Interference Decoding for Deterministic Channels

Bernd Bandemer and Abbas El Gamal
Stanford University, Information Systems Laboratory
350 Serra Mall, Stanford, CA 94305, USA
Email: bandemer@stanford.edu, abbas@ee.stanford.edu

***Abstract*—An inner bound to the capacity region of a class of three user pair deterministic interference channels is presented. The key idea is to simultaneously decode the combined interference signal and the intended message at each receiver. It is shown that this interference-decoding inner bound strictly contains the inner bound obtained by treating interference as noise, which includes interference alignment for deterministic channels. The gain comes from judicious analysis of the number of combined interference sequences in different regimes of input distributions and message rates.**

I. Introduction

Interference channels with three or more user pairs exhibit the interesting property that decoding at each receiver is impaired by the *joint* effect of interference from all other senders rather by each sender's signal separately. Consequently, coding schemes that deal directly with the effect of the *combined interference signal* are expected to achieve higher rates. One such coding scheme is the celebrated *interference alignment*, e.g., [1], [2], whereby the code is designed so that the combined interference signal at each receiver is confined (*aligned*) to a subset of the receiver signal space. Depending on the specific channel, this alignment may be achieved via linear subspaces, signal scale levels, time delay slots, or number-theoretic irrational bases. In each case, the subset that contains the combined interference is disregarded, while the desired signal is reconstructed from the complement subset.

An important aspect to note about interference alignment is that the interfering signal is treated as noise, that is, the receivers do not attempt to partially or fully decode it as in the Han–Kobayashi scheme [3] for the two user pair case. Decoding combined interference signals was first considered by Bresler et al. [4] for the many-to-one Gaussian interference channel. The authors argue that with Gaussian codes, decoding the combined interference is tantamount to decoding each interfering sender's codeword, while lattice codes can make the combined interference look essentially like codewords from a single interferer. For channels with inherent linearity such as Gaussian interference channels, it is natural to consider decoding linear combinations of individual interfering codewords. Nazer and Gastpar [5] consider Gaussian relay networks and propose decoding linear functions at intermediate nodes, thus creating a compute–forward relaying architecture.

This work is partially supported by DARPA ITMANET. Bernd Bandemer is supported by an Eric and Illeana Benhamou Stanford Graduate Fellowship.

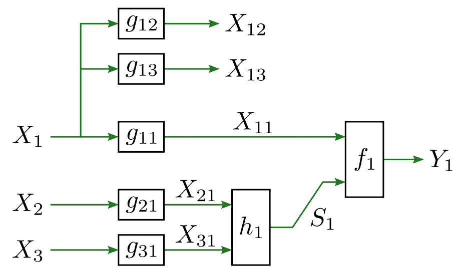

Fig. 1. The deterministic interference channel under consideration.

In this paper, we investigate interference decoding for a three receiver deterministic interference channel (3-DIC), which is an extension of the Costa–El Gamal two user pair model [6] and more general than the deterministic class studied in [7]. It consists of three sender-receiver alphabet pairs $(\mathcal{X}_k, \mathcal{Y}_k)$, loss functions g_{kl}, interference combining functions h_k, and receiver functions f_k for $k, l \in \{1, 2, 3\}$. At time i, the received symbol is $Y_{k,i} = f_k(X_{kk,i}, S_{k,i})$, where $X_{kl,i} = g_{kl}(X_{k,i})$ and $S_{1,i} = h_1(X_{21,i}, X_{31,i})$, $S_{2,i} = h_2(X_{12,i}, X_{32,i})$, and $S_{3,i} = h_3(X_{13,i}, X_{23,i})$. The channel is depicted in Figure 1 for user pair (X_1, Y_1). We assume that h_k and f_k become one-to-one when one of their arguments is fixed arbitrarily. For example, for $Y_1 = f_1(X_{11}, S_1)$, this assumption is equivalent to $H(X_{11}) = H(Y_1 \mid S_1)$ and $H(S_1) = H(Y_1 \mid X_{11})$ for every probability mass function (pmf) $p(x_{11}, s_1)$.

Each sender $k \in \{1, 2, 3\}$ wishes to convey an independent message M_k at data rate R_k to its corresponding receiver. We define a $(2^{nR_1}, 2^{nR_2}, 2^{nR_3}, n)$ code, probability of error, and achievability of a given rate triple (R_1, R_2, R_3) in the standard way [8].

We focus on this class of deterministic channels for several reasons. First, the capacity region for the two user pair version of this class [6] is known and is achieved by the Han–Kobayashi scheme. This gives some hope that an appropriate extension of Han–Kobayashi where the combined interference is decoded partially or fully may be optimal for more than two user pairs. Second, this class includes the finite field deterministic model in [9], which approximates Gaussian ICs in the high SNR regime [10]. For more than two user pairs, capacity results for this finite field deterministic model are known only in some special cases [11], [12], where interference is treated as noise. An interesting question is whether more sophisticated coding schemes can achieve higher rates for this class of channels. Finally, the combined interference signal in our channel takes values from a finite set, and therefore a certain type of alignment can be observed without resorting to complicated structured codes [13]. Of course, structured codes may still achieve even higher rates, but we do not investigate this possibility in this paper.

The main result of the paper is an inner bound to the capacity region of the 3-DIC, which is achieved via *interference decoding*. In this scheme, each receiver decodes the combined interference signal together with its intended message. We use simple randomly generated point-to-point codes without rate splitting or superposition coding. The key observation is that depending on the input pmfs and the message rates, the number of possible combined interference sequences can be equal to the number of interfering message pairs, the number of typical combined interference sequences, or some combination of the two. In our scheme, each sender does not need to know the other senders' codebooks. However, we use simultaneous decoding, which requires that the receivers know all codebooks. As in the recent characterization of the Han–Kobayashi region [14], we do not require the interference decoding to be correct with arbitrarily small probability of error. We show that interference decoding strictly outperforms treating interference as noise.

