad (24)$$
$$R_1 + R_2 \leq \min\{I(V, U_1; Y_1) + I(X_1, X_2; Y_2 | U_1, V), \quad (25)$$
$$I(X_1; Y_1 | X_2, U_2, V) + I(V, U_2, X_2; Y_2)\} \quad (26)$$

From (24) and (26), we recover a bound in [6, Thm. 3.2]:

$$R_2 \leq I(V', X_2; Y_2) \quad (27)$$
$$R_1 + R_2 \leq I(X_1; Y_1 | X_2, V') + I(V', X_2; Y_2) \quad (28)$$

where $V' = [U_2, V]$ and the probability distribution factors as

$$p(v', x_1, x_2)p(y_1, y_2 | x_1, x_2). \quad (29)$$

The bound (27)-(29) was shown to be tight under weak interference [6, Def. 2.3] and in particular for Gaussian channels with weak interference [6], [7].

We also have the following bound in strong interference.

Theorem 2: For an ICUC that satisfies

$$I(X_1; Y_1 | X_2) \leq I(X_1; Y_2 | X_2) \quad (30)$$

for all input distribution $p(x_1, x_2)$, the set of rate pairs (R_1, R_2) satisfying

$$R_1 \leq I(X_1; Y_1 | X_2) \quad (31)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_2) \quad (32)$$

for all input distributions $p(x_1, x_2)$ is an outer bound to the capacity region.

Proof: The bound (31) follows by standard methods. To prove (32), consider (10) and

$$\begin{aligned} I(U_1;Y_1|U_2,V) &\le I(U_1;Y_1,X_2|U_2,V) \\ &= I(U_1;Y_1|U_2,V,X_2) \\ &\le I(U_1,X_1;Y_1|U_2,V,X_2) \\ &= I(X_1;Y_1|U_2,V,X_2) \\ &\le I(X_1;Y_2|U_2,V,X_2) \end{aligned} \quad (33)$$

where the second step follows by the Markov chain (11), and the last step follows by (30). We similarly have

$$I(V,U_2;Y_2) \le I(U_2,V,X_2;Y_2). \quad (34)$$

Combining inequalities (10), (33) and (34) gives (32). ■

B. Inner Bound

For the achievable scheme, we will employ rate splitting. We define

$$R_1 = R_{1a} + R_{1c} \quad (35)$$

$$R_2 = R_{2a} + R_{2b} + R_{2c} \quad (36)$$

for nonnegative $R_{1a}, R_{1c}, R_{2c}, R_{2a}, R_{2b}$ and $R_c = R_{1c} + R_{2c}$.

Theorem 3: The rates (35)-(36) are achievable if

$$R_{2a} \le I(X_{2a};Y_2) \quad (37)$$

$$R_{2b} \le I(X_{2b};Y_2,U_{1c}|X_{2a}) \quad (38)$$

$$R_{1a} \le I(U_{1a};Y_1|U_{1c}) - I(U_{1a};X_{2a},X_{2b}|U_{1c}) \quad (39)$$

$$R_c \le \min\{I(U_{1c};Y_1), I(U_{1c};Y_2,X_{2a})\} - I(U_{1c};X_{2a},X_{2b}) \quad (40)$$

for some joint distribution that factors as

$$p(x_{2a},x_{2b},u_{1c},u_{1a},x_1,x_2)p(y_1,y_2|x_1,x_2). \quad (41)$$

This strategy includes the following schemes:

- The scheme of [6, Thm 3.1] for $X_{2a}=\emptyset, U_{1c}=\emptyset, X_{2b}=(X_2,U)$ and $U_{1a}=V$ achieving:

  $$R_2 \le I(X_2,U;Y_2) \quad (42)$$

  $$R_1 \le I(V;Y_1) - I(V;X_2,U) \quad (43)$$

  for $p(u,x_2)p(v|u,x_2)p(x_1|v)$.
- The scheme of [16, Lemma 4.2] for $X_{2a}=\emptyset$, $X_{2b}=X_2$, $U_{1a}=\emptyset$, and $R_1=R_c$, $R_2=R_{2b}$ as:

  $$R_2 \le I(X_2;Y_2|U_{1c})$$

  $$R_1 \le \min\{I(U_{1c};Y_1), I(U_{1c};Y_2)\} - I(U_{1c};X_2)$$

  for $p(x_2)p(u_{1c})$. The strategy is considered for the case when $I(U_{1c};Y_1) \le I(U_{1c};Y_2)$.
- Carbon-copy on dirty paper [12] for $X_{2a}=\emptyset, U_{1a}=\emptyset$.
- The strong interference case [10], [17] for $X_{2a}=X_2, X_{2b}=\emptyset, U_{1c}=X_2, U_{1a}=X_1$.

