

# Analog Network Coding in the High-SNR Regime

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**Abstract**—A node performing analog network coding simply forwards a signal it receives over a wireless channel. This allows for a (noisy) linear combination of signals simultaneously sent from multiple sources to be forwarded in the network. As such, analog network coding extends the idea of network coding to wireless networks. However, the analog network coding performance is limited by propagated noise, and we expect this strategy to perform well only in high SNR. In this paper, we formalize this intuition and determine high-SNR conditions under which analog network coding approaches capacity in a layered relay network. By relating the received SNR at the nodes with the propagated noise, we determine the rate achievable with analog network coding. In particular, when all the received powers are lower bounded by  $1/\delta$ , the propagated noise power in a network with  $L$  layers is of the order  $L\delta$ . The result demonstrates that the analog network coding approaches the cut-set bound as the received powers at relays increase. As all powers in the network increase, the analog network coding rate is within a constant gap from the upper bound. The gap depends on number of nodes. We further demonstrate by an example that analog network coding can perform close to sum-capacity also in the multicast case.

## I. INTRODUCTION

For noiseless networks on graphs, network coding achieves the *multicast* capacity, i.e., the highest rate at which a source can send information to a set of destination nodes [1]. The multicast capacity can be achieved with *linear* network coding [2]. This result implies that each node only has to send out a linear combination of its received packets. Destination nodes effectively obtain source information multiplied by a *transfer* matrix determined by a network graph, and can recover the original data provided that the matrix is invertible [3].

The capacity of wireless networks even with a single source-destination pair is still unknown. The deterministic view of wireless networks [4] led to the characterization of the network capacity within a constant; it has been shown that in a wireless network with a single source-destination pair, compress-and-forward [5] achieves the cut-set bound within a constant gap [6]. This gap does not depend on channel gains, but it increases with the number of network nodes. In [7], Kramer demonstrated that at high-SNR, decode-and-forward [5] exhibits a good scaling performance where the gap from the cut-set bound increases only logarithmically with the

number of nodes. For multiple source networks, an extension of compress-and-forward was more recently developed in [8]. It was demonstrated that the proposed scheme outperforms existing compress-and-forward strategies, without requiring Wyner-Ziv coding [9].

In a wireless channel, signals simultaneously transmitted from multiple sources add up in the air resulting in interference; a receiver obtains a noisy sum of these signals, each scaled by a channel gain. Relays exploit this interference by forwarding it through the network to their destinations. Because relays are not interested in these messages, decoding them can unnecessarily limit the transmission rates. In fact, since the receiver already receive the sum of the signals, a natural strategy, following the idea of network coding, would be to forward the received sum after clearing the noise. A technique that exploits this idea by having relays decode linear functions of sent data, *compute-and-forward*, was recently proposed and demonstrated to perform well in certain regimes [10]. In this strategy, a relay has to map the received signal sum to a lattice codeword.

A simpler strategy that alleviates this need is to amplify and forward the observed noisy signal sum. Unlike the amplify-and-forward in the relay channel, in a network, the forwarded signal carries information about signals sent by multiple sources, in that way extending the idea of network coding to the physical layer. To emphasize this fact, in this paper we will refer to this scheme as analog network coding [11]. The drawback of analog network coding is noise propagation. Consequently, at low SNRs amplify-and-forward in relay networks reduces to no relaying, i.e., it reduces to a direct transmission from the source [12], [13].

Gastpar and Vetterli showed that uncoded transmission and two-hop amplify-and-forward achieve the constant gap from the cut-set bound in the limit of large number of relays [14, Sec.VIII]. In the proposed scheme, the source transmits in the first slot and the relays amplify-and-forward the observed noisy signals in the second slot. Therefore, a message reaches the destination in two hops and the noise is propagated only for one hop. This approach avoids noise propagation through the network, but reduces the rate by half. The advantages of two-hop amplify-and-forward have also been demonstrated for multiple antenna networks and fading channels (see [15] and references therein).

