Reduced-Dimension Multiuser Detection

Yao Xie*, Yonina C. Eldar*†, Andrea Goldsmith*

*Department of Electrical Engineering, Stanford University, Stanford, CA.
†Department of Electrical Engineering, Technion, Israel Institution of Technology.
Email: yaoxie@stanford.edu, yonina@ee.technion.ac.il, andrea@wsl.stanford.edu

***Abstract*—We present a new framework for reduced-dimension multiuser detection (RD-MUD) that trades off complexity for bit-error-rate (BER) performance. This approach can significantly reduce the number of matched filter branches required by classic multiuser detection designs. We show that the RD-MUD can perform similarly to the linear MUD detector when M is sufficiently large relative to N and K, where N and K are the number of total and active users, respectively. We also study the inherent RD-MUD tradeoff between complexity (the number of correlating signals) and BER performance. This leads to a new notion of approximate sufficient statistics, whereby sufficient statistics are approximated to reduce complexity at the expense of some BER performance loss.** [1]

I. Introduction

Multiuser detection (MUD) is a classical problem in communications and signal processing: given a noisy received signal, we would like to detect all signals (out of a set of possible choices) that comprise the received signal. For instance, in a CDMA uplink system, a number of mobile users communicate simultaneously with a given base station (BS). The BS must demodulate all of these simultaneously received signals in the presence of noise and other channel impairments.

One of the conventional methods for MUD is a matched filter (MF) bank followed by a sampler and demodulator [1]. The bank of MFs correlates the received signal with all possible waveforms, and the sampler and demodulator demodulate all users' information bits simultaneously. When the waveforms are orthogonal this MF bank with a sampler and demodulator maximizes the output signal-to-noise ratio (SNR) of each individual user.

When the signals are not orthogonal, the MF bank structure suffers from the near-far problem[2], so that any user with strong power at the receiver can degrade the bit-error-rate (BER) performance of other users. In general, the MUD that minimizes BER is the maximum likelihood sequence estimator (MLSE) [1], but it is exponentially complex in the number of users even with orthogonal signals. A lower complexity sub-optimal linear MUD detector is the decorrelator, which consists of an MF bank followed by an appropriate linear transform which eliminates multiuser interference, thereby solving the near-far problem. An alternate MUD is the linear minimum mean-square error (MMSE) detector, which requires the mean-square erro between the kth user bit and the output of the kth linear transform of the MF bank be minimized [1]. An issue with all of these conventional detectors using an MF bank front end is that when the number of users is large, building correlators for all possible waveforms can be costly in terms of hardware.

In practical systems the number of active users, K, is much smaller than the total number of users N. Motivated by the idea of compressed sensing [3], we would like to exploit this sparsity in the context of MUD to reduce the number of correlators. The reduced-dimension multiuser detector (RD-MUD) we propose can reduce the number of correlators needed at the front end of MUDs. The RD-MUD correlates the received signal with a set of M correlating signals, where M is typically much smaller than N, followed by appropriate processing on the outputs. As we show, the BER performance of RD-MUD can be similar to that of standard linear MUD receivers while using fewer hardware branches.

Previous work exploited compressed sensing ideas in MUD by equating user detection with support recovery [4][5][6]. These works establish conditions for the number of correlating signals M required to achieve a zero probability-of-false-detection (PFD) when the number of signals N tends to infinity. However, this asymptotic analysis sheds little insight into system design questions such as, e.g., how many correlating signals we should use to achieve a given probability of error target, and how to choose the correlating signals.

A key aspect of our work is that we process analog signals; most existing work on exploiting compressed sensing ideas for MUD assumes discrete signals. In particular, these detectors start with discrete samples obtained by sampling the received analog signal at the Nyquist sampling rate. Some of these detectors apply compressed sensing techniques by compressing these samples via matrix multiplication [4][5][6][7][8][9][10]. The RD-MUD takes a different approach that is closely related to analog compressive sensing [11][12][13]. However, those works focus on sparse signal estimation rather than the MUD problem considered herein.

