An Achievable Rate Region for Interference Channels with One Cooperating Transmitter

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***Abstract*—A previously established inner bound on the capacity region of two-sender, two-receiver interference channels where one transmitter knows both messages is evaluated for Gaussian channels. The inner bound is based on strategies that generalize prior work, and include rate-splitting, Gel'fand-Pinsker coding and cooperative transmission. The bound is shown to outperform existing schemes. In this context, coding against the interference known at the transmitter is also discussed. It is shown that, when the interference is a codebook, superposition coding can outperform Gel'fand-Pinsker binning.**

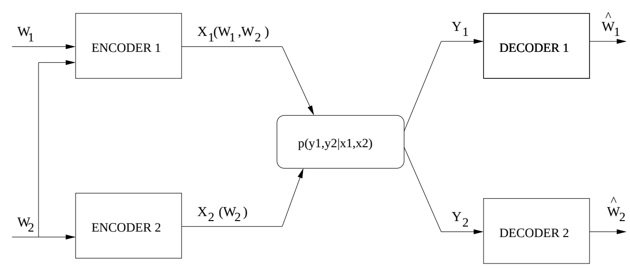

Fig. 1. Interference channel with cooperating encoder.

I. Introduction and Related Work

Two-sender, two-receiver channel models allow for various forms of transmitter cooperation. This paper considers a channel model in which one user knows the full message of the other encoder, as shown in Fig. 1. The transmitter with extra knowledge is referred to as being cognitive. This channel model was dubbed the *cognitive radio channel* and achievable rates were presented in [1], [2]. The capacity region for the Gaussian case of *weak* interference was determined in [3] and [4]. This result was extended to the Gaussian MIMO cognitive radio channel and shown to achieve the sum-capacity in [5]. The capacity region in *strong* interference was determined in [6]. The above two solved regimes do not exhaust all cases and the capacity is in general unknown. A general encoding scheme was proposed more recently in [7]. Related work can be found in [8], [9]. In [10], we proposed a scheme that generalizes those in [3] and [4]. In the scheme, as in [7] and [1], an encoder uses *rate-splitting* [11] to enable the other receiver to decode part of the interference; the cognitive transmitter cooperates in sending the other user's message to its intended receivers and uses Gel'fand-Pinsker (GP) binning [12] to reduce interference to its own receiver. The key difference of our contribution to the prior work is in the way the binning is performed. An overview of the encoding scheme is given in the next section and stated precisely in Sec. IV. In Sec. V, this scheme is specialized to the Gaussian channels, compared to the scheme of [7], and to the outer bound, [13]. Results demonstrate an improvement compared to the general scheme of [7].

In this context, we also discuss coding against the interference known at the transmitter. We show that, when the interference is a codebook, superposition coding can outperform Gel'fand Pinsker binning. Superposition coding achieves the capacity of the cognitive radio channel if receiver 1 is restricted to decode both messages, [14], [15].

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II. Overview of the Encoding Strategy

The considered channel model has elements of both the interference channel (IC) and the broadcast channel (BC). Encoding techniques developed for either of them are therefore potentially useful. If the message W_2 of encoder 2 was not known at the cognitive encoder, the considered channel would reduce to the IC. The best achievable rate region for the IC, [16], is achieved by rate-splitting [11]: each encoder divides its message into two parts and encodes each of them with a separate codebook. This allows receivers to decode one of the two sub-messages of the other user's and cancel a part of the interference that it would otherwise create. Rate-splitting in the cognitive radio channel model was applied in [1], [7]. In this paper, rate-splitting is performed at the cognitive encoder.

Additional knowledge allows the cognitive encoder to employ a number of techniques in addition to rate-splitting. To improve the rate for the noncognitive communicating pair, the cognitive encoder can *cooperate* by encoding W_2 to help convey it to the other decoder. On the other hand, any signal carrying information about W_2 creates interference to receiver 1. This interference is known at the cognitive transmitter and the precoding, i.e. GP binning and, specifically, dirty-paper coding (DPC) [17] in Gaussian channels, can be employed. In

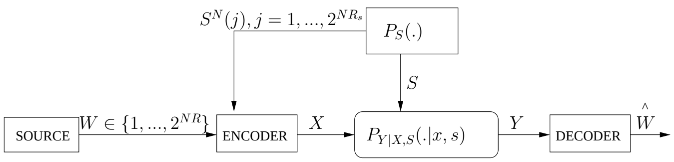

Fig. 2. Communication from the cognitive transmitter to the corresponding receiver.