In this paper, we will propose an analog network coding scheme that takes more of the network coding approach,

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where data is propagated over many intermediate nodes. The diversity-multiplexing tradeoff (DMT) of multihop amplify-and-forward when relays have multiple antennas was characterized in [16]. By considering a deterministic wireless network, the diversity and degrees-of-freedom were analyzed in [17]. For a special type of networks, DMT of this scheme was also considered in [18], [19]. In this paper, we will derive the rate achievable with the multihop amplify-and-forward scheme and show that it achieves capacity in the regime in which the propagated noise is negligible.

Intuition suggests that the noise amplification drawback of analog network coding should diminish at high SNR. In fact, it was shown that amplify-and-forward multihopping achieves full degrees of freedom of the MIMO system [20]. This intuition might seem to contradict results in [6, Sec. III] where it was demonstrated that analog network coding can have an unbounded gap from capacity in the high channel gain regime. As the main contribution in this paper we will show the intuition that, in the high-SNR regime analog network coding approaches network capacity, is correct. In fact, one of the key insights from our work is that high channel gains do not necessarily lead to the high-SNR regime in a multihop network, unlike in the point-to-point channel. In a multihop network, even for high channel gains, noise propagation can lead to low SNRs at the nodes. In this paper, we relate the received SNRs and the propagated noise powers at the nodes in the analog network coding scheme. We determine high-SNR conditions under which analog network coding approaches capacity in a layered wireless relay network. We further demonstrate that at high-SNR, analog network coding rate has a favorable scaling, i.e., it is within a constant gap from the upper bound as the received powers increase. We also demonstrate by an example that analog network coding can perform close to sum-capacity also in the multicast case.

This paper is organized as follows. The considered network model is presented in Section II. The main result on the analog network coding performance and capacity is presented in Section III. Two examples demonstrating the capacity-achieving performance of analog network coding in the high-SNR regime are presented in Section IV. Section V extends the consideration to a multicast problem. Section VI discusses extensions. Section VII concludes the paper. The proofs are omitted for the lack of space and can be found in [21].

## II. NETWORK MODEL

We consider a wireless network with one source-destination pair and  $N$  relays. The channel output at node  $k$  is

$$y_k = \sum_{j \in \mathcal{N}(k)} h_{jk} x_j + z_k \quad (1)$$

where  $h_{jk}$  is a real number representing the channel gain from node  $j$  to node  $k$ ,  $\mathcal{N}(k)$  denotes neighboring nodes of node  $k$  and  $z_k$  is the Gaussian noise with zero mean and variance 1. We assume that there is a power constraint at node  $k$ :

$$E[X_k^2] \leq P_k. \quad (2)$$

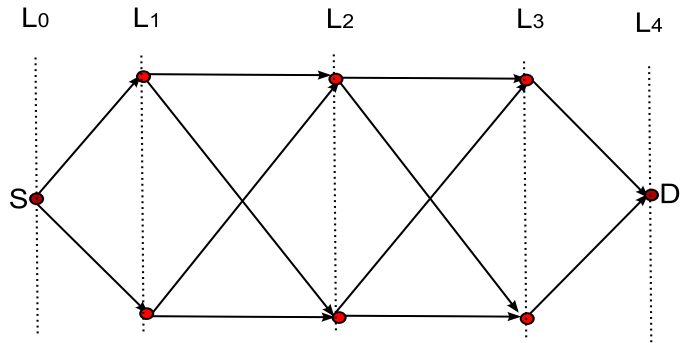


Fig. 1. Layered network with 4 layers and 2 relays at each layer.

All nodes are full-duplex. Source  $S$  wishes to send a message from a message set  $\mathcal{W} = \{1, \dots, 2^{nR}\}$  to destination node  $D$ . The encoding function at the source is given by  $X_s^n = f(W)$ . An  $(R, n)$  code consists of a message set, an encoding function at the source encoder, an encoding function at each node  $k$ , that at time  $i$  performs  $X_{k,i} = f_{k,i}(Y_k^{i-1})$ , and a decoding function at the destination node  $D$ :  $\hat{W} = g(Y_D^n)$ . The average error probability of the code is given by  $P_e = P[\hat{W} \neq W]$ . A rate  $R$  is achievable if, for any  $\epsilon > 0$ , there exists, for a sufficiently large  $n$ , a code  $(R, n)$  such that  $P_e \leq \epsilon$ .