Finally, there is another branch of compressive detection,

[1]This work supported in part by AFOSR Complex Network's program under grant FA9550-08-1-0480, IFC grant, and Stanford General Yao-Wu Wang Graduate Fellowship.

which focuses on detecting the presence of a particular signal with the signal itself being sparse [7][8][9][10]. The differences between these works and our own are as follows. First, the sparsity we employ is in the number of active signals; however, each user's signal may not be sparse. Second, their formulations primarily focus on binary detection, which corresponds to a single-user case. The PFD for a binary-detection problem was given in these previous works using the restricted-isometry-property (RIP) of correlation matrices. Although [7] showed that their research can be extended to a signal set with multiple waveforms, there was no further PFD analysis.

In this paper, we present a framework for RD-MUD that exploits the sparsity in the number of active users. Our contributions include the following: (1) We present an RD-MUD which can reduce the number of correlators compared to that of the MF bank. The RD-MUD correlates the received signal with $M \ll N$ correlating signals, followed by a linear transform of the correlation coefficients (essentially correlating the received signals with all possible discrete signal vectors). The correlating signals are linear combinations of the bi-orthogonal signals of the original signal set. (2) We characterize the tradeoff between complexity (the number of correlating signals) and BER performance, which offers a continuum of design choices for practical implementations. (3) We introduce the notion of approximate sufficient statistics: correlating with our correlating signals does not provide a set of sufficient statistics (which is obtained by correlating with N appropriate signals); however, the RD-MUD yields approximate sufficient statistics in the sense that it attains a BER performance similar to that which can be obtained by linearly processing the sufficient statistics.

The rest of the paper is organized as follows. Section II presents the system model and the RD-MUD structure. In Section III we discuss the RD-MUD noise performance and introduce the notion of approximate sufficient statistics. Section IV demonstrates the performance of the RD-MUD via several numerical examples.

Throughout, we use standard notation: $\langle x(t), y(t)\rangle = \frac{1}{T}\int_0^T x(t)y(t)dt$ denotes the inner product between two real analog signals in l_2; $\|x(t)\| = \langle x(t), x(t)\rangle^{1/2}$ is the norm of $x(t)$; $[X]_{ij}$ indicates the ijth entry of a matrix X; diag$\{x_1, \cdots, x_n\}$ denotes a diagonal matrix with the specified entries on the diagonal; $\mathbf{I}$ represents the identity matrix; $\mathbf{X}^T$, $\mathbf{X}^{-1}$, and $\mathbf{X}^\dagger$ denote the transpose, inverse, and Moore-Penrose pseudo-inverse of a matrix (or vector) $\mathbf{X}$, respectively; $\|\mathbf{x}\| = (\mathbf{x}^T\mathbf{x})^{1/2}$ is the norm of the vector $\mathbf{x}$. The function δ_{ij} is defined such that $\delta_{ij} = 1$ only when $i = j$ and equals 0 otherwise. The sign function is defined as

$$\mathbf{sign}(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \\ 0 & x = 0. \end{cases} \tag{1}$$

II. System Model and Detector Structure

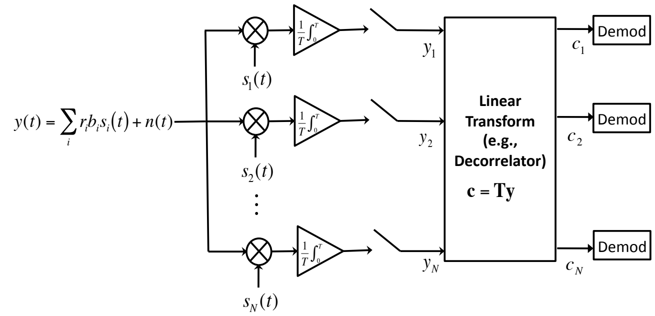

Fig. 1: Conventional MUD using a bank of MFs.

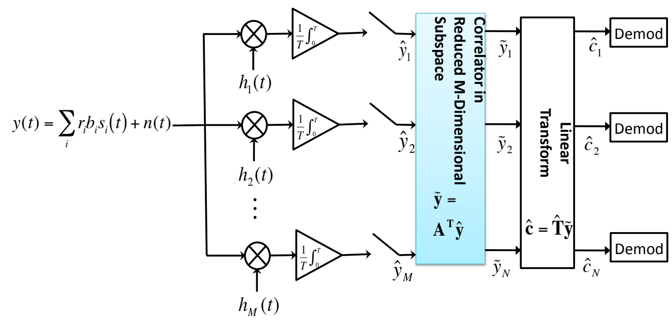

Fig. 2: The proposed RD-MUD.