Proof: **Code construction:** Choose a distribution $p(x_{2a},x_{2b},u_{1c},u_{1a},x_1,x_2)$.

- Split the rates as in (35)-(36).
- Generate $2^{NR_{2a}}$ codewords $\mathbf{x}_{2a}(w_{2a})$ using $P_{X_{2a}}(\cdot)$, $w_{2a}=1,\ldots,2^{NR_{2a}}$.
- For each w_{2a}: Generate $2^{NR_{2b}}$ codewords $\mathbf{x}_{2b}(w_{2a},w_{2b})$ using $P_{X_{2b}|X_{2a}}(\cdot|x_{2a})$, $w_{2b}=1,\ldots,2^{NR_{2b}}$, where $x_{2a}=x_{2a,i}(w_{2a})$. Similar notation is used in the rest of the code construction.
- For each pair (w_{2a},w_{2b}) : Generate a codeword $\mathbf{x}_2(w_{2a},w_{2b})$ using $P_{X_2|X_{2a}X_{2b}}(\cdot|x_{2a}x_{2b})$.
- Generate $2^{N(R_c+R'_c)}$ codewords $\mathbf{u}_{1c}(w_c,b_c)$, $w_c=1,\ldots,2^{NR_c}$, $b_c=1,\ldots,2^{NR'_c}$ using $P_{U_{1c}}(\cdot)$.
- For each $\mathbf{u}_{1c}(w_c,b_c)$: Generate $2^{N(R_{1a}+R'_{1a})}$ codewords $\mathbf{u}_{1a}(w_c,b_c,w_{1a},b_{1a})$, $w_{1a}=1,\ldots,2^{NR_{1a}}$, $b_{1a}=1,\ldots,2^{NR'_{1a}}$ using $P_{U_{1a}|U_{1c}}(\cdot|u_{1c})$. The last two codebooks are used for transmitting over a channel with interference $S=(X_{2a},X_{2b})$.
- For (w_1,w_2) : Generate $\mathbf{x}_1(w_{2a},w_{2b},w_c,b_c,w_{1a},b_{1a})$ using $P_{X_1|X_{2a}X_{2b}U_{1c}U_{1a}X_2}(\cdot|x_{2a}x_{2b}u_{1c}u_{1a}x_2)$.

Encoders: Encoder 1:

1) Split the NR_1 bits w_1 into NR_{1a} bits w_{1a} and NR_{1c} bits w_{1c}. Similarly, split the NR_2 bits w_2 into NR_{2a} bits w_{2a} and NR_{2c} bits w_{2c}. We write this as

   $$w_1=(w_{1a},w_{1c}), \qquad w_2=(w_{2a},w_{2b},w_{2c})$$

   and we define $w_c=(w_{1c},w_{2c})$.
2) Try to find a bin index b_c so that $(\mathbf{u}_{1c}(w_c,b_c),\mathbf{x}_{2a}(w_{2a}),\mathbf{x}_{2b}(w_{2a},w_{2b})) \in A_\epsilon(P_{U_{1c}X_{2a}X_{2b}})$, where $A_\epsilon(P_{XY})$ denotes jointly ϵ-typical set with respect to P_{XY}, as defined in [18]. If such b_c cannot be found, choose $b_c=1$.
3) For each (w_c,b_c): Try to find a bin index b_{1a} such that $(\mathbf{u}_{1a}(w_c,b_c,w_{1a},b_{1a}),\mathbf{x}_{2a}(w_{2a}),\mathbf{x}_{2b}(w_{2a},w_{2b}),\mathbf{u}_{1c}(w_c,b_c)) \in A_\epsilon(P_{U_{1a}X_{2a}X_{2b}U_{1c}})$. If unsuccessful, choose $b_{1a}=1$.
4) Transmit $\mathbf{x}_1$.

Encoder 2: Transmit $\mathbf{x}_2$.

