Multi-hop MIMO Relay Networks: Diversity-Multiplexing Trade-off Analysis

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Abstract

A multi-hop relay network with multiple antenna terminals in a quasi-static slow fading environment is considered. The fundamental diversity-multiplexing gain tradeoff (DMT) is analyzed in the case of half-duplex relay terminals. While decode-and-forward (DF) relaying achieves the optimal DMT in the full-duplex relay scenario, it is shown that the dynamic decode-and-forward (DDF) protocol achieves the optimal DMT if the relay is constrained to half-duplex operation. For the latter case, static protocols are considered as well, and the corresponding DMT performances are shown to fall short of the optimal performance, which indicates that dynamic channel allocation is required for optimal DMT performance over half-duplex relay networks. The optimal DMT is expressed as the solution of a convex optimization problem and explicit DMT expressions are presented for some special cases.

I. Introduction

Relays are commonly used in wireless networks to improve the performance, although the fundamental capacity limits of relay channels have yet to be fully characterized, even for simple systems [1]. Rather than focusing on capacity limits, we are interested in characterizing the tradeoff between the rate gain through multiplexing versus the robustness gain through diversity associated with multiple-antenna relays. We will focus on a multiple antenna multi-hop system in which the transmission from each terminal can be received only by the next terminal in the network, as shown in Fig. 1. We call this the multiple-input multiple-output (MIMO) multi-hop relay network. The links are assumed to be quasi-static, frequency non-selective Rayleigh fading, and the channel state information (CSI) is available only at the receiving end of each transmission.

We analyze this system in terms of the diversity-multiplexing tradeoff (DMT) in the high signal-to-noise ratio (SNR) regime introduced in [4]. DMT analysis is useful in characterizing the fundamental tradeoff

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between the reliability and the degrees of freedom of a communication system. In DMT analysis, reliability is measured in terms of the diversity gain, which characterizes the rate of decay of the error probability with increasing SNR. The number of degrees of freedom is measured by the spatial multiplexing gain, which is the rate of increase in the transmission rate with SNR. DMT analysis has also been applied to multiuser channels such as the multiple access channel [5], the relay channel [3], [9]-[12] and the two-way relay channel [6], [7]. DMT analysis is a tool to characterize the fundamental limits of a communication system in a fading environment, and it has guided the design of practical space-time codes that approach these theoretical limits [16]- [19].

In a general cooperative relay channel, the source transmission is received by both the relay and the destination terminals and the source and the relay terminals cooperate to transmit the message to the destination [2], [3]. The DMT analysis was first applied to cooperative systems in [3] in which the DMT for simple amplify-and-forward (AF) and decode-and-forward (DF) protocols was characterized. In [9], a dynamic decode-and-forward (DDF) protocol is proposed and analyzed from the DMT perspective. In DDF, the relay terminal listens to the source transmission until it can decode the message, and then starts transmitting the message jointly with the source terminal. The DMT of DDF is shown to dominate all other protocols, but it does not meet the cut-set upper bound for high multiplexing gains. In [10], the DMT achieved by the DDF protocol is improved slightly by using superposition coding. In [12], under the assumption of full CSI at the relay terminal, the compress-and-forward (CF) protocol is shown to achieve the optimal DMT performance. Recently, [13] showed that the CF protocol can indeed achieve the optimal DMT in a single relay channel without the need of channel state information at the transmitters. However, a full characterization of the DMT curve is still an open problem in the case of a multiple antenna half-duplex relay channel when the CSI is available only at the receiving terminals.

There has also been some recent interest in the DMT analysis for multi-hop relay systems; in [20] and [21] multiple single antenna relays operating in a distributed manner are considered. Due to the distributed nature of the relay nodes, amplify-and-forward relaying is considered, under which the achievable DMT is characterized. The DMT of networks of disjoint parallel relay paths is studied in [15], in which the optimal DMT is characterized for single-antenna relays in the presence of more than three paths.

In this paper, we consider a MIMO multi-hop relay network as in Fig. 1. For this model, each relay can decode the message without sacrificing degrees of freedom. We start our analysis by considering a single relay terminal [14], and then extend it to multiple relays. In the case of full-duplex relays, the classical decode-and-forward protocol achieves the optimal DMT performance [20]. In the half-duplex relay case, we first find the DMT of static protocols in which the source and the relay transmission periods are fixed, independent of the channel realization. Then we consider the DDF protocol of [9], in which the time allocation depends on the realization of the source-relay channel, and show that it achieves the optimal DMT performance.

One of our goals in this work is to isolate the effect of channel allocation on the DMT in relay networks by removing the direct link. By focusing on this multi-hop channel model and showing that the optimal cannot be achieved by static protocols, we show that the optimal operation in half-duplex relay networks require dynamic allocation of the channel resources based on instantaneous channel conditions. The benefits of dynamic resource allocation in cooperative systems is studied in [8] in the case of channel state information at the transmitters.

In the multi-hop scenario, since the relay and the source do not transmit simultaneously, they do not need to use distributed space-time codes, which are harder to realize in practice [22], [23]. Furthermore, there is no need to inform the source or the destination terminals about the relay decision time as opposed to the general relay scenario. Hence, the dynamic relaying scheme in the case of the multi-hop relay channel can be realized by using an incremental redundancy code at the source [18] and any DMT optimal space-time code at the relay. Although the DMT of DDF has been previously shown to dominate other protocols in the case of general half-duplex relay channels, here we prove its optimality in the multi-hop multiple antenna relay scenario. In a concurrent work [24], Gharan et al. prove the optimality of the DDF protocol in a single antenna multiple access relay network. In [6], we have shown that the optimal DMT in a multi-hop network can also be achieved by a dynamic CF protocol.

The rest of the paper is organized as follows. We introduce the system model in Section II. In Section

III we focus on a single relay scenario. In Section III-A we characterize the DMT of a full-duplex MIMO multi-hop relay channel. Then in Section III-B, we consider static DF protocols for the half-duplex relay case, and find their DMT curves. In Section III-C we find the DMT of the DDF protocol and show that it achieves the upper bound; hence it is optimal. We also give an explicit characterization of the DMT for some special cases. Section III-D is devoted to the comparison of the DMT's achieved by different antenna allocations among the terminals. Then we extend our analysis to the scenario with multiple relay terminals in Section IV. Finally, Section V concludes the paper followed by appendices.