Consider a multiuser system with N users, where each user is assigned a unique signal from the set $\mathcal{S} = \{s_i(t), 1 \le i \le N\}$. The signals $s_i(t)$ are linearly independent but do not have to be orthogonal. We assume for convenience that $s_i(t)$ has unit energy: $\|s_i(t)\| = 1$ for all i. The users modulate their signals using BPSK in which the information bit of user i is $b_i \in \{1, -1\}$ (the results can be extended to higher order modulation). The signal at the receiver $y(t)$ is a linear combination of the transmitted signals, plus white Gaussian noise $n(t)$ with variance σ^2:

$$y(t) = x(t) + n(t), \quad t \in [0, T], \tag{2}$$

where $x(t) = \sum_{i=1}^{N} r_i b_i s_i(t)$. The coefficient r_i captures the user's transmitting power and channel gain. For simplicity, we assume the channel gains are real and positive and that the users are synchronized so that there is no relative delay at the receiver. The nonactive users transmit at zero power, i.e., their $r_i = 0$, so with K active users, only K coefficients r_i are non-zero. Our goal is to simultaneously detect the transmitted symbols of the active users $\{b_i : r_i > 0\}$. We assume, for simplicity, that K is known.

A classical solution to this problem is a bank of matched filters (MFs) [1] followed by samplers, possibly a linear transformation on the samples, and finally by demodulators, as illustrated in Figure 1. Each MF branch correlates the received signal with a signal $s_i(t)$. For a single-user system, the MF is a maximum likelihood (ML) detector. The MF bank is an extension of an MF when there are multiple users, and it has N MFs in parallel. Using (2), the output of the MF bank can be written as:

$$y_i = \langle y(t), s_i(t)\rangle = r_i b_i + \sum_{j\neq i} [\mathbf{G}]_{ji} r_j b_j + n_i, \tag{3}$$

where $[\mathbf{G}]_{ji} = \langle s_j(t), s_i(t)\rangle$ is the Gram matrix of correlation between signals in the set. The output noise $n_i = \langle n(t), s_i(t)\rangle$ is a Gaussian random variable with zero mean and variance equal to σ^2 (since $s_i(t)$s have unit energy), and covariance $\mathbb{E}\{n_i(t), n_j(t)\} = \sigma^2[\mathbf{G}]_{ij}$ (for a derivation of the covariance see Appendix A). We can express the outputs in vector form as:

$$\mathbf{y} = \mathbf{GRb} + \mathbf{n}, \tag{4}$$

where $\mathbf{y} = [y_1, \cdots, y_N]^T$, $\mathbf{R} = \text{diag}\{r_1, \cdots, r_N\}$, and $\mathbf{b} = [b_1, \cdots, b_N]^T$. After the linear transform, the input to the demodulator is:

$$\mathbf{c} = \mathbf{Ty} = \mathbf{TGRb} + \mathbf{Tn}. \tag{5}$$

With BPSK symbols, the demodulator simply takes the sign of the input:

$$\begin{aligned} \hat{b}_i = \mathbf{sign}\left(c_i\right), &\quad \text{if user } i \text{ is active, i.e.,} \\ &|c_i| \text{ is among the largest } K \text{ elements of } \{|c_i|\}. \end{aligned} \tag{6}$$

The more general demodulator maps the output into a set of decision regions associated with the transmitted signal constellation. In this conventional MUD structure, the linear transform $\mathbf{T} \in \mathbb{R}^{N\times N}$ is used for various purposes. For example, the decorrelator detector chooses $\mathbf{T} = \mathbf{G}^{-1}$ to remove the effect of signal correlation when the signals are not orthogonal, and compensates for the difference in signal power to solve the "near-far" problem.

The RD-MUD works differently: instead of correlating the received signal $y(t)$ with each of the $s_i(t)$s, it correlates with a set of correlating signals $h_j(t)$, $j = 1, \cdots M$, as shown in Fig. 2, where M is typically much smaller than N. By using fewer correlating signals, we essentially project the received signal from a space consisting of N signals (hence N dimensional) into a lower M-dimensional subspace. The key idea is that with proper choice of the correlating signals, we can approximately preserve the information in the received signal about the transmitted symbols in the lower dimensional space.