Decoders: Decoder 1:

1) Given $\mathbf{y}_1$, try to find a pair $(\tilde{w}_c,\tilde{b}_c)$ such that $(\mathbf{u}_{1c}(\tilde{w}_c,\tilde{b}_c),\mathbf{y}_1) \in A_\epsilon(P_{U_{1c}Y_1})$. If there is such a pair, decide $\hat{w}_c=\tilde{w}_c$. If not, declare an error.
2) Try to find a pair $(\tilde{w}_{1a},\tilde{b}_{1a})$ such that $(\mathbf{u}_{1a}(\tilde{w}_c,\tilde{b}_c,\tilde{w}_{1a},\tilde{b}_{1a}),\mathbf{u}_{1c}(\tilde{w}_c,\tilde{b}_c),\mathbf{y}_1) \in A_\epsilon(P_{U_{1a}U_{1c}Y_1})$. If there is such a pair, decide $\hat{w}_{1a}=\tilde{w}_{1a}$. If not, declare an error.

Decoder 2:

1) Given $\mathbf{y}_2$, look for a unique $\hat{w}'_{2a}$ such that $(\mathbf{x}_{2a}(\hat{w}'_{2a}),\mathbf{y}_2) \in A_\epsilon(P_{X_{2a}Y_2})$. If there is no such unique $\hat{w}'_{2a}$, declare an error.
2) Try to find a pair $(\tilde{w}'_c,\tilde{b}'_c)$ such that $(\mathbf{u}_{1c}(\tilde{w}'_c,\tilde{b}'_c),\mathbf{x}_{2a}(\hat{w}'_{2a}),\mathbf{y}_2) \in A_\epsilon(P_{U_{1c}X_{2a}Y_2})$. If there is such a pair, decide $\hat{w}'_c=\tilde{w}'_c$. If not, declare an error.
3) Decide on unique $\hat{w}'_{2b}$ such that $(\mathbf{x}_{2b}(\hat{w}'_{2a},\hat{w}'_{2b}),\mathbf{u}_{1c}(\tilde{w}'_c,\tilde{b}'_c),\mathbf{x}_{2a}(\hat{w}'_{2a}),\mathbf{y}_2) \in A_\epsilon(P_{X_{2b}U_{1c}X_{2a}Y_2})$. If there is no such unique $\hat{w}'_{2b}$, declare an error.

Analysis: Assume $(w_{1a}, w_c, w_{2a}, w_{2b}) = (1,1,1,1)$ was sent. To guarantee that encoder 1 can find a b_c such that $(\mathbf{u}_{1c}(w_c, b_c), \mathbf{x}_{2a}, \mathbf{x}_{2b}) \in A_\epsilon(P_{U_{1c}X_{2a}X_{2b}})$, with probability close to 1 when N is large, requires [18, Thm.8.6.1.]

$$R_c' > I(U_{1c}; X_{2a}, X_{2b}). \tag{44}$$

To guarantee that, for a given (w_c, b_c), encoder 1 finds a b_{1a} such that $(\mathbf{u}_{1a}(w_c, b_c, w_{1a}, b_{1a}), \mathbf{x}_{2a}, \mathbf{x}_{2b}, \mathbf{u}_{1c}(w_c, b_c)) \in A_\epsilon(P_{U_{1a}X_{2a}X_{2b}U_{1c}})$, with probability close to 1 when N is large, requires

$$R_{1a}' > I(U_{1a}; X_{2a}, X_{2b}|U_{1c}). \tag{45}$$

One can easily show that the error event $\hat{w}_c \neq 1$ occurs at decoder 1 with arbitrarily small error probability if

$$R_c + R_c' < I(U_{1c}; Y_1). \tag{46}$$

The error event $\hat{w}_{1a} \neq 1$ has arbitrarily small error probability if

$$R_{1a} + R_{1a}' < I(U_{1a}; Y_1|U_{1c}). \tag{47}$$

From (45) and (47), bound (39) follows.

One can similarly show that the error event $\hat{w}_{2a}' \neq 1$ at decoder 2 occurs with arbitrarily small probability if (37) holds.