II. System Model

Here we introduce the system model. We consider a multi-hop network with $K+1$ terminals as in Fig. 1. Here, the first terminal T_1 is the *source* terminal, the last terminal T_{K+1} is the *destination* terminal, while the rest are the *relay* terminals. Terminal T_i is assumed to have M_i antennas for $i = 1, \ldots, K+1$. We call this system an $(M_1, \ldots, M_{K+1})$ multi-hop MIMO relay network. The channel from terminal T_i to terminal T_{i+1} is given by

$$\mathbf{Y}_k = \sqrt{\frac{SNR}{M_i}}\mathbf{H}_i\mathbf{X}_i + \mathbf{W}_i, \tag{1}$$

for $i = 1, \ldots, K$, respectively. Here, $\mathbf{Y}_i$ is the received signal at terminal T_{i+1}. Note that the transmission from terminal T_i is received only by terminal T_{i+1}. Channels are assumed to be frequency non-selective, quasi-static Rayleigh fading and independent of each other; that is, for $i = 1, \ldots, K$, $\mathbf{H}_i$ is an $M_{i+1} \times M_i$ channel matrix whose entries are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero means and unit variances (i.e., they are $\mathcal{CN}(0,1)$). The additive white Gaussian terms also have i.i.d. $\mathcal{CN}(0,1)$ entries. $\mathbf{X}_i$, $i = 1, \ldots, K$, are $M_i \times L$ input matrices of the terminals, where L is the total number of transmissions over which the channel is constant. We have short-term power constraints at each of the terminals given by $\mathrm{trace}(E[\mathbf{X}_i^H\mathbf{X}_i]) \leq M_iL$. For $i = 1, \ldots, K$, we define

$$M_i^* \triangleq \min\{M_i, M_{i+1}\}.$$

We assume that the receivers have perfect channel state information while the transmitters know only the channel statistics.

Following [4], for increasing SNR we consider a family of codes and say that the system achieves a multiplexing gain of r if the rate $R(SNR)$ satisfies

$$\lim_{SNR\to\infty}\frac{R(SNR)}{\log(SNR)}=r.$$

The diversity gain d of this family is defined as

$$d=-\lim_{SNR\to\infty}\frac{\log P_e(SNR)}{\log(SNR)},$$

in which $P_e(SNR)$ is the error probability. For each r, define $d(r)$ as the supremum of the diversity gain over all families of codes. The full characterization of the DMT curve for a MIMO system is given in the following theorem [4].

Theorem 1: [4] For a MIMO system with M_1 transmit and M_2 receive antennas and sufficiently long codewords, the optimal DMT curve $d_{M_1,M_2}(r)$ is given by the piecewise-linear function connecting the points $(k,d(k))$, $k=0,\ldots,\min(M_1,M_2)$, where

$$d(k)=(M_1-k)(M_2-k).$$

For the rest of the paper, we always consider codes with sufficiently long codewords so that the error event is dominated by the outage event.

III. DMT of the MIMO Multi-hop Channel with a Single Relay

In this section, we consider an (M_1,M_2,M_3) system with a single relay terminal, i.e., $K=2$. Generalization to the multiple relay scenarios follows from the results in this section, and will be considered in Section IV.

A. Full-duplex Relaying

We first consider the full-duplex relay case. The next theorem shows that the DMT tradeoff of the end-to-end system is equal to the worst-case DMT tradeoff of each link along the multi-hop path.

Theorem 2: The DMT $d^f_{M_1,M_2,M_3}(r)$ of an (M_1,M_2,M_3) full-duplex system is characterized by

$$d^f_{M_1,M_2,M_3}(r) \quad = \quad \min\{d_{M_1,M_2}(r),d_{M_2,M_3}(r)\}. \tag{2}$$

Proof: The converse is easily obtained from the cut-set bound; the capacity is bounded by the rate that can be transmitted over each hop. Hence, the end-to-end DMT is bounded by the DMT of each hop, each of which is a point-to-point MIMO channel. The achievability follows since DF relaying is in outage if any of the links is in outage. The outage event is dominated by the link that has the highest outage probability, or equivalently, the DMT is dominated by the minimum diversity gain. This result, which was shown in [20], is included here for completeness. ■

B. Static Protocols for Half-duplex Relaying

In the half-duplex relay scenario, the total L time units need to be divided among the source and the relay transmissions. We first consider static protocols where the time allocation is fixed, independent of the channel states. However, similar to the generalized decode-and-forward protocol in [25], we consider unequal division of the time slot among the source and the relay. The source transmits during the first aL channel uses, where $0 < a < 1$. The relay tries to decode the message and forwards over the remaining $(1-a)L$ channel uses. We call this protocol *decode-and-forward with fixed time allocation* (fDF), and its DMT is given in the next proposition.

Proposition 1: The DMT of the half-duplex (M_1, M_2, M_3) relay channel with fixed time allocation a $(0 < a < 1)$ is

$$d^{fDF}_{M_1 M_2 M_3}(r) \quad = \quad \min\left\{ d_{M_1,M_2}\left(\frac{r}{a}\right), d_{M_2,M_3}\left(\frac{r}{1-a}\right)\right\}. \tag{3}$$

Proof: This result follows easily from Theorem 1 with simple scaling of the DMT curve due to time division. ■

We can see from the above DMT that the highest multiplexing gain for the fDF scheme is $\min\{aM_1^*, (1-a)M_2^*\}$. On the other hand, the highest diversity gain is limited to $M_2 \min\{M_1, M_3\}$. We illustrate the DMT of a $(4, 2, 3)$ system with a fixed time allocation of $a = 0.3$ in Fig. 2.

Since different time allocations result in different DMT curves, we can optimize the time allocation based on the multiplexing gain [11], [25]. We call this protocol *DF with variable time allocation* (vDF). Note that this is still a static protocol since the time allocation variable is determined based only on the multiplexing gain and is independent of the channel realization. For each multiplexing gain r, the diversity gain is the minimum of the two diversity gains in (3), hence the optimal time allocation variable $a(r)$ is the one that satisfies

$$d^{vDF}_{M_1,M_2,M_3}(r) = d_{M_1,M_2}\left(\frac{r}{a(r)}\right) = d_{M_2,M_3}\left(\frac{r}{1-a(r)}\right). \tag{4}$$

Corollary 1: The number of degrees of freedom of an (M_1, M_2, M_3) multi-hop relay channel with the vDF protocol is $\frac{M_1^* M_2^*}{M_1^* + M_2^*}$, while the maximal diversity gain is $M_2 \min\{M_1, M_3\}$.