One way of constructing these correlating signals is to use bi-orthogonal signals $\hat{s}_i(t)$ [11]. The bi-orthogonal signals can be obtained from the original signals using the inverse Gram matrix as

$$\hat{s}_j(t) = \sum_j [\mathbf{G}^{-1}]_{ij} s_i(t). \tag{7}$$

Note that when the $s_i(t)$s are orthogonal, $\mathbf{G}$ is an identity matrix, and $\hat{s}_i(t) = s_i(t)$. Per this definition, the bi-orthogonal signals have the property that $\langle s_i(t), \hat{s}_j(t)\rangle = \delta_{i,j}$, for all i, j. The correlating signals $h_j(t)$ are constructed as a linear combination of these bi-orthogonal signals:

$$h_j(t) = \sum_{i=1}^{N} a_{ji}\hat{s}_i(t), \tag{8}$$

where a_{ji}s are the coefficients of the linear combinations. Define the correlation matrix $\mathbf{A} \in \mathbb{R}^{M\times N}$ as $[\mathbf{A}]_{ji} = a_{ji}$, and the ith column of $\mathbf{A}$ as $\mathbf{a}_i = [a_{1i}, \cdots a_{Ni}]^T$, $i = 1, \cdots N$. This matrix plays an important role in the BER performance of the RD-MUD and is our design parameter.

The outputs of the correlators in the RD-MUD are given by:

$$\begin{aligned} \hat{y}_j &= \langle h_j(t), y(t)\rangle \\ &= \left\langle \sum_{i=1}^{N} a_{ji}\hat{s}_i(t), \sum_{l=1}^{N} r_l b_l s_l(t) \right\rangle + \left\langle \sum_{i=1}^{N} a_{ji}\hat{s}_i(t), n(t) \right\rangle \\ &= \sum_{l=1}^{N} r_l b_l \sum_{i=1}^{N} a_{ji} \langle \hat{s}_i(t), s_l(t)\rangle + \hat{n}_j \\ &= \sum_{l=1}^{N} a_{jl} r_l b_l + \hat{n}_j, \end{aligned} \tag{9}$$

where the output noise

$$\hat{n}_j = \sum_{i=1}^{N} a_{ji}\langle \hat{s}_i(t), n(t)\rangle, \tag{10}$$

is a Gaussian random variable with zero-mean and variance $\sigma^2\|\mathbf{a}_j\|^2$ (or σ^2 since later we impose that $\|\mathbf{a}_j\|^2 = 1$). The covariance of the noise is given by $\mathbb{E}\{\hat{n}_j\hat{n}_k\} = \sigma^2[AG^{-1}A^T]_{jk}$ (for a derivation of this covariance see Appendix B.) In (9) we have used the property of the biorthogonal signals

$\langle s_i(t), \hat{s}_j(t) \rangle = \delta_{i,j}$. It is convenient to express (9) in vector form as

$$\hat{\mathbf{y}} = \mathbf{A}\mathbf{R}\mathbf{b} + \hat{\mathbf{n}}, \tag{11}$$

where $\hat{\mathbf{y}} = [\hat{y}_1, \cdots \hat{y}_M]^T$, $\hat{\mathbf{n}} = [\hat{n}_1, \cdots \hat{n}_M]^T$. Comparing this model with (4), we find that the RD-MUD differs from the MF bank in the correlation matrix: the $N \times N$ Gram matrix $\mathbf{G}$ in the MF bank is replaced by the $M \times N$ matrix $\mathbf{A}$ in the RD-MUD. The matrix $\mathbf{A}$ projects the correlation coefficients from the original signal space into a lower dimensional space.

