Achievable Rate Regions for Broadcast Channels With Cognitive Relays

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***Abstract*— We investigate the fundamental limits of broadcast channels with cognitive relays. Specifically, we develop a new coding strategy and the associated achievable rate regions for the broadcast channel with two receivers and two additional cognitive relays. The new achievable rate regions exhibit potentially higher rates than existing schemes. We also consider a simplified model where there is only one cognitive relay. In this scenario we develop a general coding strategy that includes most existing schemes over the cognitive radio channel. We demonstrate the advantages associated with our generalized coding scheme for a broadcast channel with a single cognitive relay in Gaussian noise.**

I. Introduction

The demand for high-speed wireless applications has made efficient spectrum utilization critical for next-generation communication systems. Cognitive radio has emerged as a promising candidate to significantly improve spectrum utilization over existing technologies. As quantifying the fundamental limits of wireless networks is a long-standing open problem, the fundamental limits of most multi-terminal channels involving cognitive radios are also unknown. Nevertheless, much work has investigated such channels from an information theoretic perspective, where various coding strategies have been proposed to exploit the use of cognitive radios and push the achievable rate region towards the capacity region. In particular, two channel models have been explicitly studied in the literature: the cognitive radio channel (CRC) (also called the interference channel with one cognitive transmitter or the interference channel with degraded message sets) [1]–[6]; and the interference channel with one cognitive relay [7], [8].

Several coding schemes that combine rate splitting [9], Gel'fand-Pinsker coding [10], and superposition coding have been developed for the CRC, and the associated achievable rate regions have been derived [1]–[6]. Furthermore, capacity regions have been characterized for a few special cases [2], [3], [6], [11]. For the interference channel with a cognitive relay, only the Gaussian case has been investigated [7], [8]. In particular, a transmission scheme that combines beamforming and dirty paper coding was proposed in [7], which was later generalized in [8] with a coding scheme combining rate splitting, dirty paper coding, and superposition coding to enable cooperation among the nodes.

The model of the interference channel with a cognitive relay includes the CRC model as a special case in terms of the mathematical channel representation, and both of these models incorporate the two-user broadcast channel. In this paper, we view the two cognitive models as a broadcast channel with a cognitive relay or two cognitive relays, and study both of these models using a unified approach. We develop two generalized coding schemes for the case with two cognitive relays first. In contrast to the scheme in [8], we split the messages into two parts only, apply Gel'fand-Pinsker coding against all the known interference, and perform simultaneous joint decoding at both receivers. We show that the derived achievable rate regions contain the discrete memoryless counterpart of the Gaussian rate region presented in [8]. We then specialize our coding scheme to the case with one cognitive relay, and derive a new achievable rate region for this channel. We use a Gaussian example to demonstrate the strengths of the proposed coding scheme compared to existing schemes.

II. Channel Model and Definitions

As shown in Fig. 1, three nodes including the sender and two cognitive relays (relay 1 and relay 2) collaborate on sending two messages M_1 and M_2 to receiver 1 and receiver 2, respectively. The sender has knowledge of both messages, while each cognitive relay only knows one of the two messages. As such, the channel can be defined as $(\mathcal{X}_0 \times \mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_0, x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$, with $\mathcal{X}_t,\ t = 0, 1, 2$, denoting the channel input alphabets of each transmitting node, $\mathcal{Y}_t,\ t = 1, 2$, denoting the channel output alphabets at the two receivers, and $p(y_1, y_2|x_0, x_1, x_2)$ denoting the channel transition probabilities. The source messages $M_t,\ t = 1, 2$, are assumed to be uniformly generated over their respective ranges: $\mathcal{M}_t = \{1, 2, ..., \|\mathcal{M}_t\|\}$, $t = 1, 2$. The channel is also assumed to be memoryless. We call this channel the broadcast channel with cognitive relays (BCCR). Note that when we remove one of the relays, the proposed channel model reduces to the CRC model [1].

Definition 2.1: An $(\|\mathcal{M}_1\|, \|\mathcal{M}_2\|, n, P_e^{(n)})$ code for the BCCR consists of an encoding function at the sender $f_0 : \mathcal{M}_1 \times \mathcal{M}_2 \mapsto \mathcal{X}_0^n$, an encoding function at each cognitive relay $f_t : \mathcal{M}_t \mapsto \mathcal{X}_t^n,\ t = 1, 2$, and a decoding function at each receiver $g_t : \mathcal{Y}_t^n \mapsto \mathcal{M}_t,\ t = 1, 2$, with the probability of decoding error defined as $P_e^{(n)} = \max\{P_{e,1}^{(n)}, P_{e,2}^{(n)}\}$,