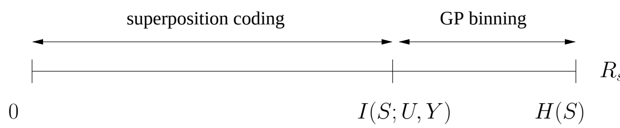

Fig. 3. Binning against a codebook.

fact, GP binning is crucial for the cognitive radio channel: together with cooperation, it leads to capacity in certain scenarios, [3], [4], [5]. It is not surprising that DPC brings gains in the Gaussian case: if the non-cognitive encoder is silent, we have the broadcast channel from the cognitive encoder to two receivers, for which DPC is optimal [18], [19].

In general, however, there are two differences at the cognitive encoder from the classical GP setting. First, the interference carries useful information for receiver 2. Second, the interference is a *codebook* of some rate and can thus have lower entropy than in the GP setting. As shown in Sec. II-B, the latter can be exploited to achieve a higher rate.

A. Summary of Techniques and Special Cases

Although the interference channel with one cooperating encoder can easily be visualized as an extension of the classical IC, a number of techniques become potentially relevant due to additional knowledge of the cognitive encoder:

- Rate splitting at encoder 1: Improves rate R_2 through interference cancelation at decoder 2.
- GP binning and binning against a codebook: Improves rate R_1 by precoding against interference. It also allows decoder 1 to decode message W_2 (or part of it) when R_2 is small, as will be shown in Sec. II-B.
- Carbon-copying onto dirty paper: further improves the rate of the common message sent at the cognitive encoder.
- Cooperation: Encoder 1 improves R_2 by encoding W_2.

A general encoding scheme that brings these techniques together is described in Section IV. There will be number of special cases for which a subset of techniques will suffice:

1) Strong interference: Both decoders can decode both messages with no rate penalty, so there is no need for either rate-splitting or binning. Superposition coding achieves capacity, [6].
2) Cognitive encoder decodes both messages: Again, there is no need for binning. Rate-splitting and superposition coding achieve capacity, [15], [14].
3) Weak interference at receiver 2: There is no need for rate-splitting. DPC and cooperation achieve capacity in Gaussian channel, [3], [4], [5].

B. Rate Improvement due to Binning Against Codebook

For the communication between the cognitive transmitter and its corresponding receiver, a codebook carrying W_2 creates interference. The situation is depicted in Fig 2, where S plays the role of the codebook of rate R_s interfering with the communication of message W at rate R. While in the GP problem the interference S is generated by a discrete memoryless source (DMS), the interference in the cognitive setting is a *codebook* of some rate, R_s. The next lemma reflects the fact that when R_s is small, this can be exploited for potential rate gains.

Lemma 1: In the communication setting of Fig. 3, the rate

$$R \le \max_{P_{U|S}, f(\cdot)} \min\{I(X;Y|S), \max\{I(U,S;Y) - R_s, I(U;Y) - I(U;S)\}\} \quad (1)$$

is achievable.

For $I(S;U,Y) \le R_s \le H(S)$, binning achieves the GP rate given by the second term in (1). For $R_s \le I(S;U,Y)$, superposition coding achieves the rate of the first term in (1). The two cases are shown in Fig. 3.

Proof: See [20]. ■

Remark 1: Rate (1) can be written as

$$R \le \max_{P_{U|S}, f(\cdot)} \{I(X,S;Y) - \max\{I(S;Y), \min\{R_s, I(U,Y;S)\}\}. \quad (2)$$

From (1) and (2), we observe that $I(S;U,Y) \le R_s \le H(S)$, corresponds to the classical GP setting. Potential rate improvement comes for $R_s \le I(S;U,Y)$. Interestingly, in this case the receiver decodes both indexes (w,j), where $w = 1,\dots,2^{NR}$ and $j = 1,\dots,2^{NR_s}$ (see Fig. 2), thus learning both its message and the interference. A related setting in which both data and the channel state information is communicated to the receiver was analyzed in [21], [22].

In the setting of Fig. 1, index j carries information about W_2. The implication is that, when R_s is small, receiver 1 will decode a part (or the whole) of W_2 without having encoder 2 rate split to send common message in the sense of [11], [16].