We now present the DMT for some special cases because a general closed form expression is not tractable. We first consider the $(M_1, 1, M_3)$ system assuming, without loss of generality, $M_1 \geq M_3$. Since the two hops for this setup are multiple-input single-output (MISO) and single-input multiple-output (SIMO) systems, respectively, the DMTs are characterized as $d_{M_1,M_2} = M_1(1-r)$ and $d_{M_2,M_3} = M_3(1-r)$, respectively. We define $A \triangleq M_1^*/M_3^*$ and $B \triangleq 1 - r - A(1 + r)$. We have $A \geq 1$ and $B \leq 0$, and we find

$$a(r) = \frac{-B + \sqrt{B^2 - 4A(A-1)r}}{2(A-1)}$$

for $A \neq 1$. We have $a(r) = 0.5$ if $A = 1$. The DMT achieved by the vDF protocol in a $(4, 1, 3)$ system is plotted in Fig. 3. In this figure, we also plot the DMT for the fDF scheme with a fixed time allocation $a = 0.5$.

If we have $M_1 = M_3 = M$, then the optimal time allocation is $a = 0.5$ independently of the multiplexing gain, and the DMT is given by $d^{vDF}_{M,M_2,M}(r) = d_{M,M_2}(2r)$.

C. Dynamic Decode-and-Forward Protocol for Half-duplex Relaying

In [9], Azarian et al. proposed the dynamic decode-and-forward protocol for the cooperative relay channel with single antennas. In DDF for the relay channel, the source transmits during the entire timeslot using an incremental redundancy type codebook. This code design enables the relay to decode the message after receiving only a portion of the codeword, and hence the relay decodes the message when the accumulated mutual information over the source-relay channel is sufficient for the transmission rate. Thus, the relay decoding time becomes a random variable which depends on the source-relay channel quality. As soon as the relay decodes the message, it starts transmitting.

The achievable DMT of the DDF scheme in the case of the single antenna cooperative relay channel is characterized in [9], where it is shown to dominate the DMTs of amplify-and-forward (AF) and decode-and-forward (DF) based protocols and, more strikingly, to achieve the DMT upper bound for multiplexing gains $r \leq 0.5$. Hence, DDF is DMT optimal in this range of low multiplexing gains for the single antenna cooperative relay channel. An improved DDF scheme is presented in [10] in which the source terminal transmits two data streams rather than one, and only one of these streams benefits from cooperation. Although this improves the achievable DMT performance, it still does not meet the MISO upper bound for $r > 0.5$.

Recently, it has been shown [13] that the optimal DMT for a single antenna relay channel is achieved by the compress-and-forward (CF) scheme with a fixed and equal time allocation between the relay listen and transmit times. For this system, the DMT of this fixed time allocation CF scheme meets the 2×1 MISO upper bound; however, this is not the case when we have a different numbers of antennas at the source and the destination [12]. Our goal here is to isolate the effect of time allocation on the DMT and, hence, we consider a multi-hop MIMO channel.

We consider using the DDF protocol for the multi-antenna multi-hop relay channel, and show that it achieves the upper bound, that is, DDF is DMT optimal in this setting. The intuitive explanation behind the optimality of DDF in this setting is as follows: In the multi-hop relay scenario, the message needs to be decoded at the relay terminal, since otherwise the destination would not be able to decode it either, due to the data processing inequality. However, any fixed time allocation scheme either wastes multiplexing gain since it cannot exploit the good states of the source-relay channel, or results in outages in the case of a poor quality source-relay channel. On the other hand, by enforcing decoding at the relay and dynamically allocating the source transmission time based on the source-relay channel state, DDF achieves the optimal DMT performance. Our results, apart from characterizing the optimal DMT for multi-hop relay networks, highlight the importance of dynamic operation over fading relay networks.

Theorem 3: For the (M_1, M_2, M_3) system with a half-duplex relay and rate $R = r \log SNR$, the outage probability of the dynamic decode-and-forward protocol is given by

$$P_{out}(r) \doteq SNR^{-d^{DDF}(r)}$$

where

$$d^{DDF}(r) = \inf_{(\boldsymbol{\alpha_1},\boldsymbol{\alpha_2})\in\tilde{\mathcal{O}}_2} \sum_{i=1}^{2}\sum_{j=1}^{M_i^*}(2j-1+|M_i - M_{i+1}|)\alpha_{i,j} \tag{5}$$

and

$$\begin{aligned}\tilde{\mathcal{O}}_2 \;\; &\triangleq \;\; \Big\{(\boldsymbol{\alpha_1},\boldsymbol{\alpha_2}) \in \mathcal{R}^{M_1^*+} \times \mathcal{R}^{M_2^*+} \Big| \\ & \qquad \alpha_{i,1} \geq \cdots \geq \alpha_{i,M_i^*} \geq 0, r > \frac{S_1(\boldsymbol{\alpha_1})S_2(\boldsymbol{\alpha_2})}{S_1(\boldsymbol{\alpha_1})+S_2(\boldsymbol{\alpha_2})}\Big\}\end{aligned}$$

in which we have defined

$$S_i(\boldsymbol{\alpha_i}) \triangleq \sum_{j=1}^{M_i^*}(1-\alpha_{i,j})^+, \text{ for } i = 1, 2. \tag{6}$$

Proof: The proof of the theorem can be found in Appendix A. ■

Let $d^h_{M_1,M_2,M_3}(r)$ denote the optimal DMT for an (M_1, M_2, M_3) half-duplex relay system. In the following theorem, we prove that this optimal DMT performance is achieved by the DDF protocol. Here, as opposed to the general relay channel where a direct link exists between the source and the destination, the optimal DMT does not meet the MISO bound. We characterize the DMT upper bound by taking into account the time allocation between the listen and transmit times [27].

Theorem 4: DDF is DMT optimal for MIMO multi-hop half-duplex relay channels, i.e.,

$$d^h_{M_1,M_2,M_3}(r) = d^{DDF}_{M_1,M_2,M_3}(r).$$

Proof: The proof of the theorem can be found in Appendix B. ■

Corollary 2: The number of degrees of freedom of an (M_1, M_2, M_3) multi-hop relay channel is $\frac{M_1^* M_2^*}{M_1^*+M_2^*}$, while the maximal diversity gain is $M_2 \min\{M_1, M_3\}$. Hence, the end-points of the optimal DMT curve can also be achieved by static relaying, i.e., with fixed time allocation.

It can be seen from Theorem 3 that the DMT of a half-duplex multi-hop relay channel is not a piecewise-linear function any more as opposed to a point-to-point MIMO channel. The non-linearity in the DDF is due to the requirement of dynamic time allocation which itself is a random variable.

We do not provide a general closed form expression for the DMT of MIMO multi-hop channels. Next we show that for given M_1, M_2 and M_3 and a fixed multiplexing gain r, the optimization problem in (5) can be cast into a convex optimization problem, and hence can be solved efficiently by interior point methods [26].