Next consider

$$P[\hat{w}_c' \neq 1] = \sum_{w_c=2}^{2^{NR_c}} \sum_{b_c=1}^{2^{NR_c'}} P[(\mathbf{U}_{1c}(w_c, b_c), \mathbf{X}_{2a}(1), \mathbf{Y}_2) \in A_\epsilon(P_{U_{1c}X_{2a}Y_2})].$$

Consider

$$\begin{aligned} P[(\mathbf{U}_{1c}(w_c, b_c), \mathbf{X}_{2a}(1), \mathbf{Y}_2) \in A_\epsilon(P_{U_{1c}X_{2a}Y_2})] \\ = \sum_{(\mathbf{u}_{1c}, \mathbf{x}_{2a}, \mathbf{y}_2) \in A_\epsilon} P[\mathbf{u}_{1c}] P[\mathbf{x}_{2a}, \mathbf{y}_2] \\ \leq 2^{-NI(U_{1c}; Y_2, X_{2a})} \end{aligned} \tag{48}$$

requiring

$$R_c + R_c' < I(U_{1c}; Y_2, X_{2a}). \tag{49}$$

From (44), (46) and (49), the bound (40) follows.

Finally we consider the error event $\hat{w}_{2b}' \neq 1$.

$$P[\hat{w}_{2b}' \neq 1] = \sum_{w_{2b}=2}^{2^{NR_{2b}}} P[(\mathbf{X}_{2b}(1, \hat{w}_{2b}), \mathbf{U}_{1c}(1,1), \mathbf{X}_{2a}(1), \mathbf{Y}_2) \in A_\epsilon(P_{X_{2b}U_{1c}X_{2a}Y_2})].$$

We have

$$\begin{aligned} P[(\mathbf{X}_{2b}(1, \hat{w}_{2b}), \mathbf{U}_{1c}(1,1), \mathbf{X}_{2a}(1), \mathbf{Y}_2) \in A_\epsilon(P_{X_{2b}U_{1c}X_{2a}Y_2})] \\ = \sum_{(\mathbf{x}_{2b}, \mathbf{u}_{1c}, \mathbf{x}_{2a}, \mathbf{y}_2) \in A_\epsilon} P[\mathbf{x}_{2a}] P[\mathbf{x}_{2b}|\mathbf{x}_{2a}] P[\mathbf{u}_{1c}, \mathbf{y}_2|\mathbf{x}_{2a}] \\ \leq 2^{-NI(X_{2b}; Y_2, U_{1c}|X_{2a})} \end{aligned} \tag{50}$$

requiring

$$R_{2b} < I(X_{2b}; Y_2, U_{1c}|X_{2a}). \tag{51}$$

Note that the transmitted U_{1c} and the received Y_2 are independent of not-transmitted X_{2b} when conditioned on the transmitted X_{2a}. ■

III. Gaussian Channel

We consider the Gaussian interference channel

$$Y_1 = X_1 + aX_2 + Z_1 \tag{52}$$
$$Y_2 = bX_1 + X_2 + Z_2 \tag{53}$$

where $Z_t \sim [0,1]$ and $E[X_k^2] \leq P_k$. $k = 1, 2$. In the case of weak interference, i.e., $b \leq 1$, the capacity region was determined in [6], [7].

A. Outer Bound

In the Gaussian case, Thm. 2 yields the following bound.

Corollary 1: When $b \geq 1$, any achievable rate pair (R_1, R_2) satisfies

$$R_1 \leq C((1-\rho^2)P_1) \tag{54}$$
$$R_1 + R_2 \leq C(P_1 + b^2 P_2 + 2\rho\sqrt{b^2 P_1 P_2}) \tag{55}$$

for some ρ, $0 \leq \rho \leq 1$, where

$$C(x) = \frac{1}{2}\log(1+x). \tag{56}$$

Remark: The bound reflects the fact that, because decoder 2 experiences strong interference, it can decode W_1 with no rate penalty.

B. Inner Bound

We start with a simple encoding scheme that is a special case of Thm. 3. We choose X_2 according to $\mathcal{N}[0, P_2]$ and

$$X_1 = X_{1c} + \sqrt{\frac{\beta P_1}{P_2}} X_2 \tag{57}$$

where X_{1c} is Gaussian, independent of X_2 and has variance $\bar{\beta}P_1$. We denote $c = \sqrt{\beta P_1/P_2}$ and choose

$$U_{1c} = X_{1c} + \alpha(a+c)X_2 \tag{58}$$

where $\alpha = \bar{\beta}P_1/(\bar{\beta}P_1 + 1)$.