Recall that, due to rate-splitting, encoder 1 uses two codebooks to send a common and a private index, respectively denoted (U_{1c}^N, U_{1a}^N). We distinguish four cases depending on whether they are generated using binning or superposition:

1) Binning: Both (U_{1c}^N, U_{1a}^N) are binned against the codebook of the non-cognitive encoder, X_2^N.
2) Superposition coding: (U_{1c}^N, U_{1a}^N) superimposed on X_2^N.
3) Binning then superposition coding: U_{1c}^N is binned against X_2^N, and U_{1a}^N is superimposed on (X_2^N, U_{1c}^N).

4) Superposition coding then binning: U_{1c}^N superimposed on X_2^N; U_{1a}^N on U_{1c}^N and binned against X_2^N.

In the last two cases, decoder 1 can decode W_2 due to superposition coding of U_{1a}^N or U_{1c}^N on X_2^N, as shown in Lemma 1. The setting thus corresponds to the *cognitive radio with the degraded message sets* for which superposition coding achieves the capacity [15], [14]. The two last cases can therefore bring no improvement; the achievable region is the union of two rate regions, achieved by binning or superposition coding. We state these regions after formally defining the problem in the next section. We remark that in the above scheme, U_{1a}^N is superimposed on U_{1c}^N. The other encoding choice would be to use binning for U_{1a}^N against the codebook U_{1c}^N.

As the final point, we note that encoder 2 also uses rate-splitting and forms two codebooks (X_{2a}^N, X_{2b}^N) using superposition coding. When encoder 1 is binning against both codebooks, it is not decoding a part of W_2. An interesting next step would be to choose respective rates R_{2a} and R_{2b} following Lemma 1 such that (U_{1a}^N, U_{1c}^N) are binned against one of the two codebooks, but superimposed on the other. That would facilitate decoding a part of W_2 at receiver 1.

III. Channel Model

Consider a channel with finite input and output alphabets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_1, \mathcal{Y}_2$, and a probability distribution $p(y_1, y_2|x_1, x_2)$, where $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$ are channel inputs and $(y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2$ are channel outputs. Each encoder t, $t = 1, 2$, wishes to send a message $W_t \in \{1, \ldots, M_t\}$ to decoder t in N channel uses. Message W_2 is also known at encoder 1 (see Fig. 1). The channel is memoryless and time-invariant.

An (M_1, M_2, N, P_e) code has two encoding functions $X_1^N = f_1(W_1, W_2)$, $X_2^N = f_2(W_2)$, two decoding functions $\hat{W}_t = g_t(Y_t^N)$, and an error probability $P_e = \max\{P_{e,1}, P_{e,2}\}$ where, for $t = 1, 2$, we have

$$P_{e,t} = \sum_{(w_1,w_2)} \frac{1}{M_1 M_2} P\left[g_t(Y_t^N) \neq w_t|(w_1, w_2) \text{ sent}\right]. \quad (3)$$

A rate pair (R_1, R_2) is achievable if, for any $\epsilon > 0$, there is an (M_1, M_2, N, P_e) code such that $M_t \geq 2^{NR_t}$, $t = 1, 2$, and $P_e \leq \epsilon$. The capacity region is the closure of the set of all achievable pairs (R_1, R_2).

IV. Achievable Rate Region

To obtain an inner bound, we employ rate splitting. We let

$$R_1 = R_{1a} + R_c \quad (4)$$

$$R_2 = R_{2a} + R_{2b} \quad (5)$$

for nonnegative $R_{1a}, R_c, R_{2a}, R_{2b}$ which we now specify.

In the encoding scheme, encoder 2 uses superposition coding with two codebooks X_{2a}^N, X_{2b}^N. Encoder 1 repeats the steps of encoder 2 and adds binning: it encodes the split message W_1 with two codebooks which are Gel'fand-Pinsker precoded against X_{2a}^N, X_{2b}^N. In particular:

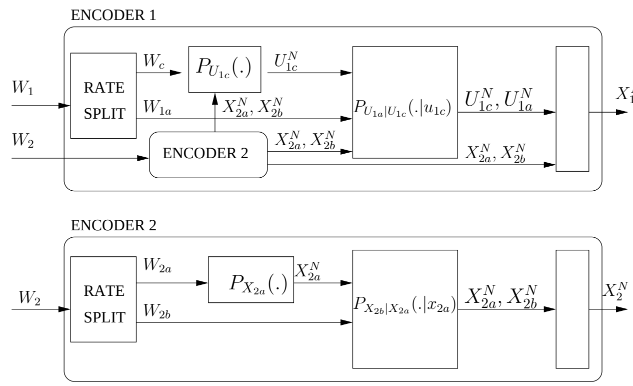

Fig. 4. Encoding structure.