Define $\beta_{i,j} \triangleq \alpha_{i,j} - 1$. Then the optimal diversity gain can be found by solving the following optimization problem:

$$\min \sum_{i=1}^{2} \sum_{j=1}^{M_i^*} (2j - 1 + |M_i - M_{i+1}|)(1 + \beta_{i,j}) \tag{7a}$$

$$\text{such that} \quad \beta_{i,j} \leq 0 \tag{7b}$$

$$-\beta_{i,j} - 1 \leq 0 \tag{7c}$$

$$-\frac{\sum_{j=1}^{M_1^*} \beta_{1,j} \sum_{j=1}^{M_2^*} \beta_{2,j}}{\sum_{j=1}^{M_1^*} \beta_{1,j} + \sum_{j=1}^{M_2^*} \beta_{2,j}} - r \leq 0, i = 1, \dots, M_1^*, j = 1, \dots, M_2^*. \tag{7d}$$

For $\boldsymbol{\beta} = [\beta_{1,1} \cdots \beta_{1,M_1^*} \quad \beta_{2,1} \cdots \beta_{2,M_2^*}]^T$, define

$$h(\boldsymbol{\beta}) \triangleq -\frac{\sum_{j=1}^{M_1^*} \beta_{1,j} \sum_{j=1}^{M_2^*} \beta_{2,j}}{\sum_{j=1}^{M_1^*} \beta_{1,j} + \sum_{j=1}^{M_2^*} \beta_{2,j}} - r.$$

It is possible to show that

$$h(\boldsymbol{\beta}) \nabla^2 h(\boldsymbol{\beta}) \succeq \nabla h(\boldsymbol{\beta}) \nabla h(\boldsymbol{\beta})^T,$$

that is, $h(\boldsymbol{\beta})$ is log-convex, hence, also convex. Then, we conclude that the optimization problem in (7) is convex.

Here we give an explicit characterization of the DMT for some special cases. We first consider the cases when the relay terminal has a single antenna, i.e., $M_2 = 1$, or when both the source and the destination terminals have a single antenna, i.e., $M_1 = M_3 = 1$.

Corollary 3: The DMT of an $(M_1, 1, M_3)$ system is

$$d^h_{M_1,1,M_3}(r) = \min(M_1, M_3) \frac{1 - 2r}{1 - r}$$

for $0 \leq r \leq 1/2$, and 0 elsewhere.

Corollary 4: The DMT of a $(1, M_2, 1)$ system is

$$d^h_{1,M_2,1}(r) = M_2 \frac{1-2r}{1-r}$$

for $0 \leq r \leq 1/2$, and 0 elsewhere.

Proof: In both cases, both of the hops have only a single degree-of-freedom. From (5), we have

$$d^{DDF}(r) = \min \alpha_{1,1} + \alpha_{2,1}$$

such that $\alpha_{i,1} \geq 0$, $i = 1, 2$ and

$$\frac{(1-\alpha_{1,1})^+(1-\alpha_{2,1})^+}{(1-\alpha_{1,1})^+ + (1-\alpha_{2,1})^+} > r.$$

The optimal solution satisfies $\alpha_{i,1} \leq 1$, $i = 1, 2$. Then we can write the Karush-Kuhn-Tucker (KKT) conditions and obtain the diversity gain given in the corollary. Due to the convexity of the optimization problem, the KKT conditions are also sufficient, hence the diversity gain we find characterizes the optimal DMT. ■

In Fig. 3 we illustrate the DMT of the $(4, 1, 3)$ multi-hop MIMO relay channel, which is achieved by the DDF protocol. We see that the DDF dominates the static protocols at all multiplexing gains but the end-points. As stated in Corollary 2, these end-points can be achieved by static time allocation as well.

Next, we give an explicit expression for the optimal DMT for the $(2, 2, 2)$ multi-hop MIMO relay channel.

Corollary 5: The DMT of the $(2, 2, 2)$ system with a half-duplex relay is given by

$$d^h_{2,2,2}(r) = \begin{cases} \frac{2(4-5r)}{2-r} & \text{if} \quad 0 \leq r < 1/2 \\ \frac{3-4r}{1-r} & \text{if} \quad 1/2 \leq r < 2/3 \\ \frac{4(1-r)}{2-r} & \text{if} \quad 2/3 \leq r \leq 1. \end{cases} \tag{8}$$

The DMT of the $(2, 2, 2)$ system is illustrated in Fig. 5. The topmost curve in the figure is the DMT of a 2×2 MIMO system, which can be achieved by a full-duplex relay. The lowest curve is the DMT of the vDF protocol. Note that for this symmetric scenario vDF reduces to fDF with $a = 0.5$.

D. Antenna Allocation

In a practical multi-hop relay system, adding antennas to the mobile terminals is costly, hence, a relevant problem is how to allocate a given number of antennas among the terminals in the network. It is also possible that, we are given three terminals with a fixed number of antennas, and asked to assign these terminals as the source, the relay and the destination for optimal performance. Since error probability serves as an appropriate performance measure for many scenarios, our results in this paper on the optimal DMT of the multi-hop relay system provide design insights for these problems based on the outage probability performance which closely approximates the error probability in the high SNR regime.

As an example, consider a total of 5 antennas available to us. We can have 4 different antenna allocations among the three terminals: $(2, 2, 1)$, $(2, 1, 2)$, $(3, 1, 1)$ or $(1, 3, 1)$. The optimal DMT of these 4 different systems, achieved by DDF, is plotted in Fig. 4. As we can see from the figure, the optimal antenna allocation, in terms of the diversity gain, depends on the operating multiplexing gain. The $(3, 1, 1)$ systems dominates at low multiplexing gains, while the $(2, 2, 1)$ system dominates at higher multiplexing gains. The DMTs of the $(3, 1, 1)$ and the $(2, 1, 2)$ systems are dominated by the other two systems due to the single antenna available at the relay. We can prove that, given three terminals with any fixed number of antennas, assigning the one with the maximum number of antennas as the relay terminal will always maximize the optimal DMT. On the other hand, for more than a total of 5 antennas assigning the antennas equally among the terminals will achieve a better DMT performance even at lower multiplexing gains. For a total of $M = 3m$ antennas ($m \in \mathbb{Z}^+$), the highest diversity gain achievable by the $(1, 3m - 2, 1)$ system is $M - 2$, while the (m, m, m) system can achieve a diversity gain of m^2, which surpasses $3m - 2$ for $m > 2$.

IV. MULTIPLE RELAYS

In this section, we extend our results to a multi-hop MIMO network with multiple half-duplex relay terminals. We consider an $(M_1, \ldots, M_{K+1})$ network with $K + 1$ terminals as introduced in Section II, where T_1 is the source node, T_{K+1} is the destination and the remaining nodes $T_1, \ldots, T_K$ are the relays.