### A. Layered Networks

As in [6], we will initially consider layered networks in which each path from the source to the destination has the same number of hops. We consider the source node to be at layer 0 and the destination at layer  $L$ . We denote number of relays at layer 1 as  $n_l$ , hence  $\sum_{l=1}^{L-1} n_l = N$ . A layered network with  $L = 4$  layers and 2 relays at each layer is shown in Fig. 1. In a layered network with  $L$  layers, the input-output relationship is simplified because all copies of a source input traveling on different paths arrive at a given destination with an  $L - 1$  time delay. For that reason, from now on we drop the time index in the notation. We denote a transmitted vector at layer  $l$  as  $\mathbf{x}_l = [x_{l,1} \dots x_{l,n_l}]^T$  where  $x_{l,i}$  denotes  $x_i$  when  $i \in l$ . We accordingly define the received signal  $\mathbf{y}_l$  and noise  $\mathbf{z}_l$  at layer  $l$ . We let  $\mathbf{H}_l$  denote the channel matrix between layers  $l$  and  $l + 1$ . An element  $\mathbf{H}_l(i, j)$  is the channel gain from node  $i$  at layer  $l$  to node  $j$  at level  $l + 1$ . As observed in [20], the received vector at layer  $l + 1$  can then be written as

$$\mathbf{y}_{l+1} = \mathbf{H}_l \mathbf{x}_l + \mathbf{z}_{l+1}. \quad (3)$$

## III. MAIN RESULT

### High-SNR Regime

We are interested in the regime in which nodes transmit with high enough power so that the noise propagated by analog network coding is low. When each node  $j$  transmits with  $P_j$  given in (2), we denote the power received at node  $k \in l$  as

$$P_{R,k} = \left( \sum_{j \in \mathcal{N}(k)} h_{jk} \sqrt{P_j} \right)^2, \quad k \in l. \quad (4)$$

*Definition.* A wireless network is in the high-SNR regime if

$$\frac{1}{\min_{k \in l} P_{R,k}} \leq \delta, \quad l = 1, \dots, L-1 \quad (5)$$

for some small  $\delta \geq 0$ .

*Remark 1:* Condition (5) implies that the received SNR at every transmitting node  $k$  is large, i.e.,

$$P_{R,k} \gg 1, \quad k \in l, \quad l = 1, \dots, L-1. \quad (6)$$

For brevity and with a slight abuse of notation, we denote the received power at the destination as  $P_D$ :

$$P_D = \left( \sum_{i \in \mathcal{N}(D)} h_{iD} \sqrt{P_i} \right)^2. \quad (7)$$

We observe that the received SNR at the destination does not need to satisfy (5) and (6). In that case, the bottleneck on the data transfer is on the multi-access (MAC) side of the network. The MAC cut-set bound [22] evaluates to

$$C_{\text{MAC}} = \frac{1}{2} \log(1 + P_D). \quad (8)$$

We will address both cases: 1)  $P_D = \text{const.}$  as  $\delta \rightarrow 0$  and thus the MAC at the destination is a bottleneck; 2)  $P_D$  increases as  $\delta \rightarrow 0$  such that  $\delta P_D = \text{const.}$  This is the case when, for example, all transmit powers increase at the same rate.

#### Analog Network Coding

In the considered transmission scheme, the source node encodes using the Gaussian codebook  $X_s \sim \mathcal{N}[0, P_s]$  where  $\mathcal{N}[0, \sigma^2]$  denotes normal distribution with zero mean and variance  $\sigma^2$ . Each network node  $j$  at layer  $l \in \{1, \dots, L-1\}$  performs analog network coding, i.e., at time  $i$  transmits:

$$x_{l,j}(i) = \beta_{l,j} y_{l,j}(i-1) \quad (9)$$

where the amplification gain  $\beta_{l,j}$  is chosen such that the power constraint (2) is satisfied. In a layered network, this corresponds to the transmit vector at layer  $l$ :