1) Binning against X_{2a}^N, X_{2b}^N is used to create a codebook U_{1c}^N of common rate R_c.
2) Binning against X_{2a}^N, X_{2b}^N conditioned on U_{1c} is used to create a codebook U_{1a}^N with private rate R_{1a}.

The encoding structure is shown in Fig. 4. We have the following result, [10].

Theorem 1: (joint decoding) Rates (4)-(5) are achievable if

$$R_{1a} \leq I(U_{1a}; Y_1|U_{1c}, Q) - I(U_{1a}; X_{2a}, X_{2b}|U_{1c}, Q) \quad (6)$$

$$R_1 \leq I(U_{1c}, U_{1a}; Y_1|Q) - I(U_{1c}, U_{1a}; X_{2a}, X_{2b}|Q) \quad (7)$$

$$R_2 \leq I(X_2; Y_2, U_{1c}|Q) \quad (8)$$

$$R_2 + R_c \leq I(X_2, U_{1c}; Y_2|Q) \quad (9)$$

$$R_{2b} \leq I(X_{2b}; Y_2, U_{1c}|X_{2a}, Q) \quad (10)$$

$$R_{2b} + R_c \leq I(X_{2b}, U_{1c}; Y_2|X_{2a}, Q) \quad (11)$$

for some joint distribution that factors as

$$p(q)p(x_{2a}, x_{2b}, u_{1c}, u_{1a}, x_1, x_2|q)p(y_1, y_2|x_1, x_2) \quad (12)$$

and for which all the right-hand sides are nonnegative.

Proof: See [20]. ∎

Remark 2: Thm. 1 includes the following schemes:

- The scheme of [3, Thm 3.1] for $X_{2a} = \emptyset, U_{1c} = \emptyset, X_{2b} = (X_2, U)$ and $U_{1a} = V$.
- The scheme of [4, Lemma 4.2] for $X_{2a} = \emptyset$, $X_{2b} = X_2$, $U_{1a} = \emptyset$, and $R_1 = R_c$, $R_2 = R_{2b}$ and $p(x_2)p(u_{1c})$. The scheme in [4] assumes $I(U_{1c}; Y_1) \leq I(U_{1c}; Y_2)$.
- Carbon-copy on dirty paper [23] for $X_{2a} = \emptyset, U_{1a} = \emptyset$.

For $X_{2a} = \emptyset$, our scheme closely resembles the scheme in [7]. The first difference in our scheme is that two binning steps are not done independently which brings potential improvements. The second difference is in the evaluation of error events.

The scheme in [1] performs rate splitting at both users, and two-step binning at the cognitive user. Each user sends a private index decoded by its receiver, and a common index decoded by both. Again, one difference in our scheme is that two binning steps are not independent. The other is that in our scheme the cognitive encoder cooperates by encoding W_2.

The next rate region is obtained by exploiting Lemma 1.

A. An Achievable Rate Region with Superposition Coding

Consider a joint distribution (12) and rate R_2 that satisfy

$$R_2 \le I(X_2; U_{1c}, Y_1) \quad (13)$$
$$R_2 \le I(X_2; U_{1a}, Y_1|U_{1c}). \quad (14)$$

From Lemma 1, we know that under respective conditions (13) and (14), superposition of U_{1c}^N and U_{1a}^N with X_2^N should be used instead of binning. The encoding scheme of the cognitive encoder reduces to rate-splitting and superposition coding. The scheme and the rates reduce to that of [7, Thm.5]. We restate the result for completeness. Achievable rates (R_1, R_2) satisfy

$$\begin{aligned} R_{1a} &\le I(X_1; Y_1|X_2, U_{1c}) \\ R_1 &\le I(X_1; Y_1|X_2) \\ R_1 + R_2 &\le I(X_1, X_2; Y_1) \\ R_c + R_2 &\le I(U_{1c}, X_2; Y_2) \end{aligned} \quad (15)$$

for some joint input distribution $p(x_2, u_{1c}, x_1)$.

Remark 3: The above region can further be simplified using Fourier-Motzkin elimination [24] as in [15]. This region is the capacity region for the cognitive radio with degraded message sets: the converse follows from [14] where a more general case of confidential messages is analyzed. The result follows by considering the special case of no security.