To demodulate the transmitted symbols, we need to recover information about the transmitted symbols from the projected subspace. This is achieved by using a linear transform

$$\tilde{\mathbf{y}} = \mathbf{A}^T \hat{\mathbf{y}}. \tag{12}$$

This linear transform essentially takes the inner products of the correlation vector $\hat{\mathbf{y}}$ with all N columns of $\mathbf{A}$, which span the lower M-dimensional subspace. Hence we can view this block in the RD-MUD, shown in Fig. 2, as a discrete correlator in the lower dimensional space. Finally, the linear transform $\hat{\mathbf{T}} \in \mathbb{R}^{M\times M}$ in the RD-MUD that precedes the demodulators plays a similar role as the linear transform $\mathbf{T}$ in the conventional MF bank: $\hat{\mathbf{T}}$ may compensate for users' power difference and alleviate the "near-far" problem. For instance, if we choose $\hat{\mathbf{T}} = [\mathbf{A}^T(\mathbf{A}\mathbf{G}^{-1}\mathbf{A}^T)\mathbf{A}]^{-1}\mathbf{A}^T(\mathbf{A}\mathbf{G}^{-1}\mathbf{A}^T)^{-1}$ then the distortions due to channel and correlating signals will be compensated for. In summary, the RD-MUD demodulates symbols for the K active users as follows:

$$\begin{aligned} &\hat{b}_i = \mathbf{sign}\left(\hat{c}_i\right), \quad \text{if user } i \text{ is active, i.e.,} \\ &\quad \text{if } |\hat{c}_i| \text{ is among the largest } K \text{ elements of } \{|\hat{c}_i|\}, \end{aligned} \tag{13}$$

where $\hat{c}_i$ is the ith element of the vector:

$$\hat{\mathbf{c}} = \hat{\mathbf{T}}\tilde{\mathbf{y}} = \hat{\mathbf{T}}\mathbf{A}^T\mathbf{A}\mathbf{R}\mathbf{b} + \hat{\mathbf{T}}\mathbf{A}^T\hat{\mathbf{n}}. \tag{14}$$

Similarly, the more general demodulator maps the output into a set of decision regions associated with the transmitted signal constellation.

The correlation matrix $\mathbf{A}$ plays an important role in the performance of the RD-MUD and is a design parameter which we can control. Comparing (14) with (5), we see that the RD-MUD is identical to a MF bank detector when $\mathbf{A}^T\mathbf{A} = \mathbf{I}$, $\mathbf{G} = \mathbf{I}$, and $\hat{\mathbf{T}} = \mathbf{T}$. However, this is not possible unless $\mathbf{A}$ is a square matrix. The intuition behind the choice of $\mathbf{A}$ is that if we can choose $\mathbf{A}$ such that $\mathbf{A}^T\mathbf{A}$ is approximately an identity matrix, then the BER performance of our RD-MUD will be similar to that of detectors based on the MF bank. Indeed, the choice of the matrix $\mathbf{A}$ links our RD-MUD to compressed sensing. In the compressed sensing literature, various conditions on a matrix $\mathbf{A}$ that yields $\mathbf{A}^T\mathbf{A}$ an approximate identity matrix have been derived. A common measure is the restricted isometry property (RIP) [3]. In the following we consider two types of random matrices $\mathbf{A}$ that possess this RIP property:

(1) The Gaussian matrix, with entries a_{ij} i.i.d. $\mathcal{N}(0,1)$, and then normalized to have unit column norm;

(2) The partial orthogonal matrix. An example of this type of matrix is the partial discrete Fourier transform (DFT) matrix, which is formed by selecting uniformly at random the rows of a DFT matrix F: $[F]_{lp} = e^{j\frac{2\pi}{N}lp}$, and then normalizing the columns of the matrix.

As we demonstrate in Section IV, the partial orthogonal matrix outperforms the Gaussian matrix in terms of BER.

The following theorem shows that if there is only one active user, then with only $M = 2$ correlating signals, the RD-MUD can demodulate the symbols of the active user perfectly in the absence of noise:

Theorem 1. *In the absence of noise, if there is only one active user $K = 1$, and if the correlation matrix A satisfies (i) the unit column norm, (ii) any column $\boldsymbol{a}_i$ is not a scalar multiple of any of the other columns $\boldsymbol{a}_j$, $i \neq j$. Then, with $M \geq 2$ correlating signals and $\hat{\boldsymbol{T}} = \boldsymbol{I}$, the RD-MUD can demodulate the symbol of the active user with zero BER.*

Proof: The demodulation of RD-MUD is based on $\hat{\mathbf{c}}$, which is given by (14). With $K = 1$, only one of the N users is active. Suppose user k is active, so that $r_i = 0$ for all $i \neq k$. With $\hat{\mathbf{T}} = \mathbf{I}$, and $n(t) = 0$, from (14) we have

$$\hat{\mathbf{c}} = \mathbf{A}^T\mathbf{A}\mathbf{R}\mathbf{b} = r_k b_k \mathbf{A}^T\mathbf{a}_k. \tag{15}$$