In the case of full-duplex relays, the result for a single relay in Theorem 2 directly extends to the multiple relay scenario by using the DF protocol for relaying. The DMT is given in the following corollary [20].

Corollary 6: The DMT $d^f_{M_1,\ldots,M_K}(r)$ of an $(M_1,\ldots,M_K)$ full-duplex system is characterized by

$$d^f_{M_1,\ldots,M_K}(r) \;\; = \;\; \min_{i=1,\ldots,K-1} d_{M_i,M_{i+1}}(r). \tag{9}$$

Next, we consider the more interesting case of half-duplex relays. We will show that the DDF protocol achieves the optimal DMT performance in a multiple relay network as well. Similar to the single relay setup, the source terminal starts transmitting the first message, and the first relay listens until it accumulates enough mutual information to decode this message. Once it decodes this message, it starts forwarding it to the next relay. Note that, due to the half-duplex constraint, the source terminal cannot transmit until the first relay terminal is done with forwarding the message to the next relay. However, if the channel from the first relay to the second relay is in a deep fade, this transmission might take a long time, which in the end delays the transmission of all the following messages. To mitigate this problem of propagating delays over the messages, we restrict the transmission time of each message over two consecutive hops. If it was not decoded, than this message is dropped and an outage is declared. We have the following result.

Theorem 5: The DMT $d^h_{M_1,\ldots,M_K}(r)$ of an $(M_1,\ldots,M_K)$ half-duplex system is characterized by

$$d^h_{M_1,\ldots,M_K}(r) \;\; = \;\; \min_{i=1,\ldots,K-2} d^h_{M_i,M_{i+1},M_{i+2}}(r), \tag{10}$$

and this optimal DMT performance is achieved by the DDF protocol. Note that $d^h_{M_1,M_2,M_3}(r)$ is the optimal DMT of a single half-duplex relay characterized in Theorem 4.

Proof: The proof of the theorem can be found in Appendix C. ∎

V. Conclusions

We have derived the diversity-multiplexing tradeoff of MIMO multi-hop half-duplex relay networks. For full-duplex relays, the decode-and-forward protocol achieves the optimal DMT, which is simply the minimum of the DMTs of the links. In the case of half-duplex relays, we have shown that the dynamic decode-and-forward protocol, in which the relay listens until decoding and then forwards the message, achieves the optimal DMT, which is no longer a piecewise-linear function of the multiplexing gain. We have shown that the optimal DMT for any given multiplexing gain can be found by solving a convex optimization problem. We have also shown that this optimal DMT performance cannot be achieved by static time allocation. Finally, we have provided explicit expressions for the DMT of some classes of half-duplex multi-hop relay systems, and compared the achievable performance with fixed and dynamic time allocation.

APPENDIX A
PROOF OF THEOREM 3

For the achievability scheme, we assume that the inputs at both the source and the relay are Gaussian with identity covariance matrices. Let the transmission rate be $R = r\log SNR$, and define

$$C_i(\mathbf{H}_i) \triangleq \log\det\left(\mathbf{I} + \frac{\mathrm{SNR}}{M_i}\mathbf{H}_i\mathbf{H}_i^{\dagger}\right). \tag{11}$$

The relay listens for aT channel uses until it decodes the message. Hence, we have $a = \frac{r\log\mathrm{SNR}}{C_1}$.

If $a \geq 1$ then the relay is in outage, which leads to an outage for the whole system. If $a < 1$, then the relay transmits during the rest of the timeslot for $(1-a)T$ channel uses. Conditioned on successful decoding at the relay with $a < 1$, the outage probability over the second hop is given by

$$\begin{aligned} P\{r\log\mathrm{SNR} > (1-a)C_2(\mathbf{H}_2)\} &= P\left\{r\log\mathrm{SNR} > \left(1 - \frac{r\log\mathrm{SNR}}{C_1(\mathbf{H}_1)}\right)C_2(\mathbf{H}_2)\right\} \\ &= P\left\{r\log\mathrm{SNR} > \frac{C_1(\mathbf{H}_1)C_2(\mathbf{H}_2)}{C_1(\mathbf{H}_1)+C_2(\mathbf{H}_2)}\right\}. \end{aligned} \tag{12}$$

Let $\lambda_{i,1},\ldots,\lambda_{1,M_i^*}$ be the nonzero eigenvalues of $\mathbf{H}_i\mathbf{H}_i^{\dagger}$ for $i = 1,2$. Suppose $\lambda_{i,j} = \mathrm{SNR}^{-\alpha_{i,j}}$ for $j = 1,\ldots,M_i^*$, $i = 1,2$. We have[1]

$$\begin{aligned} C_i(\mathbf{H}_i) &= \log\prod_{j=1}^{M_i^*}\left(1 + \frac{\mathrm{SNR}}{M_i}\lambda_{i,j}\right) \\ &\doteq \log\prod_{j=1}^{M_i^*}\mathrm{SNR}^{(1-\alpha_{i,j})^+} \end{aligned} \tag{13}$$

[1]Define the exponential equality as $f(SNR) \doteq SNR^c$, if $\lim_{SNR\to\infty}\frac{\log f(SNR)}{\log SNR} = c$. The exponential inequalities $\dot{\leq}$ and $\dot{\geq}$ are defined similarly.

where $(x)^+ \triangleq \max\{0, x\}$. Using these exponential equalities, we can rewrite (12) as follows

$$\begin{aligned} P\{r\log\mathrm{SNR} > (1-a)C_2(\mathbf{H}_2)\} &= P\left\{r\log\mathrm{SNR} > \frac{C_1(\mathbf{H}_1)C_2(\mathbf{H}_2)}{C_1(\mathbf{H}_1)+C_2(\mathbf{H}_2)}\right\} \\ &\doteq P\left\{\log\mathrm{SNR}^r > \frac{\log\mathrm{SNR}^{S_1(\boldsymbol{\alpha_1})}\log\mathrm{SNR}^{S_2(\boldsymbol{\alpha_2})}}{\log\mathrm{SNR}^{S_1(\boldsymbol{\alpha_1})}+\log\mathrm{SNR}^{S_2(\boldsymbol{\alpha_2})}}\right\} \\ &= P\left\{r > \frac{S_1(\boldsymbol{\alpha_1})S_2(\boldsymbol{\alpha_2})}{S_1(\boldsymbol{\alpha_1})+S_2(\boldsymbol{\alpha_2})}\right\} \end{aligned}$$

where we have $S_i(\boldsymbol{\alpha_i}) = \sum_{j=1}^{M_i^*}(1-\alpha_{i,j})^+$.