$$\mathbf{x}_l = \mathbf{B}_l \mathbf{y}_l \quad (10)$$

where  $\mathbf{B}_l = \text{diag}\{\beta_{l,1} \dots \beta_{l,n_l}\}$ . From (3) and (10), the received signal at any layer  $l$  is given by

$$\mathbf{y}_l = \mathbf{H}_{l-1} \mathbf{B}_{l-1} \dots \mathbf{H}_1 \mathbf{B}_1 \mathbf{H}_0 x_s + \sum_{i=1}^{l-1} \mathbf{H}_{l-1} \mathbf{B}_{l-1} \dots \mathbf{H}_i \mathbf{B}_i \mathbf{z}_i + \mathbf{z}_l. \quad (11)$$

Each term in the sum is noise propagated from layer  $i$  to layer  $l$ . We choose the amplification gain at node  $j$  at level  $l$  as

$$\beta_{l,j}^2 = \frac{P_j}{(1+\delta)P_{R,j}}, \quad j \in l. \quad (12)$$

For brevity, we use  $\beta_j$  instead of  $\beta_{l,j}$  when specified that  $j \in l$ .

*Lemma 1:* At every node performing analog network coding with the amplification gain (12), the power constraint (2) is satisfied.

Both Lemma 1 and Theorem 1 rely on the following key lemma.

*Lemma 2:* At any node  $j$  at layer  $l$ , noise propagated from a layer  $l-d$ ,  $d \in \{1, \dots, l-1\}$  via analog network coding, has a power

$$P_{Z,j}^{(l-d)} \leq \frac{\delta P_{R,j}}{(1+\delta)^d}. \quad (13)$$

*Remark 2:* From (13), it follows that the total noise propagated to level  $L$  (i.e., the destination) is

$$P_{Z,D} = \sum_{d=1}^{L-1} P_{Z,D}^{(l-d)} = \delta P_D \sum_{d=1}^{L-1} \frac{1}{(1+\delta)^d} \leq L \delta P_D. \quad (14)$$

The following theorem is the main result of our paper.

*Theorem 1:* In a layered relay network (1) in the high-SNR regime (5), analog network coding achieves the rate

$$R = \frac{1}{2} \log \left( 1 + \frac{1}{(1+\delta)^{L-1}} \frac{P_D}{P_{Z,D} + 1} \right) \quad (15)$$

where  $P_{Z,D}$  is given by (14).

*Remark 3:* For  $\delta \rightarrow 0$ , and  $P_D = \text{const.}$ , (14) implies that  $P_{Z,D} \rightarrow 0$ ; the achievable rate (15) then approaches the MAC cut-set bound (8), and thus the capacity.

*Remark 4:* From (15), we also obtain the scaling behavior of analog network coding when all received powers increase at the same rate, i.e.,  $\delta \rightarrow 0$ , and  $\delta P_D = \text{const.}$  By comparing (8) and (15) in that regime, we have that the rate is within  $1/2 \log(L \text{ const})$  from the MAC cut-set bound.

*Remark 5:* From (15), we also obtain the first-order approximation as

$$R = \frac{1}{2} \log(1 + P_D) - O(\delta). \quad (16)$$

Proofs are omitted. The key part in showing the above result is in proving Lemma 2. We next illustrate the above result for several networks.

## IV. EXAMPLES

### Example 1: Diamond Network

It was observed in [6] that analog network coding in a diamond network (first analyzed in [12] and shown in Fig. 2 for the choice of channel gains as in [6]) cannot achieve the cut-set bound when  $a$  is large and transmit powers are set to 1. Rather, the gap between the analog network coding rate and cut-set bound increases as  $a$  increases. We consider a different regime and show that, for any value of  $a$ , there is a range of power  $P_s$  for which analog network achieves capacity. The MAC cut-set bound (8) in a diamond network evaluates to

$$C_{\text{MAC}} = \frac{1}{2} \log(1 + h_{1D}^2 P_1 + h_{2D}^2 P_2 + 2h_{1D} h_{2D} \sqrt{P_1 P_2}) \quad (17)$$

where  $h_{iD}$  is the channel gain from relay  $i$  to the destination. With analog network coding, the SNR at the destination is

$$SNR_D = \frac{h_{1D}^2 P_1 + h_{2D}^2 P_2 + 2h_{1D} h_{2D} \sqrt{P_1 P_2}}{\frac{h_{1D}^2 P_1}{h_{s1}^2 P_s} + \frac{h_{2D}^2 P_2}{h_{s2}^2 P_s} + 1} \quad (18)$$