Equivalently, $\hat{c}_i = r_k b_k \mathbf{a}_i^T\mathbf{a}_k$. For any $i \neq k$, because of condition (ii), we have from the Cauchy-Schwartz inequality:

$$|\hat{c}_i| = |r_k b_k| \mathbf{a}_i^T\mathbf{a}_k < |r_k b_k| \|\mathbf{a}_i\| \|\mathbf{a}_k\| = |r_k b_k| \|\mathbf{a}_k\|^2 = |\hat{c}_k|. \tag{16}$$

Hence the active user is correctly detected as the kth user. Finally, the demodulator detects his transmitted symbol as:

$$\mathbf{sign}(\hat{c}_k) = \mathbf{sign}(r_k b_k \|\mathbf{a}_k\|^2) = \mathbf{sign}(b_k) = b_k, \tag{17}$$

which completes the proof. ■

III. Noise Performance and Approximate Sufficient Statistics

In the presence of noise, for the RD-MUD to correctly demodulate the multiple users, we need several conditions. First, the non-active users are not detected as active users, i.e., the noise is not misinterpreted as transmitted symbols; the active users are not classified as non-active users; and third, the active users, when correctly identified as active, must have their symbols demodulated correctly. All of these conditions involves noise analysis. The third condition is satisfied when the noise is not too large that it exceeds the decision boundary of the demodulator. The first two conditions involve correct active user detection, for which we have the following insights from a geometric point of view: with the appropriate choice of M correlating signals constructed using bi-orthogonal signals, RD-MUD projects the received signal as well as the original N analog signals onto an M-dimensional vector subspace. The MF bank and RD-MUD perform detection in the original and projection spaces, respectively. The projection is illustrated in Fig. 3.

- MF bank: Project the received signal $y(t)$ onto the space spanned by the signals $s_i(t)$ by correlation $\langle y(t), s_i(t)\rangle$, $i = 1, \cdots N$; then detect K signals with the largest inner products to the received signal as active users: $\langle y(t), s_i(t)\rangle$.
- RD-MUD: Project both the received signal $y(t)$ and all signals $s_i(t)$ onto the space spanned by the correlating signals $h_j(t)$, $j = 1, \cdots M$: $\hat{y}_j = \langle h_j(t), y(t)\rangle$, $\hat{s}_{ij} = \langle h_j(t), s_i(t)\rangle$; then detect K projected signals with the largest inner products to the projected received signal in the lower dimensional space as active users.

In the presence of noise, if the inner product can be approximately preserved in the projection space, the RD-MUD will have a performance similar to that of the MF bank. This explains why we choose $\mathbf{A}$ to have the RIP property: when the correlation matrix $\mathbf{A}$ has the RIP, the inner products in the original space when projected by the matrix $\mathbf{A}$ will be preserved in the lower dimensional space.

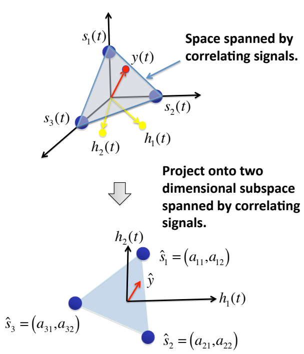

Fig. 3: The projection performed by RD-MUD. The figure illustrate the case $N = 3$, $M = 2$, and the received signal is due to two active users.

All the information in $y(t)$ about the transmitted symbols is captured by the sufficient statistics for the transmitted symbols [1]. The MF bank yields a set of sufficient statistics about the transmitted symbols $\{b_i\}$ given $y(t)$ [1]. Clearly, the output from RD-MUD does not constitute a set of sufficient statistics. However, as we will show in the numerical examples, the performance of the RD-MUD is similar to that of a MF bank if M is sufficiently large relative to N and K. In this sense, the RD-MUD yields a set of "approximate sufficient statistics" in that it approximates the BER performance of the sufficient statistics for MUD.