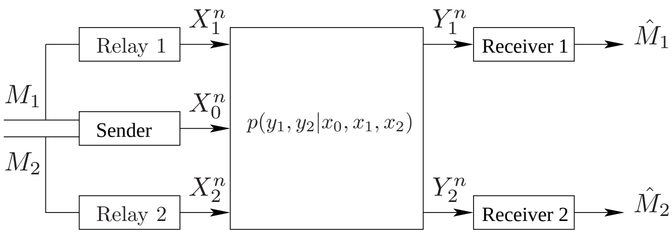

Fig. 1. Broadcast channel with two cognitive relays.

where the individual error probability at each receiver is computed as $P_{e,t}^{(n)} = \frac{1}{\|\mathcal{M}_1\|\|\mathcal{M}_2\|}\sum_{M_1\in\mathcal{M}_1, M_2\in\mathcal{M}_2} P(g_t(Y_t^n) \neq M_t|(M_1, M_2) \text{ were sent}),\ t = 1, 2.$

Definition 2.2: A non-negative rate pair (R_1, R_2) is achievable for the BCCR if there exists a sequence of codes $(2^{nR_1}, 2^{nR_2}, n, P_e^{(n)})$ for the channel such that $P_e^{(n)}$ approaches 0 as $n \to \infty$.

The capacity region of the BCCR is defined as the closure of the set of all achievable rate pairs, and the capacity region consists of the set of all achievable rate pairs.

III. Main Results

In this section, we first present two new achievable rate regions for the discrete memoryless BCCR. These new rate regions comprise the main results of this paper. We then compare these rate regions with the existing regions derived in [7], [8].

A. New Achievable Rate Regions for the BCCR

Let U_1, V_1, W_1, U_2, V_2, and W_2 be auxiliary random variables defined on the arbitrary finite alphabets, $\mathcal{U}_1$, $\mathcal{V}_1$, $\mathcal{W}_1$, $\mathcal{U}_2$, $\mathcal{V}_2$, and $\mathcal{W}_2$, respectively. Let $\mathcal{P}^*$ denote the set of all joint probability distributions defined over the auxiliary random variables and the channel inputs and outputs, each factoring in the form

$$p(u_1)p(u_2)p(v_1)p(v_2)p(w_1, w_2|v_1, v_2)p(x_1|u_1, v_1)p(x_2|u_2, v_2) \times p(x_0|w_1, w_2, v_1, v_2, u_1, u_2)p(y_1, y_2|x_0, x_1, x_2).$$

For each joint distribution $p(\cdot) \in \mathcal{P}^*$, define $\mathcal{R}^*(p(\cdot))$ as the set of all non-negative rate pairs $(R_{12} + R_{11}, R_{21} + R_{22})$ such that

$$R_{11} - L_{11} \le -I(W_1; V_2|V_1), \quad (1)$$
$$R_{22} - L_{22} \le -I(W_2; V_1|V_2), \quad (2)$$
$$R_{11} - L_{11} + R_{22} - L_{22} \le -(I(W_1; V_2|V_1) + I(W_2; V_1|V_2) + I(W_2; W_1|V_1, V_2)), \quad (3)$$
$$R_{12} \le I(U_1; Y_1|V_1, W_1, U_2), \quad (4)$$
$$R_{12} + L_{11} - R_{11} \le I(U_1, W_1; Y_1|V_1, U_2), \quad (5)$$
$$L_{11} \le I(V_1, W_1; Y_1|U_1, U_2), \quad (6)$$
$$R_{12} + L_{11} \le I(U_1, V_1, W_1; Y_1|U_2), \quad (7)$$
$$L_{11} + R_{21} \le I(V_1, W_1, U_2; Y_1|U_1), \quad (8)$$
$$R_{12} + R_{21} \le I(U_1, U_2; Y_1|V_1, W_1), \quad (9)$$
$$R_{12} + R_{21} + L_{11} - R_{11} \le I(U_1, U_2, W_1; Y_1|V_1), \quad (10)$$
$$R_{12} + L_{11} + R_{21} \le I(U_1, V_1, W_1, U_2; Y_1), \quad (11)$$
$$R_{21} \le I(U_2; Y_2|V_2, W_2, U_1), \quad (12)$$
$$R_{21} + L_{22} - R_{22} \le I(U_2, W_2; Y_2|V_2, U_1), \quad (13)$$
$$L_{22} \le I(V_2, W_2; Y_2|U_2, U_1), \quad (14)$$
$$R_{21} + L_{22} \le I(U_2, V_2, W_2; Y_2|U_1), \quad (15)$$
$$L_{22} + R_{12} \le I(V_2, W_2, U_1; Y_2|U_2), \quad (16)$$
$$R_{21} + R_{12} \le I(U_2, U_1; Y_2|V_2, W_2), \quad (17)$$
$$R_{21} + R_{12} + L_{22} - R_{22} \le I(U_2, U_1, W_2; Y_2|V_2), \quad (18)$$
$$R_{21} + L_{22} + R_{12} \le I(U_2, V_2, W_2, U_1; Y_2). \quad (19)$$

Theorem 3.1: Any rate pair $(R_1, R_2) \in \mathcal{R}^* \triangleq \bigcup_{p(\cdot)\in\mathcal{P}^*} \mathcal{R}^*(p(\cdot))$ is achievable for a BCCR.