V. Gaussian Channel

To illustrate the obtained results more concretely, we next consider the Gaussian interference channel described by

$$Y_1 = X_1 + aX_2 + Z_1 \quad (16)$$
$$Y_2 = bX_1 + X_2 + Z_2 \quad (17)$$

where $Z_t \sim \mathcal{N}[0, 1]$ and $E[X_t^2] \le P_t$, $t = 1, 2$. For $b \le 1$, the capacity region was determined in [3], [4].

We next evaluate the rates of Thm. 1 for the special case $X_{2a} = \emptyset$, $Q = \emptyset$. To simplify them, we express the conditional entropies in terms of joint entropies, recall that $R_1 = R_c + R_{1a}$, and apply Fourier-Motzkin elimination to obtain

$$\begin{aligned} R_1 &\le I(U_{1c}, U_{1a}; Y_1) - I(U_{1c}, U_{1a}; X_2) \\ R_2 &\le I(X_2; Y_2, U_{1c}) \\ R_2 &\le I(X_2, U_{1c}; Y_2) \\ R_1 + R_2 &\le I(X_2, U_{1c}; Y_2) + I(U_{1a}; Y_1, U_{1c}) \\ &\quad - I(U_{1a}; X_2, U_{1c}). \end{aligned} \quad (18)$$

We evaluated region (18) for

$$X_2 \sim \mathcal{N}[0, P_2], \quad X_{1c} \sim \mathcal{N}[0, \alpha\beta P_1], \quad X_{1a} \sim \mathcal{N}[0, \alpha\bar{\beta} P_1]$$

$$\begin{aligned} &U_{1c} = X_{1c} + \lambda_1 X_2, \quad U_{1a} = X_{1a} + \lambda_2 X_2 \\ &X_1 = X_{1c} + X_{1a} + \sqrt{\frac{\bar{\alpha} P_1}{P_2}} X_2 \end{aligned} \quad (19)$$

where $0 \le \alpha, \beta \le 1$ and $0 \le \lambda_1, \lambda_2$. Parameters $\bar{\alpha}$ and β determine the amount of power that the cognitive user uses respectively for cooperation and for the common message.

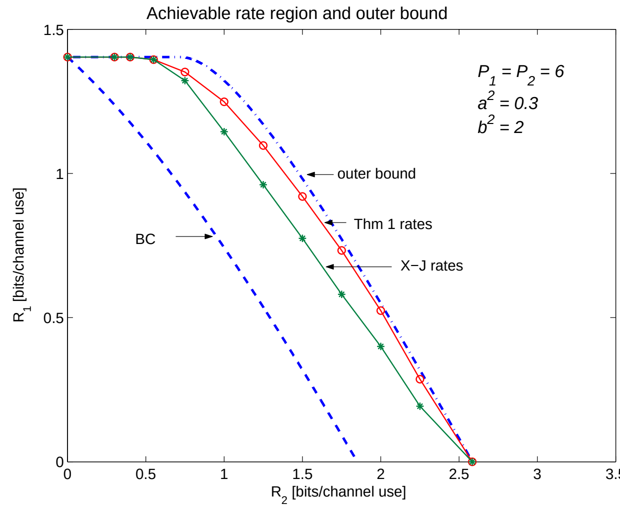

Fig. 5. Rates of Thm. 1 and [7, Thm.5] and outer bound of Cor. 1. Also shown is the capacity region of BC from the cooperative encoder, $P_2 = 0$.

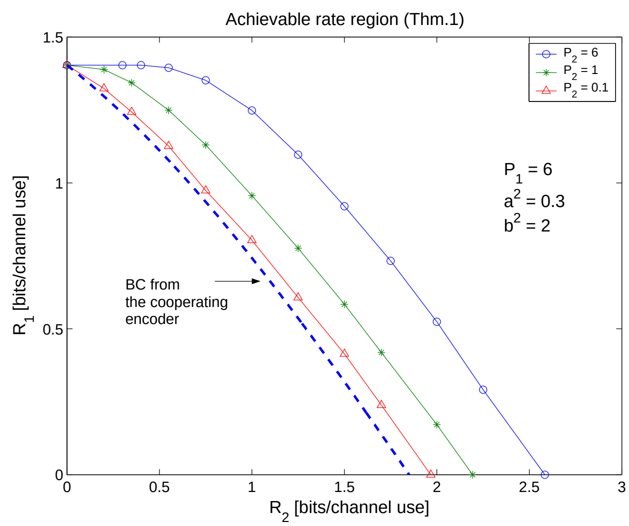

Fig. 6. Achievable rates for different values of P_2.