In the presence of noise, we can prove that when the number of correlating signals in RD-MUD is at least on the order of:

$$M \gtrsim \frac{K}{\text{SNR}(1+\text{SIR})} \log\left(\frac{N}{K}\right), \tag{18}$$

and the correlating matrix $\mathbf{A}$ satisfies the RIP, the BER performance of RD-MUD using A can approach that of the MF bank (when the original waveforms are orthogonal) or the decorrelator (when the original waveforms are non-orthogonal), with high probability. Here

$$\text{SNR} = \max_k \left\{\frac{r_k}{\sigma}\right\} \tag{19}$$

is the ratio of strongest signal amplitude over the square-root of noise power, and

$$\text{SIR} = \max_k \left\{\sum_{j\neq k} \frac{r_j[\mathbf{G}]_{jk}}{r_k}\right\} \tag{20}$$

can be interpreted as the strongest interference-to-signal ratio among all the users. The statement of the theorem and proof will be presented in our future journal paper.

IV. Numerical Examples

Next we present some numerical examples demonstrating the performance of RD-MUD. We use an average BER over all users as a performance metric. We assume there are $N = 8$ users, and $K = 2$ active users in the system, the users' signals $s_i(t)$ are orthogonal, and BPSK modulation is used. All the following examples are obtained from 50000 Monte Carlo trials.

Example 1: Multiuser Detection

We assume SNR = 15 dB, and use a Gaussian or partial DFT correlation matrix for RD-MUD. Fig. 4 shows the BER versus M. Note that when M approaches N, the RD-MUD has a BER performance very similar to that of the MF bank. If we allow a target BER of 10^{-2} then with RD-MUD $M = 5$ will achieve this goal. Also note that the partial DFT matrix outperforms the Gaussian matrix in terms of BER.

Fig. 4: BER versus M for $N = 8$ and $K = 2$ active users.

Example 2: Scaling of M vs. N

The second example studies how many correlating signals M are needed for the RD-MUD to achieve ($\leq 10\%$) BER performance degradation relative to that of the MF bank. There is only one active user, i.e., $K = 1$. We use SNR = 20 dB and a partial DFT matrix. Fig. 5 shows that under this setting M is at most $N/2$. This means that a 50% saving of signal correlators is possible with the RD-MUD. Also note that this saving in complexity increases with N.

Fig. 5: M vs. N for a BER performance degradation (compared with MF) less than 10%.

V. Conclusions and Future Work

We presented a general framework for reduced-dimension multiuser detection (RD-MUD), which employs the user sparsity in a multiuser system to achieve lower complexity than the conventional matched filter (MF) bank for multiuser detection. We proved theoretically and demonstrated via numerical examples that RD-MUD can perform similarly to a MF bank when the number of correlating signals is sufficiently large. We also introduced the new notion of approximate sufficient statistics. This tradeoff can hopefully shed more insights into the practical multiuser detection system design tradeoffs.

References

[1] S. Verdu, *Multiuser Detection*. Cambridge University Press, 1998.

[2] A. Duel-Hallen, J. Holtzman, and Z. Zvonar, “Multiuser detection for CDMA systems,” *IEEE Personal Communications*, pp. 46–58, April 1995.

[3] E. J. Candes and T. Tao, “Near-optimal signal recovery from random projections: universal encoding strategies?,” *IEEE Transactions on Information Theory*, vol. 52, pp. 5406–5424, Dec. 2006.

[4] S. R. A. K. Fletcher and V. K. Goyal, “On-off random access channels: A compressed sensing framework,” *submitted to IEEE Trans. Information Theory and arXived.*, March 2010.

[5] S. R. A. K. Fletcher and V. K. Goyal, “Necessary and sufficient conditions on sparsity pattern recovery,” *IEEE Trans. Information Theory*, Jan. 2009.

[6] Y. Jin, Y.-H. Kim, and B. D. Rao, “Support recovery of sparse signals,” *submitted to IEEE Trans. Information Theory*, March 2010.

[7] M. A. Davenport, P. T. Boufounos, M. B. Wakin, and R. G. Baraniuk, “Signal processing with compressive measurements,” *to appear in Journal of Selected Topics in Signal Processing*, 2010.

[8] M. A. Davenport, M. B. Wakin, and R. G. Baraniuk, “Detection and estimation with compressive measurements,” *Tech. Rep. TREE 0610, Department of Electrical and Computer Engineering, Rice University*, 2007.