Proof: Due to lack of space, we only present the coding scheme, without error probability analysis [12]. First, fix a joint distribution $p(\cdot) \in \mathcal{P}^*$. The messages are split into two messages each, i.e., $m_1 = (m_{12}, m_{11})$ and $m_2 = (m_{21}, m_{22})$. Each message m_{ab} is assumed to be of rate R_{ab}, $a, b \in 1, 2$.

[**Codebook generation.**] Generate $2^{nR_{12}}$ independent codewords of length n: $\mathbf{U}_1(m_{12})$, $m_{12} \in \{1, 2, ..., 2^{nR_{12}}\}$, according to $\prod_{i=1}^n p(u_{1i})$. Generate $2^{nR_{11}}$ independent codewords: $\mathbf{V}_1(m_{11})$, $m_{11} \in \{1, 2, ..., 2^{nR_{11}}\}$, according to $\prod_{i=1}^n p(v_{1i})$. For each codeword pair $(\mathbf{U}_1(m_{12}), \mathbf{V}_1(m_{11}))$, generate one codeword, $\mathbf{X}_1(m_{12}, m_{11})$, according to $\prod_{i=1}^n p(x_{1i}|v_{1i}(m_{11}), u_{1i}(m_{12}))$. In a symmetric manner, generate codewords $\mathbf{U}_2(m_{21})$, $\mathbf{V}_2(m_{22})$, and $\mathbf{X}_2(m_{21}, m_{22})$, with $m_{21} \in \{1, 2, ..., 2^{nR_{21}}\}$ and $m_{22} \in \{1, 2, ..., 2^{nR_{22}}\}$.

For each codeword $\mathbf{V}_1(m_{11})$, generate $2^{n(L_{11}-R_{11})}$ independent codewords: $\mathbf{W}_1(m_{11}, l_{11})$, $l_{11} \in \{1, 2, ..., 2^{n(L_{11}-R_{11})}\}$, according to $\prod_{i=1}^n p(w_{1i}|v_{1i}(m_{11}))$. For each codeword $\mathbf{V}_2(m_{22})$, generate $2^{n(L_{22}-R_{22})}$ independent codewords: $\mathbf{W}_2(m_{22}, l_{22})$, $l_{22} \in \{1, 2, ..., 2^{n(L_{22}-R_{22})}\}$, according to $\prod_{i=1}^n p(w_{2i}|v_{2i}(m_{22}))$. For each codeword tuple $(\mathbf{U}_1(m_{12}), \mathbf{U}_2(m_{21}), \mathbf{V}_1(m_{11}), \mathbf{V}_2(m_{22}), \mathbf{W}_1(m_{11}, l_{11}), \mathbf{W}_2(m_{22}, l_{22}))$, generate one codeword $\mathbf{X}_0(m_{12}, m_{21}, m_{11}, m_{22}, l_{11}, l_{22})$, according to

$$\prod_{i=1}^n p(x_{0i}|w_{1i}(m_{11}, l_{11}), w_{2i}(m_{22}, l_{22}), v_{1i}(m_{11}), v_{2i}(m_{22}), u_{1i}(m_{12}), u_{2i}(m_{21})).$$

[**Encoding and transmission.**] Suppose that messages $m_1 = (m_{12}, m_{11}) = (i_1, j_1)$ and $m_2 = (m_{21}, m_{22}) = (i_2, j_2)$ are to be transmitted. The two cognitive relays, relay 1 and relay 2, simply transmit codewords $\mathbf{X}_1(i_1, j_1)$ and $\mathbf{X}_2(i_2, j_2)$, respectively. The sender will first look for a pair of indices $(\hat{l}_{11}, \hat{l}_{22})$ such that $\left(\mathbf{V}_1(j_1), \mathbf{V}_2(j_2), \mathbf{W}_1(j_1, \hat{l}_{11}), \mathbf{W}_2(j_2, \hat{l}_{22})\right) \in \mathcal{A}_\epsilon^{(n)}$, where $\mathcal{A}_\epsilon^{(n)}$ denotes the typical set [13]. If found, the sender transmits the codeword $\mathbf{X}_0(i_1, i_2, j_1, j_2, \hat{l}_{11}, \hat{l}_{22})$; otherwise, it transmits the codeword $\mathbf{X}_0(i_1, i_2, j_1, j_2, 1, 1)$.