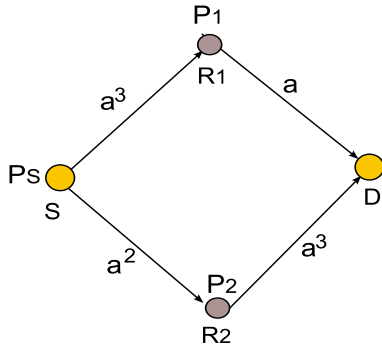


Fig. 2. Diamond network.

where, due to (5), we can approximate  $\beta_i^2 = P_i/(h_{si}^2 P_s)$ ,  $i = 1, 2$ . Condition (5) implies that

$$SNR_D \geq \frac{h_{1D}^2 P_1 + h_{2D}^2 P_2 + 2h_{1D}h_{2D}\sqrt{P_1 P_2}}{\delta(h_{1D}^2 P_1 + h_{2D}^2 P_2) + 1} \quad (19)$$

and the achievable rate is in the first-order approximation,

$$R = \frac{1}{2} \log \left( 1 + (h_{1D}\sqrt{P_1} + h_{2D}\sqrt{P_2})^2 \right) - O(\delta). \quad (20)$$

Hence, for the class of diamond networks in the high-SNR regime, the capacity can be achieved with analog network coding. In terms of powers, from (17) and (18) we see that rate (18) approaches capacity in diamond networks that satisfy

$$\frac{h_{1D}^2 P_1}{h_{s1}^2 P_s} + \frac{h_{2D}^2 P_2}{h_{s2}^2 P_s} \ll 1. \quad (21)$$

For the specific choice of channel gains as in Fig. 2 and relay powers  $P_1 = P_2 = 1$ , the MAC bound (17) for large  $a$  is

$$C_{MAC} = 3 \log(a). \quad (22)$$

For  $P_s \geq a^2$  and large  $a$ , the achievable rate, determined by (18), is

$$R = 3 \log(a) - 1. \quad (23)$$

Furthermore, as  $P_s$  increases, the  $SNR_D$  in (18) approaches the SNR in the cut-set bound (17).

The comparison of the analog network coding rate with the MAC cut-set bound is shown in Fig. 3. The capacity within a bit is achieved for  $P_s \geq 35$ . For  $P_s = 1$ , we recover the example from [6], and indeed observe a gap from the capacity.

We also examine scaling of the achievable rate and the cut-set bound when all transmit powers in the network increase, while their ratio is kept constant. This is equivalent to  $\delta P_D = \text{const}$ . Channel gains are fixed as given by the network topology. From (17) and (18), the difference between the two bounds for any value of gains, and for large powers evaluates to a constant

$$C_{MAC} - R = \frac{1}{2} \log c \quad (24)$$

where  $c$  is the denominator in (18). The behavior is shown in Fig. 4 when all transmit powers are chosen equal (denoted  $P$ ). All channel gains equal 1. The difference between (17) and (18) for this choice of parameters is  $0.5 \log 3$ , for large  $P$ .

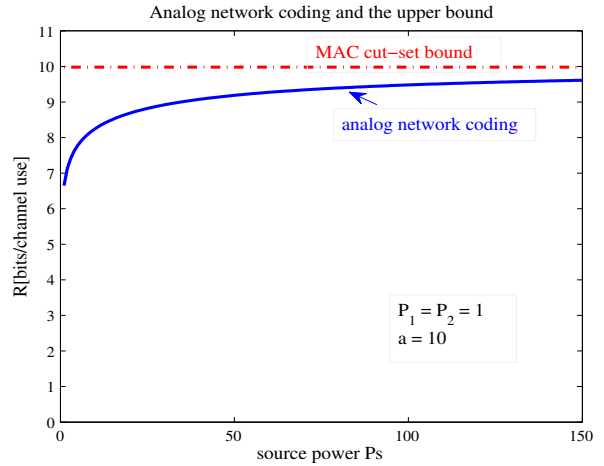


Fig. 3. Analog network coding rate and the MAC cut-set bound in a diamond network for  $a = 10$ . We observe that the achievable rate approaches the cut-set bound as the source power increases.