[9] J. Haupt and R. Nowak, “A generalized restricted isometry property,” *Tech. Rep. ECE-07-1, University of Wisconsin*, 2007.

[10] J. Haupt and R. Nowak, “Compressive sampling for signal detection,” *IEEE International Conference on Acoustics Speech and Signal Processing*, vol. 3, pp. 1509–1512, 2007.

[11] Y. C. Eldar, “Compressed sensing of analog signals in shift-invariant spaces,” *IEEE Trans. on Signal Processing*, vol. 57, pp. 2986–2997, August 2009.

[12] M. Mishali, Y. C. Eldar, and A. Elron, “Xampling: Signal acquisition and processing in union of subspaces,” *arXiv: 0911.0519*, 2009.

[13] M. Mishali and Y. C. Eldar, “From theory to practice: Sub-Nyquist sampling of sparse wideband analog signals,” *IEEE Journal of Selected Topics in Signal Processing,*, vol. 4, pp. 375 – 391, April 2010.

Appendix A
Covariance of MF Bank Output Noise

Consider the covariance of the output noise at the ith and jth MFs in the MF bank:

$$\begin{aligned}\mathbb{E}\{n_i n_j\} &= \mathbb{E}\left\{\int_0^T\int_0^T s_i(t)s_j(u)n(t)n(u)dtdu\right\}\\ &= \int_0^T\int_0^T s_i(t)s_j(u)\mathbb{E}\left\{n(t)n(u)\right\}dtdu\\ &= \int_0^T\int_0^T s_i(t)s_j(u)\sigma^2\delta(t-u)dtdu\\ &= \sigma^2\int_0^T s_i(t)s_j(t)dt = \sigma^2[G]_{ij}.\end{aligned} \tag{21}$$

Appendix B
Covariance of RD-MUD Output Noise

Consider the covariance of the output noise at the ith and jth branches in the RD-MUD after the linear transform $\mathbf{A}^T$:

$$\begin{aligned}\mathbb{E}\{\hat{n}_j\hat{n}_k\} &= \mathbb{E}\left\{\sum_{i=1}^N\sum_{l=1}^N a_{ji}a_{kl}\langle\hat{s}_i(t),n(t)\rangle\langle\hat{s}_l(t),n(t)\rangle\right\}\\ &= \sum_{i=1}^N\sum_{l=1}^N a_{ji}a_{kl}\mathbb{E}\{\langle\hat{s}_i(t),n(t)\rangle\langle\hat{s}_l(t),n(t)\rangle\}.\end{aligned} \tag{22}$$

We now look at $\mathbb{E}\{\langle\hat{s}_i(t),n(t)\rangle\langle\hat{s}_l(t),n(t)\rangle\}$:

$$\begin{aligned}&\mathbb{E}\{\langle\hat{s}_i(t),n(t)\rangle\langle\hat{s}_l(t),n(t)\rangle\}\\ &=\int_0^T\int_0^T \hat{s}_i(t)\hat{s}_l(t)\mathbb{E}\{n(t)n(s)\}dtds\\ &=\int_0^T\int_0^T \hat{s}_i(t)\hat{s}_l(t)\sigma^2\delta(t-s)dtds\\ &=\sigma^2\int_0^T \hat{s}_i(t)\hat{s}_l(t)dt\\ &=\sigma^2\left\langle\sum_j[G^{-1}]_{ij}s_j(t),\sum_k[G^{-1}]_{lk}s_k(t)\right\rangle\\ &=\sigma^2\sum_j\sum_k[G^{-1}]_{ij}[G^{-1}]_{lk}\langle s_j(t),s_k(t)\rangle\\ &=\sigma^2\sum_j\sum_k[G^{-1}]_{ij}[G^{-1}]_{lk}[G]_{jk}\\ &=\sigma^2[G^{-1}]_{il}\end{aligned} \tag{23}$$

Plugging this back into (22), we have

$$\begin{aligned}\mathbb{E}\{\hat{n}_j\hat{n}_k\} &= \sigma^2\sum_{i=1}^N\sum_{l=1}^N a_{ji}a_{kl}[G^{-1}]_{il}\\ &= \sigma^2[AG^{-1}A^T]_{jk}.\end{aligned} \tag{24}$$