[**Decoding.**] Let $\mathbf{Y}_1$ denote the channel output sequence at receiver 1 when the transmission of one block is completed. Receiver 1 first looks for all message quadruples $(\hat{m}_{12}, \hat{m}_{11}, \hat{m}_{21}, \bar{l}_{11})$ such that

$$\left(\mathbf{U}_1(\hat{m}_{12}), \mathbf{V}_1(\hat{m}_{11}), \mathbf{W}_1(\hat{m}_{11}, \bar{l}_{11}), \mathbf{U}_2(\hat{m}_{21}), \mathbf{Y}_1\right) \in \mathcal{A}_\epsilon^{(n)}.$$

If all the quadruples refer to a common and unique message pair $(\hat{m}_{12}, \hat{m}_{11})$, receiver 1 declares that the decoded message is $\hat{m}_1 = (\hat{m}_{12}, \hat{m}_{11})$; otherwise, a decoding error is declared. The decoding process at receiver 2 is performed in a symmetrical manner.

The achievability of the rate region $\mathcal{R}^*(p(\cdot))$ follows from the analysis of error probabilities. ■

Remark 3.1: The advantage of this scheme is that the relations between the messages and the corresponding random variables can be easily observed. However the rate region description is relatively complex. Nevertheless, by with successive superposition encoding on codewords $\mathbf{V}_i$ and $\mathbf{W}_i$ by using codewords $\mathbf{U}_i$ as cloud centers, we obtain a potentially larger achievable rate region with a simpler description as follows.

With a slight abuse of notation, we reuse the notations of the respective auxiliary random variables. Define $\mathcal{P}$ as the set of all joint distributions, each factoring in the following form

$$\begin{aligned} &p(u_1)p(v_1|u_1)p(x_1|v_1,u_1)p(u_2)p(v_2|u_2)p(x_2|v_2,u_2) \\ &\times p(w_1,w_2|v_1,v_2,u_1,u_2)p(x_0|w_1,w_2,v_1,v_2,u_1,u_2) \\ &\times p(y_1,y_2|x_0,x_1,x_2). \qquad (20) \end{aligned}$$

For each joint distribution $p(\cdot) \in \mathcal{P}$, define $\mathcal{R}(p(\cdot))$ as the set of all the non-negative rate pairs $(R_{12}+R_{11}, R_{21}+R_{22})$ such that

$$R_{11} - L_{11} \le -I(W_1;V_2|V_1,U_1,U_2), \qquad (21)$$
$$R_{22} - L_{22} \le -I(W_2;V_1|V_2,U_1,U_2), \qquad (22)$$
$$\begin{aligned} R_{11} - L_{11} + R_{22} - L_{22} \le &-(I(W_1;V_2|V_1,U_1,U_2) \\ &+I(W_2;V_1|V_2,U_1,U_2) + I(W_2;W_1|V_1,V_2,U_1,U_2)), \qquad (23) \end{aligned}$$
$$L_{11} \le I(V_1,W_1;Y_1|U_1,U_2), \qquad (24)$$
$$R_{12} + L_{11} \le I(U_1,V_1,W_1;Y_1|U_2), \qquad (25)$$
$$L_{11} + R_{21} \le I(V_1,W_1,U_2;Y_1|U_1), \qquad (26)$$
$$R_{12} + L_{11} + R_{21} \le I(U_1,V_1,W_1,U_2;Y_1), \qquad (27)$$
$$L_{22} \le I(V_2,W_2;Y_2|U_2,U_1), \qquad (28)$$
$$R_{21} + L_{22} \le I(U_2,V_2,W_2;Y_2|U_1), \qquad (29)$$
$$L_{22} + R_{12} \le I(V_2,W_2,U_1;Y_2|U_2), \qquad (30)$$
$$R_{21} + L_{22} + R_{12} \le I(U_2,V_2,W_2,U_1;Y_2). \qquad (31)$$

Theorem 3.2: Any rate pair $(R_1, R_2) \in \mathcal{R} \triangleq \bigcup_{p(\cdot)\in\mathcal{P}} \mathcal{R}(p(\cdot))$ is achievable for a BCCR.