Then the overall outage probability can be written as

$$\begin{aligned} P_{out}(r) &\doteq P\left\{r \geq S_1(\boldsymbol{\alpha_1})\right\} + P\left\{S_1(\boldsymbol{\alpha_1}) > r > \frac{S_1(\boldsymbol{\alpha_1})S_2(\boldsymbol{\alpha_2})}{S_1(\boldsymbol{\alpha_1})+S_2(\boldsymbol{\alpha_2})}\right\} \\ &= P\left\{r > \frac{S_1(\boldsymbol{\alpha_1})S_2(\boldsymbol{\alpha_2})}{S_1(\boldsymbol{\alpha_1})+S_2(\boldsymbol{\alpha_2})}\right\}, \end{aligned}$$

since we have

$$S_1(\boldsymbol{\alpha_1}) \geq \frac{S_1(\boldsymbol{\alpha_1})S_2(\boldsymbol{\alpha_2})}{S_1(\boldsymbol{\alpha_1})+S_2(\boldsymbol{\alpha_2})}$$

for all $(\boldsymbol{\alpha_1}, \boldsymbol{\alpha_2})$.

We define

$$\mathcal{O} \triangleq \left\{(\boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}) : r > \frac{S_1(\boldsymbol{\alpha_1})S_2(\boldsymbol{\alpha_2})}{S_1(\boldsymbol{\alpha_1})+S_2(\boldsymbol{\alpha_2})}\right\}.$$

Then using the joint probability of the eigenvalues of $\mathbf{H}_i\mathbf{H}_i^\dagger$ given in [4], the outage probability can be computed as

$$\begin{aligned} P_{out}(r) &\doteq \int_{\mathcal{O}} p(\boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}) d\boldsymbol{\alpha_1} d\boldsymbol{\alpha_2} \\ &\doteq \int_{\mathcal{O}'} \prod_{i=1}^{2}\prod_{j=1}^{M_i^*} SNR^{-(2j-1+|M_i-M_{i+1}|)\alpha_{i,j}} d\boldsymbol{\alpha_1} d\boldsymbol{\alpha_2} \end{aligned}$$

where $\mathcal{O}' \triangleq \mathcal{O} \cap (\mathcal{R}^{M_1^*+}, \mathcal{R}^{M_2^*+})$.

Using Laplace's method as in [4], we obtain the exponential behavior of the outage probability as $P_{out}(r) \doteq SNR^{-d^{DDF}(r)}$, where

$$d^{DDF}(r) = \inf_{(\boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}) \in \tilde{\mathcal{O}}} \sum_{i=1}^{2}\sum_{j=1}^{M_i^*}(2j-1+|M_i-M_{i+1}|)\alpha_{i,j}, \tag{14}$$

with

$$\tilde{\mathcal{O}} \triangleq \Big\{(\boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}) \in \mathcal{R}^{M_1^*+} \times \mathcal{R}^{M_2^*+} | \\ \alpha_{i,1} \geq \cdots \geq \alpha_{i,M_i^*} \geq 0 \text{ for } i = 1, 2, \text{ and } r > \frac{S_1(\boldsymbol{\alpha_1})S_2(\boldsymbol{\alpha_2})}{S_1(\boldsymbol{\alpha_1}) + S_2(\boldsymbol{\alpha_2})}\Big\}.$$

APPENDIX B
PROOF OF THEOREM 4

We give an upper bound for the DMT of the MIMO multi-hop half duplex relay channel, and show that the DDF DMT given in Theorem 3 matches this upper bound. Let $a \in (0, 1]$ be the portion of the source transmit time, i.e., the source transmits over the first aL channel uses. Hence, the relay transmits over the remaining $(1 - a)L$ channel uses. Here we assume that the time allocation is independent of the message, i.e., it cannot be used for information transmission. As shown in [12] this does not affect the DMT of the system.

From the two cut-set bounds, the instantaneous capacity $C(\mathbf{H}_1, \mathbf{H}_2)$ is upper bounded by [27]

$$\max_{\substack{a, P_{\mathbf{X}_1} P_{\mathbf{X}_2} \\ 0<a\leq 1, tr(\mathbf{Q}_i)\leq P_i}} \min\{aI(\mathbf{X}_1; \mathbf{Y}_1|\mathbf{H}_1), (1 - a)I(\mathbf{X}_2; \mathbf{Y}_2|\mathbf{H}_2)\}.$$

where $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are the input covariance matrices at the source and the relay terminals, respectively.

Since the capacity is maximized with Gaussian inputs, and $\log\det(\cdot)$ is an increasing function over the cone of positive semi-definite Hermitian matrices, the instantaneous capacity can be bounded as

$$C(\mathbf{H}_1, \mathbf{H}_2) \leq \max_a \min\{a\bar{C}_1(\mathbf{H}_1), (1 - a)\bar{C}_2(\mathbf{H}_2)\},$$

where we define

$$\bar{C}_i(\mathbf{H}_i) \triangleq \log\det(\mathbf{I} + \mathrm{SNR}\mathbf{H}_i\mathbf{H}_i^\dagger), \tag{15}$$

which is the capacity random variable corresponding to an input covariance matrix of $M_i\mathbf{I}$.

We can further upper bound the capacity by assuming optimal time allocation at each channel realization. The instantaneous capacity is maximized at each channel realization for

$$a(\mathbf{H}_1, \mathbf{H}_2) = \frac{\bar{C}_2(\mathbf{H}_2)}{\bar{C}_1(\mathbf{H}_1) + \bar{C}_2(\mathbf{H}_2)},$$

and the corresponding upper bound is

$$C(\mathbf{H}_1,\mathbf{H}_2) \le \frac{\bar{C}_1(\mathbf{H}_1)\bar{C}_2(\mathbf{H}_2)}{\bar{C}_1(\mathbf{H}_1)+\bar{C}_2(\mathbf{H}_2)}.$$

For a transmission rate of $R = r\log SNR$, the outage probability lower bound is given by

$$P_{out}(r) \ge P\left\{ r\log \mathrm{SNR} > \frac{\bar{C}_1(\mathbf{H}_1)\bar{C}_2(\mathbf{H}_2)}{\bar{C}_1(\mathbf{H}_1)+\bar{C}_2(\mathbf{H}_2)} \right\}. \tag{16}$$

Using the characterization of the eigenvalues of the channel matrices given in Appendix A, we obtain

$$P_{out}(r) \;\; \dot{\ge} \;\; P\left\{ r > \frac{S_1(\boldsymbol{\alpha}_1)S_2(\boldsymbol{\alpha}_2)}{S_1(\boldsymbol{\alpha}_1)+S_2(\boldsymbol{\alpha}_2)} \right\}$$

in which $S_i(\boldsymbol{\alpha}_i)$ is as defined before. Then the outage probability is lower bounded by

$$\begin{aligned} P_{out}(r) \;\; &\dot{\ge} \;\; \int_{\mathcal{O}} p(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2) d\boldsymbol{\alpha}_1 d\boldsymbol{\alpha}_2 \\ &\doteq \;\; \int_{\mathcal{O}'} \prod_{i=1}^{2}\prod_{j=1}^{M_i^*} SNR^{-(2j-1+|M_i-M_{i+1}|)\alpha_{i,j}} d\boldsymbol{\alpha_1} d\boldsymbol{\alpha_2}. \end{aligned}$$

Note that the outage probability upper bound has the same diversity gain function as the DDF protocol found in Appendix A. Hence, DDF is DMT optimal.