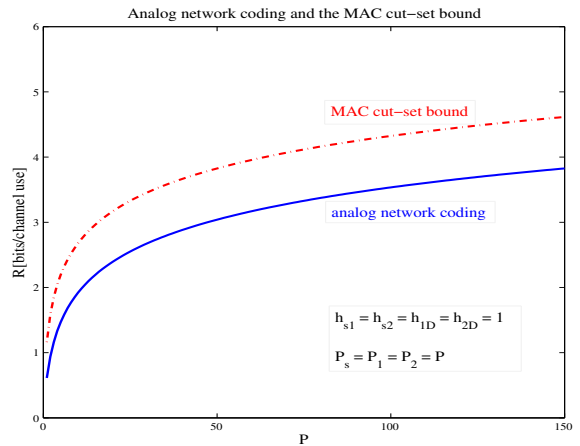


Fig. 4. Analog network coding rate and the MAC cut-set bound in a diamond network as transmit powers increase. We observe the same trend in the two bounds. Analog network coding is within a constant gap from the cut-set bound.

### Example 2: 3-Layer Network

We next consider the 3-layer network shown in Fig. 5. For analog network coding using amplification (12), the SNR at the destination in high-SNR can be shown [21] to satisfy

$$SNR_D > \frac{1}{(1 + \delta)^2} \frac{P_D}{\frac{\delta P_D}{(1 + \delta)^2} + \frac{\delta P_D}{1 + \delta} + 1}. \quad (25)$$

We observe the following:

- 1) For  $P_D$  constant,  $SNR_D$  approaches  $P_D$  as  $\delta \rightarrow 0$ , and analog network coding achieves the MAC cut-set bound (8). This behavior is shown in Fig. 6. We observe that the analog network coding rate approaches the capacity to within one bit as  $P_s > 30$ , and is within a small fraction of a bit for  $P_s > 200$ .
- 2) Fig. 7 shows the rate and the cut-set bound when all powers increase, and channel gains are fixed. In this

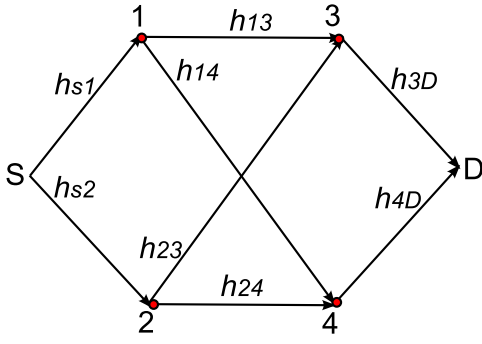


Fig. 5. 3-layer network.

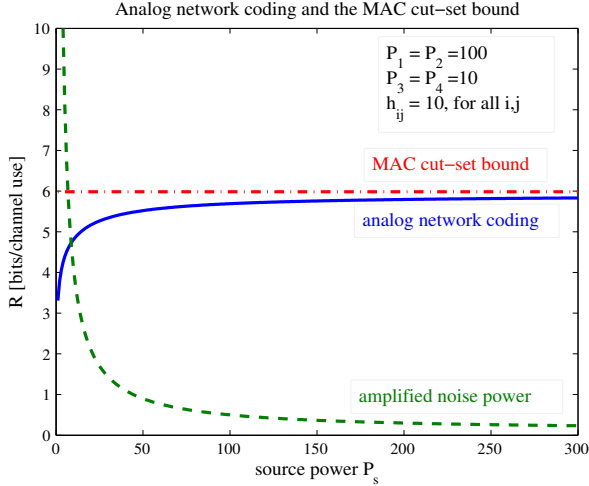


Fig. 6. Analog network coding rate and the MAC cut-set bound in a 3-layer network.

case,  $\delta \rightarrow 0$  and  $\delta P_D = \text{const}$ . As in the previous example, we observe a constant gap between the rate and the cut-set bound.