Proof: The underlying coding idea is the same as the one in Theorem 3.1, except that we perform successive superposition for the encoding. The proof is thus omitted. ■

Remark 3.2: The rate region $\mathcal{R}$ contains the rate region $\mathcal{R}^*$ in Theorem 3.1, i.e., $\mathcal{R}^* \subseteq \mathcal{R}$. The outline of the proof is as follows. Suppose that a set of random variables, $(U_1', V_1', U_2', V_2', W_1', W_2')$, satisfy a joint distribution $p'(\cdot) \in \mathcal{P}^*$. Construct the following random variables: $U_1 = U_1'$, $U_2 = U_2'$, $V_1 = (V_1', U_1')$, $V_2 = (V_2', U_2')$, $W_1 = (W_1', U_1', U_2')$, and $W_2 = (W_2', U_2', U_1')$. It is easy to verify that their joint distribution, denoted as $\tilde{p}(\cdot)$, belongs to $\mathcal{P}$. Next, evaluate the rate region $\mathcal{R}(\tilde{p}(\cdot))$ with respect to the joint distribution of the constructed random variables, over which we can easily verify that $\mathcal{R}^*(p'(\cdot)) \subseteq \mathcal{R}(\tilde{p}(\cdot))$. Since for each $p'(\cdot) \in \mathcal{P}^*$ we can construct one $\tilde{p}(\cdot) \in \mathcal{P}$, we conclude that $\mathcal{R}^* \subseteq \mathcal{R}$. However, whether the inclusion is strict is yet to be determined.

Remark 3.3: The derived achievable rate regions, $\mathcal{R}$ and $\mathcal{R}^*$, include the best known achievable rate regions for the general interference channel [9], [14] and the best achievable rate region for the general broadcast channel [15] as special cases. Taking $\mathcal{R}^*$ as an example, by setting $\mathcal{W}_1 = \mathcal{W}_2 = \emptyset$, the rate region reduces to the simplified Han-Kobayashi region for the interference channel [14]. On the other hand, by setting $\mathcal{V}_1 = \mathcal{V}_2 = \emptyset$ and merging U_1 and U_2 into one random variable W_0, which can be assigned to carry the partial information of either m_{11} or m_{22}, i.e., $W_0 = U_1 = U_2$, the rate region reduces to Marton's region for the broadcast channel [15].

B. Comparisons with Existing Rate Regions for the BCCR

In this section, we show that the new achievable rate regions presented in the previous section include the one in [8], which already contains the rate region in [7]. To illustrate this point, we first present the discrete memoryless counterpart of the Gaussian achievable rate region presented in [8].

For each joint distribution $p(\cdot) \in \mathcal{P}^*$, define $\mathcal{R}'(p(\cdot))$ as the set of all non-negative rate pairs $(R_{12}+R_{11a}+R_{11b}, R_{21}+R_{22a}+R_{22b})$ such that

$$R_{12} \le I(U_1;Y_1|V_1,U_2), \qquad (32)$$
$$R_{11a} \le I(V_1;Y_1|U_1,U_2), \qquad (33)$$
$$R_{21} \le I(U_2;Y_1|U_1,V_1), \qquad (34)$$
$$R_{12} + R_{11a} \le I(U_1,V_1;Y_1|U_2), \qquad (35)$$
$$R_{11a} + R_{21} \le I(V_1,U_2;Y_1|U_1), \qquad (36)$$
$$R_{12} + R_{21} \le I(U_1,U_2;Y_1|V_1), \qquad (37)$$
$$R_{12} + R_{11a} + R_{21} \le I(U_1,V_1,U_2;Y_1), \qquad (38)$$
$$R_{21} \le I(U_2;Y_2|V_2,U_1), \qquad (39)$$
$$R_{22a} \le I(V_2;Y_2|U_2,U_1), \qquad (40)$$
$$R_{12} \le I(U_1;Y_2|U_2,V_2), \qquad (41)$$
$$R_{21} + R_{22a} \le I(U_2,V_2;Y_2|U_1), \qquad (42)$$
$$R_{22a} + R_{12} \le I(V_2,U_1;Y_2|U_2), \qquad (43)$$
$$R_{21} + R_{12} \le I(U_2,U_1;Y_2|V_2), \qquad (44)$$
$$R_{21} + R_{22a} + R_{12} \le I(U_2,V_2,U_1;Y_2), \qquad (45)$$
$$R_{11b} - L_{11b} \le -I(W_1;V_2|V_1), \qquad (46)$$
$$R_{22b} - L_{22b} \le -I(W_2;V_1|V_2), \qquad (47)$$
$$L_{11b} \le I(W_1;Y_1|U_1,V_1,U_2), \qquad (48)$$
$$L_{22b} \le I(W_2;Y_2|U_2,V_2,U_1), \qquad (49)$$
$$\begin{aligned} R_{11b} - L_{11b} + R_{22b} - L_{22b} \le &-(I(W_1;V_2|V_1) \\ &+I(W_2;V_1|V_2) + I(W_2;W_1|V_1,V_2)). \qquad (50) \end{aligned}$$

Theorem 3.3: Any rate pair $(R_1, R_2) \in \mathcal{R}' \triangleq \bigcup_{p(\cdot)\in\mathcal{P}'} \mathcal{R}'(p(\cdot))$ is achievable for a BCCR.