Appendix C
Proof of Theorem 5

For each $i = 1,\dots,K-1$, consider the two consecutive hops from terminal T_i to T_{i+1} and then from T_{i+1} to T_{i+2}. Assume a genie aided scheme where the messages are provided to terminal T_i, and the channel outputs of terminal T_{i+2} are made available to terminal T_{K+1}. The DMT of this genie aided setup will be an upper bound on the DMT of the $(M_1,\dots,M_{K+1})$ system for each i. The DMT of the two-hop channel from T_i to T_{i+2} is found in Theorem 4 as $d^h_{M_i,M_{i+1},M_{i+2}}(r)$, from which we obtain

$$d^h_{M_1,\dots,M_K}(r) \le \min_{i=1,\dots,K-2} d^h_{M_i,M_{i+1},M_{i+2}}(r).$$

Next we prove that the DDF algorithm achieves this upper bound. In the achievable scheme, each message is transmitted over each hop until the receiver accumulates enough mutual information to successfully decode this message. Once the receiving terminal decodes this message and forwards it to the next terminal in line, the transmitter starts sending the next message. Thus, each terminal in the network starts forwarding a message after it decodes it successfully and when the next terminal in the sequel finishes forwarding its own message.

We constrain the total time each terminal spends for a message, i.e., the total time for listen and transmit modes, to some fixed value, which is small compared to L, yet still large enough to achieve the instantaneous capacity. Hence, the message will be in outage if and only if the total time spent over two consecutive hops is beyond this constant. We define the outage event over terminals T_i, T_{i+1} and T_{i+2} as

$$P_{out}^i(r) \triangleq \Pr\left\{\frac{r\log SNR}{C_i(\mathbf{H}_i)} + \frac{r\log SNR}{C_{i+1}(\mathbf{H}_{i+1})} > 1\right\} \tag{17}$$

for $i = 1, \ldots, K-1$, where $C_i(\mathbf{H}_i)$ is the instantaneous capacity as defined in (15). Hence, the system will be in outage if there is an outage over any of the consecutive hops over the terminals. From the union bound, we have

$$P_{out}(r) \le \sum_{i=1}^{K-1} P_{out}^i(r). \tag{18}$$

Since the exponential behavior of the outage probability will be dominated by the slowest decaying term in the summation, the achievable DMT will be dominated by the minimum. This concludes the proof of the theorem.

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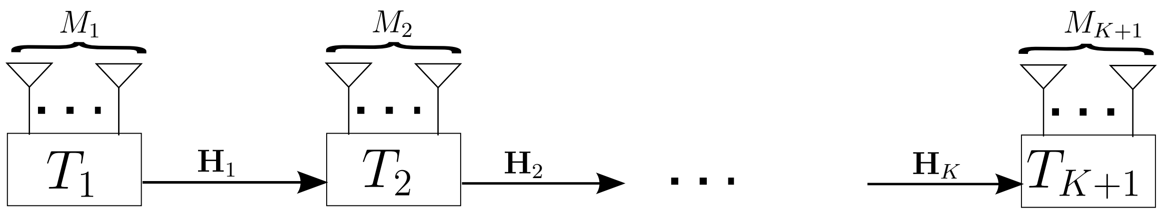

Fig. 1. The $(M_1, M_2, \ldots, M_{K+1})$ MIMO multi-hop relay network. Each terminal can receive only the signal transmitted by the terminal preceding itself in the network.

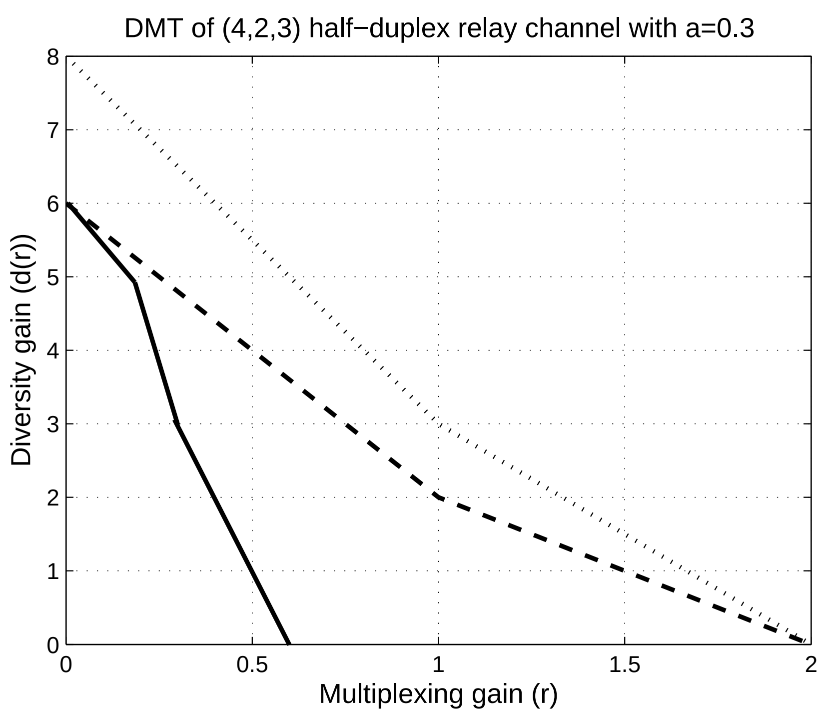

Fig. 2. The dotted and the dashed curves correspond to $d_{4,2}(r)$ and $d_{2,3}(r)$, respectively. Note that the dashed curve also corresponds to the DMT in the case of a full-duplex relay terminal. The solid curve is the DMT curve of a $(4, 2, 3)$ half-duplex multi-hop relay with fDF protocol and $a = 0.3$.