Channel outputs (52)-(53) can be written as

$$Y_1 = U_{1c} + (1-\alpha)(a+c)X_2 + Z_1 \tag{59}$$
$$Y_2 = bU_{1c} + (1 + bc - \alpha b(a+c))X_2 + Z_2. \tag{60}$$

When

$$I(U_{1c}; Y_1) \leq I(U_{1c}; Y_2) \tag{61}$$

decoder 2 can decode W_1 as in [11], and form an observation

$$\hat{Y}_2 = Y_2 - bU_{1c}(\hat{W}_1') = (1 + bc - \alpha b(a+c))\, X_2 + Z_2. \tag{62}$$

From (59) and (62), the achievable rates are

$$R_1 = \frac{1}{2}\log_2(1 + \bar{\beta}P_1) \tag{63}$$

$$R_2 = I(X_2; \hat{Y}_2, U_{1c}) = \frac{1}{2}\log(1 + hP_2) \tag{64}$$

where

$$h = \frac{1}{(\bar{\beta}P_1 + 1)^2}\left((1 + bc + \bar{\beta}P_1(1-ab))^2 + (a+c)^2\bar{\beta}P_1\right).$$

When (61) does not hold, decoder 2 cannot decode W_1 from (62), before decoding W_2. Then, rate splitting at encoder 1, as in Thm. 3, can help. Also, following the approach in Thm. 3, rate splitting is done at encoder 2. We next present the details of the encoding scheme.

We choose X_{2a} and X_{2b} to be independent and distributed according to $\mathcal{N}[0, \eta P_2]$ and $\mathcal{N}[0, \bar{\eta} P_2]$, respectively, where $0 \leq \eta \leq 1$. We have

$$X_2 = X_{2a} + X_{2b}. \tag{65}$$

The channel input at the encoder 1 is chosen as

$$X_1 = X_{1c} + X_{1a} + cX_{2b} \tag{66}$$

where $c = \sqrt{\beta P_1 / \bar{\eta} P_2}$, X_{1a} is independent of X_{2a}, X_{2b}, X_{1c} and distributed according to $\mathcal{N}[0, \bar{\gamma}\bar{\beta}P_1]$. We explain how to choose X_{1c} next. We also let

$$U_{1a} = X_{1a} + \alpha_{1a}[aX_{2a} + (a+c)X_{2b}] \tag{67}$$

where $\alpha_{1a} = \bar{\gamma}\bar{\beta}P_1/(\bar{\gamma}\bar{\beta}P_1 + 1)$.

We remark that although the encoding scheme may seem involved, the role of each codebook can be clearly identified as:

- Transmitting X_{2a} lets decoder 2 decode part of W_2 to reduce the interference before decoding W_c. X_{2a} further serves as an observation when decoding W_c, as suggested by the expression $I(U_{1c}; Y_2, X_{2a})$ in (40).
- X_{2b} is a part that is beamformed to decoder 2 from two encoders. Encoder 1 dedicates βP_1 portion of its power for X_{2b}.
- X_{1a} is dirty paper coded (DPC) against interference X_{2a}, X_{2b} at decoder 1. It has power $\bar{\gamma}\bar{\beta}P_1$.
- X_{1c} carries common message W_c and is precoded against interference. As two channels experience different interference, the method of [12] is used.

Using (66) and (65), the received signals (52)-(53) become:

$$Y_1 = X_{1c} + (a+c)X_{2b} + aX_{2a} + X_{1a} + Z_1 \tag{68}$$

$$Y_2' = \frac{Y_2}{b} = X_{1c} + \frac{1}{b}X_{2a} + (\frac{1}{b} + c)X_{2b} + X_{1a} + \frac{Z_2}{b} \tag{69}$$

Index W_c is sent to both receivers by precoding against the interference

$$S_1 = (a+c)X_{2b} + aX_{2a} \tag{70}$$

$$S_2 = (\frac{1}{b} + c)X_{2b} \tag{71}$$

and treating X_{1a} as additional noise. We denote

$$Z_1' = X_{1a} + Z_1 \tag{72}$$

$$Z_2' = X_{1a} + Z_2/b \tag{73}$$

and $N_k' = E[Z_k'^2]$. Note that decoder 2 decodes W_{2a} prior to decoding W_c and can therefore subtract X_{2a} from its received signal. Using (70)-(73), in (68)-(69) yields