Proof: The full proof is omitted due to lack of space. Instead, an outline of the coding scheme is provided as follows. Rate splitting is performed on both messages, i.e.,

$m_1 = (m_{12}, m_{11a}, m_{11b})$ and $m_2 = (m_{21}, m_{22a}, m_{22b})$. The messages (m_{12}, m_{11a}) and (m_{21}, m_{22a}) are coded with the original Han-Kobayashi scheme, and the remaining messages m_{11b} and m_{22b} are coded with Marton's binning scheme (against each other) and Gel'fand-Pinsker coding by treating the codewords carrying m_{22a} and m_{11a} as known interference. The decoding at the receivers consists of two steps: 1) Decode $(m_{12}, m_{11a}, m_{21})$ at receiver 1 and decode $(m_{21}, m_{22a}, m_{12})$ at receiver 2; 2) Upon successful decoding in step 1), decode m_{11b} and m_{22b} at receivers 1 and 2, respectively. This scheme results in the achievable rate region $\mathcal{R}'$. ■

Remark 3.4: It is straightforward to extend $\mathcal{R}'$ to the Gaussian case, which leads to the achievable rate region presented in [8, Theorem 1].

Theorem 3.4: The rate region $\mathcal{R}'$ is a subset of the rate regions, $\mathcal{R}^*$ and $\mathcal{R}$, i.e., $\mathcal{R}' \subseteq \mathcal{R}^* \subseteq \mathcal{R}$.

Proof: To show that $\mathcal{R}' \subseteq \mathcal{R}^*$, it is sufficient to show that for any $p(\cdot) \in \mathcal{P}^*$, we have $\mathcal{R}'(p(\cdot)) \subseteq \mathcal{R}^*(p(\cdot))$. To do so, we fix a joint distribution $p(\cdot) \in \mathcal{P}^*$, and show that for any rate tuple $(\dot{R}_{12}, \dot{R}_{11a}, \dot{R}_{11b}, \dot{R}_{21}, \dot{R}_{22a}, \dot{R}_{22b}, \dot{L}_{11b}, \dot{L}_{22b})$ satisfying (32)–(50), there exists a corresponding rate tuple $(\ddot{R}_{12}, \ddot{R}_{11}, \ddot{R}_{21}, \ddot{R}_{22}, \ddot{L}_{11}, \ddot{L}_{22})$ satisfying (1)–(19) such that $\ddot{R}_{12} + \ddot{R}_{11} = \dot{R}_{12} + \dot{R}_{11a} + \dot{R}_{11b}$ and $\ddot{R}_{21} + \ddot{R}_{22} = \dot{R}_{21} + \dot{R}_{22a} + \dot{R}_{22b}$. This can be shown with the following mapping: $\ddot{R}_{12} = \dot{R}_{12}$, $\ddot{R}_{11} = \dot{R}_{11a} + \dot{R}_{11b}$, $\ddot{R}_{12} = \dot{R}_{12}$, $\ddot{R}_{22} = \dot{R}_{22a} + \dot{R}_{22b}$, $\ddot{L}_{11} = \dot{L}_{11b} + \dot{R}_{11a}$, and $\ddot{L}_{22} = \dot{L}_{22b} + \dot{R}_{22a}$. It can be verified that as long as the rate tuple $(\dot{R}_{12}, \dot{R}_{11a}, \dot{R}_{11b}, \dot{R}_{21}, \dot{R}_{22a}, \dot{R}_{22b}, \dot{L}_{11b}, \dot{L}_{22b})$ satisfies (32)–(50), the mapped rate tuple $(\ddot{R}_{12}, \ddot{R}_{11}, \ddot{R}_{21}, \ddot{R}_{22}, \ddot{L}_{11}, \ddot{L}_{22})$ always satisfies (1)–(19). The detailed steps are omitted. ■

Remark 3.5: Compared with the coding scheme to achieve $\mathcal{R}'$, our new coding schemes developed to achieve $\mathcal{R}^*$ and $\mathcal{R}'$ perform simultaneous joint decoding of the messages at each receiver rather than the two-step sequential decoding. In addition, 1) in our coding scheme, messages are split into two parts, i.e., $R_1 = R_{11} + R_{12}$, while in [8], messages are split into three parts, $R_1 = R_{11a} + R_{11b} + R_{12}$; 2) the number of inequalities involved in $\mathcal{R}$ is also fewer than in $\mathcal{R}'$.