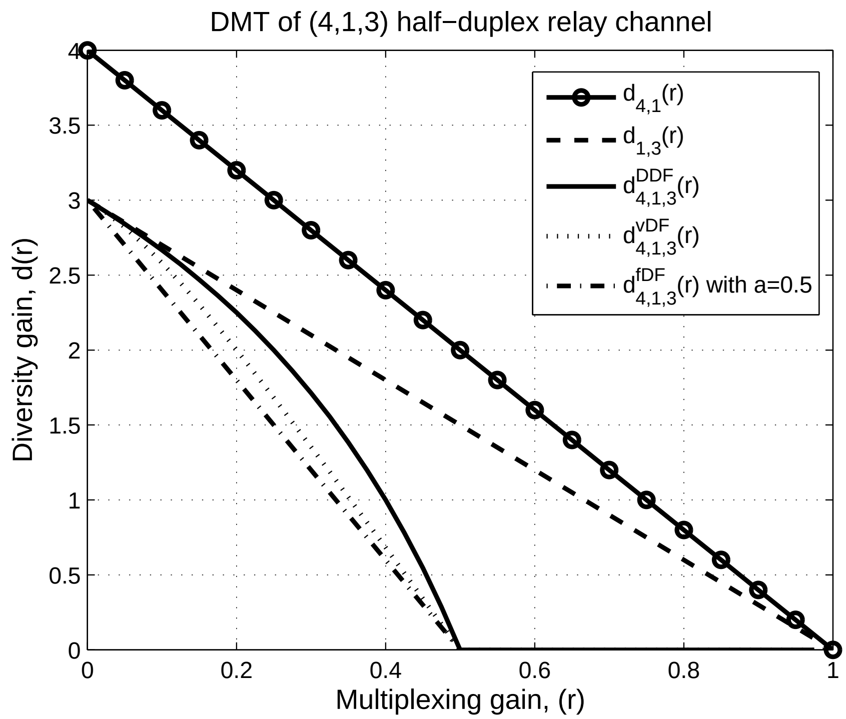

Fig. 3. The DMT curve of a $(4, 1, 3)$ multi-hop relay channel. The two topmost curves correspond to the cut-set bounds, where the dashed curve is also the DMT for a full-duplex relay. The DDF, vDF and fDF protocol with $a = 0.5$ are also illustrated, where the DDF curve is the optimal DMT with half-duplex relaying.

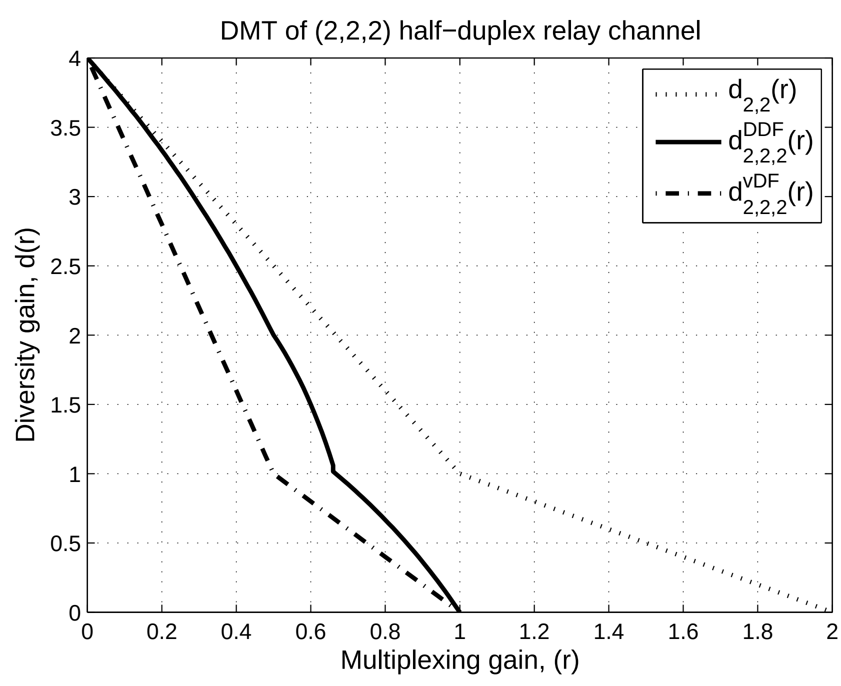

Fig. 4. The DMT of a $(2, 2, 2)$ system. From top to bottom, the three curves correspond to the full-duplex relay DMT, the half-duplex relay DMT which is achievable by DDF protocol, and the DMT of the static protocol with $a = 0.5$.

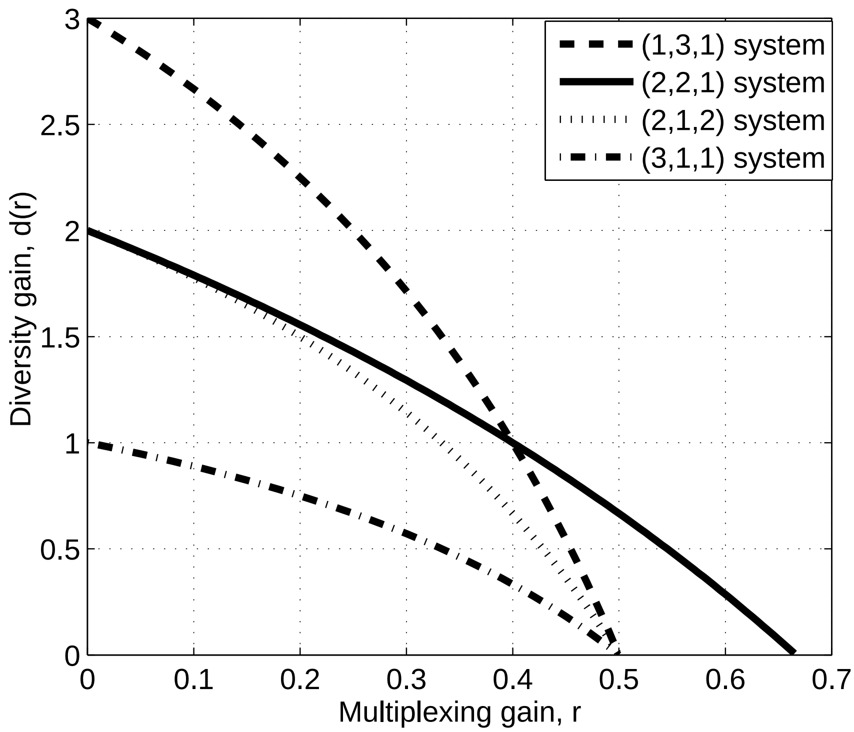

Fig. 5. The DMT of all 2-hop multi-hop MIMO systems that are possible with a total of 5 antennas.