We compared the achievable region (18) to the outer bound of [13, Thm. 2] which in Gaussian channels evaluates to:

Corollary 1: When $b \ge 1$, any achievable rate pair (R_1, R_2) satisfies

$$\begin{aligned} R_1 &\le C((1 - \rho^2)P_1) \\ R_1 + R_2 &\le C(b^2 P_1 + P_2 + 2\rho\sqrt{b^2 P_1 P_2}) \end{aligned} \quad (20)$$

for some ρ, $0 \le \rho \le 1$, where $C(x) = \log(1 + x)/2$.

Fig. 5 shows the rate region (18) and the outer bound (20) for $a^2 = 0.3$, $b^2 = 2$ and equal powers $P_1 = P_2 = 6$. We observe higher rates of Thm. 1 compared to that of [7, Thm. 5], which uses the same Gaussian distributions as given by (19). When the encoder 2 does not transmit (i.e. $P_2 = 0$), the channel reduces to the BC in which only the cooperating encoder transmits to two receivers. The rates achieved in the

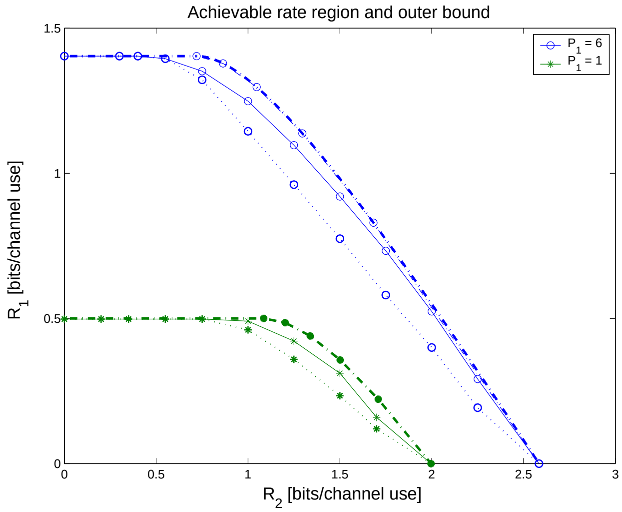

Fig. 7. Impact of reduced power of the cognitive transmitter to achievable rates. Rates achieved with Thm. 1 are shown in solid lines and rates of [7] are shown with dotted line. Dash-dotted line shows the outer bound.

BC are also shown. Unlike the BC rate region, the IC with one cooperating encoder region is flat for smaller values of R_2, reflecting the fact that the cognitive transmitter does not need to cooperate. It can instead use its full power to precode and transmit W_1 at the single-user rate as if the second user was not present. It starts cooperating only for higher R_2. At $R_1 = 0$, the cooperating encoder fully helps encoder 2, i.e. $\alpha = 0$ and user 2 benefits from the coherent combining gain. The achievable rates come close to the outer bound, especially for larger values of R_2, when the cognitive encoder dedicates more of its power to cooperate. Fig. 6 shows rates for different values of P_2 and fixed P_1. As P_2 decreases, the performance gets closer to rates in the BC with the cooperating encoder transmitting to two receivers. Since in the BC encoder 2 is not present, the rate region, given by the dashed line, does not depend on P_2. Fig. 7 shows the effect of reducing P_1 as compared to Fig. 5, with P_2 constant. This has a higher impact, drastically reducing R_1.

VI. Conclusions and Future Work

We evaluated an encoding strategy for the Gaussian interference channel with one cooperating encoder, that generalizes previously proposed encoding strategies. We compared it to the outer bound that we developed for the strong interference regime. It is unclear whether our strategy generalizes the scheme in [1], or whether a combination of the two techniques would achieve higher rates. The cognitive radio channel shares some characteristics of both interference channels and broadcast channels. Combining encoding strategies developed for either of the two channel models therefore seems a natural approach. The encoding scheme presented in this paper is a combination of rate-splitting, GP binning and superposition coding. However, the optimality of a particular encoding scheme seems to be in part dictated by the channel conditions. We believe that this general encoding scheme may be capacity-achieving for certain special cases related to the channel or specific encoding/decoding constraints. Finding such special cases is a topic of ongoing investigation.

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