IV. A New Achievable Rate Region for the CRC

By removing one of the cognitive relays, e.g. relay 2, the BCCR reduces to a CRC [1], [4], [5]. In the CRC model, relay 1 is regarded as the primary user, which transmits m_1 to receiver 1; the sender is the cognitive user, which knows both the messages and sends m_2 to receiver 2. By setting $\mathcal{V}_2 = \emptyset$ to specialize the coding scheme in Theorem 3.2, we obtain a new achievable rate region for the CRC as follows.

Define $\mathcal{P}_{\mathrm{CRC}}$ as the set of all joint distributions each factoring in the following form

$$p(u_1)p(v_1|u_1)p(x_1|v_1,u_1)p(u_2)p(w_1,w_2|v_1,u_1,u_2)$$
$$\times\, p(x_0|w_1,w_2,v_1,u_1,u_2)p(y_1,y_2|x_0,x_1). \tag{51}$$

For each joint distribution $p(\cdot) \in \mathcal{P}_{\mathrm{CRC}}$, define $\mathcal{R}_{\mathrm{CRC}}(p(\cdot))$ as the set of all the non-negative rate pairs $(R_{12} + R_{11}, R_{21} + R_{22})$ such that

$$\begin{aligned}
R_{11} - L_{11} &\le 0,\\
R_{22} - L_{22} &\le -I(W_2;V_1|U_1,U_2),\\
R_{11} - L_{11} + R_{22} - L_{22} &\le -I(W_2;W_1,V_1|U_1,U_2),\\
L_{11} &\le I(V_1,W_1;Y_1|U_1,U_2),\\
R_{12} + L_{11} &\le I(U_1,V_1,W_1;Y_1|U_2),\\
L_{11} + R_{21} &\le I(V_1,W_1,U_2;Y_1|U_1),\\
R_{12} + L_{11} + R_{21} &\le I(U_1,V_1,W_1,U_2;Y_1),\\
L_{22} &\le I(W_2;Y_2|U_2,U_1),\\
R_{21} + L_{22} &\le I(U_2,W_2;Y_2|U_1),\\
L_{22} + R_{12} &\le I(W_2,U_1;Y_2|U_2),\\
R_{21} + L_{22} + R_{12} &\le I(U_2,W_2,U_1;Y_2).
\end{aligned}$$

Theorem 4.1: Any rate pair $(R_1, R_2) \in \mathcal{R}_{\mathrm{CRC}} \triangleq \bigcup_{p(\cdot)\in\mathcal{P}_{\mathrm{CRC}}} \mathcal{R}_{\mathrm{CRC}}(p(\cdot))$ is achievable for a CRC.

Proof: Set V_2 as a constant in $\mathcal{R}$, and the achievability of $\mathcal{R}_{\mathrm{CRC}}$ follows immediately. ■

Remark 4.1: The new achievable rate region $\mathcal{R}_{\mathrm{CRC}}$ can be easily reduced to several existing rate regions that have been shown to be the capacity regions of the respective special-case CRCs [2], [3], [11], [16]. This new region also incorporates Marton's region for the broadcast channel as a special case. Note that existing achievable rate regions could not generalize Marton's region except for a recent result in [6]. Nevertheless, our scheme easily generalizes the scheme proposed in [6] as a special case if we perform Gel'fand-Pinsker coding on message m_{21}, by treating the codewords carrying message m_{11} as the known interference.

In what follows, we first introduce a special configuration of the Gaussian CRC. We then demonstrate that our coding scheme performs Costa's dirty paper coding twice in a specific order, resulting in an achievable rate region with a simple and explicit description.

Example 4.1: As shown in Fig. 2, receiver 1 consists of two component receivers, and the channel has the following input-output relationship

$$Y_1^{(1)} = X_1 + Z_1^{(1)},\ Y_1^{(2)} = bX_0 + Z_1^{(2)}, \tag{52}$$
$$Y_2 = X_0 + aX_1 + Z_2, \tag{53}$$

where X_1 and X_0 are real channel inputs subject to power constraints P_1 and P_0, respectively; $Z_1^{(1)}$, $Z_1^{(2)}$, and Z_2 denote additive white Gaussian noises of zero mean and unit variance; and a together with b denote real scalar channel gains. We restrict our attention to the case of $b > 1$.

We first specialize the coding scheme in Theorem 4.1 by setting $\mathcal{U}_1 = \mathcal{U}_2 = \emptyset$ and letting $m_{11} = m_1$ and $m_{22} = m_2$. We have the following rate region achievable for the CRC channel.