## V. MULTICAST

We next illustrate by an example that analog network coding can be efficient in terms of the sum-rate, for the multicast traffic for the network in the high-SNR regime. We consider a 2-layer network (see Fig. 8) with two sources (nodes 1 and 2) performing analog network coding (9), and two destinations (nodes 5 and 6). Each source wishes to multicast a message to both destinations. Respective channel inputs at sources 1 and 2 are  $x_1$  and  $x_2$ . The received signals at nodes 3 and 4 are

$$y_k = h_{1k}x_1 + h_{2k}x_2 + z_k, \quad k = 3, 4. \quad (26)$$

Note that  $x_1$  and  $x_2$  are independent and hence there is no coherent combining at the receivers. Therefore, amplification gains at nodes 3 and 4 in the high-SNR regime can be approximated as  $\beta_k^2 \leq P_k / (h_{1k}^2 P_1 + h_{2k}^2 P_2)$ ,  $k = 3, 4$ . The received signal at node 5 is:

$$\begin{aligned} y_5 &= h_{35}x_3 + h_{45}x_4 + z_5 \\ &= h_{1,eq}x_1 + h_{2,eq}x_2 + h_{35}\beta_3 z_3 + h_{45}\beta_4 z_4 + z_5 \end{aligned} \quad (27)$$

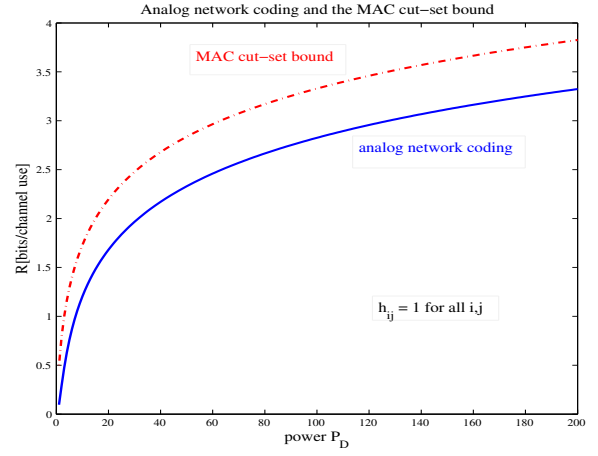


Fig. 7. The analog network coding rate and the MAC cut-set bound for the 3-layer network as all the powers increase.

where we denote

$$h_{j,eq} = h_{35}h_{j3}\beta_3 + h_{45}h_{j4}\beta_4, \quad j = 1, 2. \quad (28)$$

Equivalent relationship can be obtained at node 6. Eq. (28) describes a multiaccess (MAC) channel. The MAC capacity [22] determines the rates achievable at node 5 as

$$\begin{aligned} R_1 &\leq \frac{1}{2} \log\left(1 + \frac{h_{1,eq}^2 P_1}{1 + P_{Z,eq}}\right) \\ R_2 &\leq \frac{1}{2} \log\left(1 + \frac{h_{2,eq}^2 P_2}{1 + P_{Z,eq}}\right) \\ R_1 + R_2 &\leq \frac{1}{2} \log\left(1 + \frac{h_{1,eq}^2 P_1 + h_{2,eq}^2 P_2}{1 + P_{Z,eq}}\right) \end{aligned} \quad (29)$$

where  $P_{Z,eq}$  is the power of amplified noise in (27) given by

$$P_{Z,eq} = \frac{h_{35}^2 P_3}{h_{13}^2 P_1 + h_{23}^2 P_2} + \frac{h_{45}^2 P_4}{h_{14}^2 P_1 + h_{24}^2 P_2}. \quad (30)$$