$$Y_1 = X_{1c} + S_1 + Z_1' \tag{74}$$

$$Y_2'' = Y_2' - \frac{X_{2a}}{b} = X_{1c} + S_2 + Z_2' \quad . \tag{75}$$

We next generalize the approach of [12] to allow for correlated interference, and different variance of interference and noise, as in (74)-(75). We choose S, V_1, V_2 such that

$$S_1 = S + V_1 \tag{76}$$

$$S_2 = S + V_2 \tag{77}$$

$$E[(V_1 + Z_1')^2] = E[(V_2 + Z_2')^2] \tag{78}$$

to obtain

$$Y_1 = X_{1c} + S + V_1 + Z_1' \tag{79}$$

$$Y_2'' = X_{1c} + S + V_2 + Z_2'. \tag{80}$$

Following [12], we split $w_c = (w_s, w_v)$ and let

$$X_{1c} = X_s + X_v \tag{81}$$

where X_s and X_v are independent, Gaussian with respective variances P_s and P_v, and $P_s + P_v = \gamma\bar{\beta}P_1$. We choose

$$U_s = X_s + \alpha_s S \tag{82}$$

achieving the rates at two decoders

$$R_{s1}(\alpha_s) = I(U_s; Y_1) - I(U_s; S) \tag{83}$$

$$R_{s2}(\alpha_s) = I(U_s; Y_2'', X_{2a}) - I(U_s; S). \tag{84}$$

Note that decoder 2 uses observation X_{2a} as in [11].

Both decoders decode w_s, reconstruct $\mathbf{u}_s(w_s)$ and form observations

$$\hat{Y}_1 = Y_1 - u_s = X_v + (1-\alpha_s)S + V_1 + Z_1' \tag{85}$$

$$\hat{Y}_2 = Y_2'' - u_s = X_v + (1-\alpha_s)S + V_2 + Z_2'. \tag{86}$$

From (85)-(86) we see that, with respect to signal X_v, a decoder k experiences interference $S_{vk} = (1-\alpha_s)S + V_k$. Hence, encoder 1 chooses for $k = 1, 2$

$$U_{vk} = X_v + \alpha_{vk} S_{vk} \tag{87}$$

where $\alpha_{vk} = P_v/(P_v + N_k')$ and time shares between the two codebooks. Decoders decode W_v based on (85)-(86).

This procedure results in a common rate

$$R_c = \max_{0 \leq \alpha_s \leq 1} \min\{R_{s1}(\alpha_s), R_{s2}(\alpha_s)\} + \max_{0 \leq t \leq 1} \min\{tR_{v1}, \bar{t}R_{v2}\} \tag{88}$$

where, for $k = 1, 2$, R_{sk} are given by (83)-(84) and

$$R_{vk} = \frac{1}{2}\log\left(1 + \frac{P_v}{N_k'}\right). \tag{89}$$

Note that α_s will be chosen such that $R_{s1}(\alpha_s) = R_{s2}(\alpha_s)$. Using (82), rate (83) evaluates to

$$R_{s1}(\alpha_s) = \tag{90}$$

$$\frac{1}{2}\log\left(\frac{P_s(P_s + Q + N + 2\rho_1\sqrt{QN})}{P_sQ\bar{\alpha}_s^2 + N(P_s + \alpha_s^2 Q) - \alpha_s^2\rho_1^2 QN + 2P_s\rho_1\bar{\alpha}_s\sqrt{QN}}\right)$$

where $Q = E[S^2]$, $N = E[(X_v + V_1 + Z_1')^2] = E[(X_v + V_2 + Z_2')^2]$ and ρ_1 is the correlation coefficient of S and 'noise' $X_v + V_1 + Z_1'$ at receiver 1. For $\rho_1 = 0$, (90) reduces to rate as [14, Eq. (6)]. We can similarly evaluate the rate (84).

Rate R_{2a} in (37) evaluates to

$$R_{2a} \leq C\left(\frac{\eta P_2}{b^2\bar{\beta}P_1 + (1+bc)^2\bar{\eta}P_2 + 1}\right). \tag{91}$$

After decoding w_c the encoder 1 achieves the rate as if the interference was not present:

$$R_{1a} \leq C(\bar{\gamma}\bar{\beta}P_1). \tag{92}$$

From (38) we can similarly evaluate rate R_{2b}.

The achievable rates can now be optimized over the choice of different power allocations. In our future work, we plan to compare the presented achievable strategy and outer bounds.

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