Corollary 4.1: Any rate pair $(R_1, R_2) \in \mathcal{R}^*_{\mathrm{CRC}}$ is achiev-

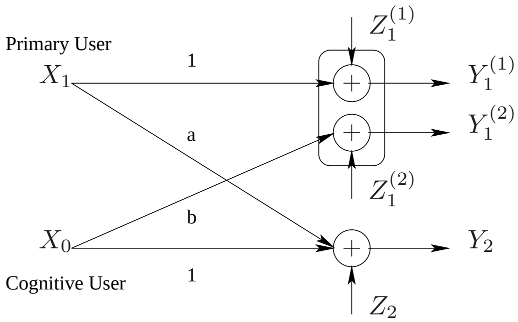

Fig. 2. A special case of the Gaussian cognitive radio channel.

able for a CRC with

$$\mathcal{R}^*_{\rm CRC} \triangleq \bigcup_{p(\cdot)\in\mathcal{P}^*_{\rm CRC}} \{(R_1,R_2): R_1 \le I(V_1,W_1;Y_1),$$
$$R_2 \le I(W_2;Y_2) - I(W_2;V_1),$$
$$R_1+R_2 \le I(V_1,W_1;Y_1)+I(W_2;Y_2)-I(W_2;W_1,V_1)\},$$

where $\mathcal{P}^*_{\rm CRC}$ denotes the set of all joint distributions:

$$p(v_1)p(w_1,w_2|v_1)p(x_1|v_1)p(x_0|w_1,w_2,v_1)p(y_1,y_2|x_1,x_0).$$

Define $\mathcal{R}_{\rm SGCRC}(\alpha)$ as the set of rate pairs (R_1,R_2):

$$R_1 \le \frac{1}{2}\log_2(1+P_1)+\frac{1}{2}\log_2(1+(1-\alpha)b^2P_0),$$
$$R_2 \le \frac{1}{2}\log_2\left(1+\frac{\alpha P_0}{(1-\alpha)P_0+1}\right),$$

for $0<\alpha<1$.

Corollary 4.2: The rate region $\mathcal{R}_{\rm SGCRC} \triangleq \bigcup_{0<\alpha<1}\mathcal{R}_{\rm SGCRC}(\alpha)$ is achievable for the Gaussian CRC defined by (52)–(53).

Proof: First, we further specialize $\mathcal{R}^*_{\rm CRC}$ to our channel configuration:

$$R_1 \le I(V_1;Y_1^{(1)})+I(W_1;Y_1^{(2)}|V_1),$$
$$R_2 \le I(W_2;Y_2)-I(W_2;V_1),$$
$$R_1+R_2 \le I(V_1;Y_1^{(1)})+I(W_1;Y_1^{(2)}|V_1)-I(W_1;W_2|V_1)$$
$$+I(W_2;Y_2)-I(W_2;V_1).$$

Next, with $\mathcal{N}(0,\sigma^2)$ denoting the Gaussian distribution with zero mean and variance σ^2, we choose the following mappings for the respective random variables: 1) $V_1 \sim \mathcal{N}(0,P_1)$, 2) $W_2=\tilde{W}_2+\beta_1V_1$ with $\tilde{W}_2\sim\mathcal{N}(0,(1-\alpha)P_0)$, 3) $W_1=V_1+\tilde{W}_1+\beta_2\tilde{W}_2$ with $\tilde{W}_1\sim\mathcal{N}(0,\alpha P_0)$, 4) $X_1=V_1$, and 5) $X_0=\tilde{W}_1+\tilde{W}_2$. By choosing $\beta_1=a(1-\alpha)P_0/(1+P_0)$ and $\beta_2=b^3\alpha P_0/(1+b^2\alpha P_0)$ as the dirty paper coding coefficients, we have $I(W_2;Y_2)-I(W_2;V_1)$ and $I(W_1;Y_1^{(2)}|V_1)-I(W_1;W_2|V_1)$ both maximized, and the achievability follows. ■

Remark 4.2: Among the existing coding schemes developed for the CRCs and the BCCRs, only the scheme developed in [8] can be specialized to this channel model to achieve the above rate region. The coding schemes developed in [1]–[5] allow no dirty paper coding on the primary user message against the codewords carrying the cognitive user message. The coding schemes proposed in [6], [7] allow no simultaneous involvement of the cognitive user message in the two dirty paper coding processes: 1) Encoding m_2 as $\mathbf{W}_2$ by treating the codewords $\mathbf{V}_1$ as the known interference; 2) Encoding m_1 as $\mathbf{W}_1$ by treating $\mathbf{W}_2$ as the known interference.

V. Conclusions

New coding schemes have been developed for the broadcast channel with cognitive relays and the associated achievable rate regions are derived. The new achievable rate regions are shown to include all previously-derived rate regions. One of the new rate regions is then specialized to the case of a single cognitive relay, which results in a new achievable rate region for the well-studied cognitive radio channel. A Gaussian example is provided to illustrate the domination of the new coding scheme over several existing ones.

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