In the high-SNR regime,  $P_{Z,eq} \rightarrow 0$  and hence the total noise power (and the denominators in (29)) is identity. Therefore, by substituting  $\beta_k$ ,  $k = 3, 4$  and (28) in (29), and by using (30), we obtain that the achievable sum-rate satisfies

$$R_1 + R_2 > 1/2 \log(1 + h_{35}^2 P_3 + h_{45}^2 P_4). \quad (31)$$

We next evaluate the MAC cut-set bound at node 5 as

$$\begin{aligned} C_{MAC} &= I(X_3, X_4; Y_5) \\ &= 1/2 \log(1 + (h_{35}\sqrt{P_3} + h_{45}\sqrt{P_4})^2). \end{aligned} \quad (32)$$

Following the same steps, we can evaluate the achievable rate and the MAC cut-set bound at node 6. By comparing the sum-rate lower bound (31) and the MAC cut-set bound (32), we observe that the gap is in the coherent combining gain, and hence at most 1/2 bit. Therefore, when the considered network is in the high-SNR regime, the sum-rate achievable with analog network coding and the cut-set bound differ due to the coherent combining gain gap by at most 1/2 bit.

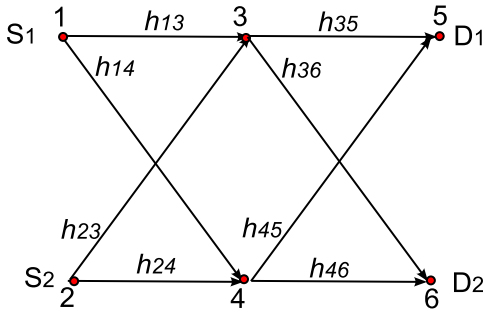


Fig. 8. A two-source network. Sources 1 and 2 wish to multicast independent data to nodes 5 and 6.

## VI. EXTENSIONS

### Non-layered Networks.

So far, we have analyzed layered networks. In non-layered networks, the input-output channel effectively behaves as an intersymbol interference channel that, at time  $i$ , is given by

$$y_D(i) = h_0 x_s(i) + \sum_{j=1}^{K_1} q_{j,1} x_s(i-1) + \dots + \sum_{j=1}^{K_L} q_{j,L} x_s(i-L) + z_e(i) \quad (33)$$

where  $h_0$  is the channel gain on the direct link,  $K_l$  is the number of routes of length  $l$ , and  $L$  is the length of the longest route. Equivalent channel gains  $q_{j,i}$  depend on the network topology; each  $q_{j,i}$  contains accumulated channel and amplification gains on a source-destination route.  $z_e(i)$  denotes the total noise at the destination at time  $i$ .

### Modeling Assumption.

Model (1) assumes directed links between nodes. As such, this model does not accurately apply to a wireless network where channels are typically reciprocal. This model can still be appropriate in some wireless networks, such as networks with sectorized antennas at the nodes. When the links are undirected, i.e., the nodes overhear each other's transmissions, the strategy need to be modified to avoid creating loops.

## VII. CONCLUSION

We characterized the behavior of analog network coding in the high-SNR regime. In particular, we related the received powers at nodes with the propagated noise, to determine the rate achievable with analog network coding. When all received powers are lower bounded by  $1/\delta$ , the propagated noise power in a network with  $L$  layers is of the order  $L\delta$ . The result demonstrates that the analog network coding approaches the MAC cut-set bound as the received powers at relays increase. As all powers in the network increase, analog network coding scaling is such that the achieved rate is within a constant gap from the upper bound. The gap depends on number of nodes. Similar behavior was observed for decode-and-forward in a large network [7], and compress-and-forward [6]. As discussed in the previous section, this result assumes directed links between nodes and hence does not consider creation of loops

due to analog network coding when nodes are full-duplex. Relaxing this assumption is a topic of our future work. In high-SNR regime, analog network coding seems as a natural choice of the coding strategy for both unicast and multicast traffic, as it allows data that is already mixed in the wireless channel to be jointly forwarded in a simple manner. Furthermore, analog network coding does not require any decoding, which reduces the rate both in decode-and-forward and compute-and-forward schemes; it does not induce a block delay (which is present in the case of decoding); and finally, as demonstrated, the penalty of amplifying noise is small in the high-SNR regime characterized by large received powers at the nodes.

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