The inner bound is presented in the next section, and examples are given in Section III. Details of the proofs are available in an extended version of this paper posted on Arxiv. Notation and basic definitions follow [8].

II. Interference-Decoding Inner Bound

Fix $(Q, X_1, X_2, X_3) \sim p(q)p(x_1|q)p(x_2|q)p(x_3|q)$, where Q is a time sharing random variable with arbitrary cardinality. Define the region $\mathscr{R}_1(Q, X_1, X_2, X_3)$ to consist of the rate triples (R_1, R_2, R_3) such that

$$\begin{aligned} R_1 &< H(X_{11} \,|\, Q), \\ R_1 + \min\{R_2, H(X_{21} \,|\, Q)\} &< H(Y_1 \,|\, X_{31}, Q), \\ R_1 + \min\{R_3, H(X_{31} \,|\, Q)\} &< H(Y_1 \,|\, X_{21}, Q), \\ R_1 + \min\{R_2 + R_3, H(S_1 \,|\, Q), & \\ R_2 + H(X_{31} \,|\, Q), H(X_{21} \,|\, Q) + R_3\} &< H(Y_1 \,|\, Q). \end{aligned}$$

Similarly define the regions $\mathscr{R}_2(Q, X_1, X_2, X_3)$ and $\mathscr{R}_3(Q, X_1, X_2, X_3)$ by making the subscript replacements $1 \mapsto 2 \mapsto 3 \mapsto 1$ and $1 \mapsto 3 \mapsto 2 \mapsto 1$ in $\mathscr{R}_1(Q, X_1, X_2, X_3)$, respectively.

We now establish our main result.

Theorem 1: The region

$$\mathscr{R}_{\mathrm{ID}} = \bigcup_{(Q,X_1,X_2,X_3)} \bigcap_{k=1}^{3} \mathscr{R}_k(Q, X_1, X_2, X_3),$$

where $(Q, X_1, X_2, X_3) \sim p(q)p(x_1|q)p(x_2|q)p(x_3|q)$ is an inner bound to the capacity region of the 3-DIC.

Region $\mathscr{R}_1(Q, X_1, X_2, X_3)$, which ensures decodability at the first receiver, is plotted in Figure 2 for Example 2 in Section III. The region is unbounded in the R_2 and R_3 directions. This is intuitive, since regardless of the values of R_2 and R_3, S_1 can always be treated as noise to achieve a non-zero rate. However, as R_2 and R_3 become smaller, the proposed scheme takes advantage of the structure in S_1 and can thereby increase R_1.

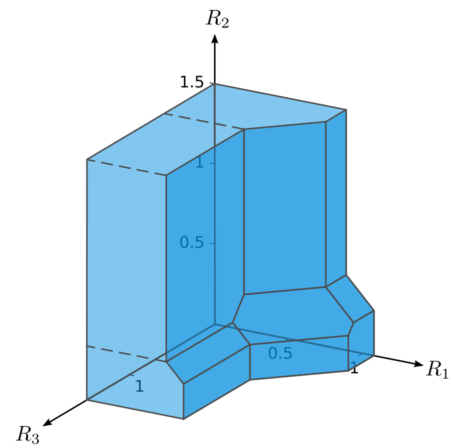

Fig. 2. Region $\mathscr{R}_1$ for Theorem 1, showing allowable rate triples for decodability at the first receiver.

In the following subsection we present two key lemmas that are needed for the proof of Theorem 1. The proof itself is presented in Subsection II-B. In Subsection II-C, we show that treating interference as noise is a special case of Theorem 1.

A. Key lemmas

The following lemma generalizes the packing lemma stated in [8].

Lemma 1 (Packing lemma for pairs): Let $(U, A, B, C) \sim p(u)p(a|u)p(b|u)p(c|a, b, u)$. Let $U^n \sim \prod_{i=1}^{n} p_U(u_i)$. For each $m \in [1 : 2^{nR_A}]$, let $A^n(m) \sim \prod_{i=1}^{n} p_{A|U}(a_i \,|\, u_i)$. For each $l \in [1 : 2^{nR_B}]$, let $B^n(l) \sim \prod_{i=1}^{n} p_{B|U}(b_i \,|\, u_i)$, conditionally independent of each $A^n(m)$ given U^n. Let $C^n \sim \prod_{i=1}^{n} p_{C|U}(c_i \,|\, u_i)$, conditionally independent of each $A^n(m)$ and $B^n(l)$ given U^n. There exists a $\delta(\varepsilon)$ with $\lim_{\varepsilon \to 0} \delta(\varepsilon) = 0$ such that if

$$\begin{aligned} \min\{R_A, H(A \,|\, U)\} + \min\{R_B, H(B \,|\, U)\} \\ < I(A, B; C \,|\, U) - \delta(\varepsilon), \end{aligned}$$

then $\mathrm{P}\{(U^n, A^n(m), B^n(l), C^n) \in \mathcal{T}_\varepsilon^{(n)}$ for some $m, l\} \to 0$ as $n \to \infty$, where typicality, entropies and mutual information are with respect to $p(u, a, b, c)$.

The proof of Lemma 1 is deferred to the extended version of this paper. An intuitive interpretation of the first $\min$ term is that the number of A^n sequences *saturates* when the rate R_A becomes larger than $H(A \,|\, U)$. At this point, the entire set of conditionally typical sequences is exhausted, and generating more sequences does not increase the probability of joint typicality. The same reasoning applies to the second $\min$ expression.

The following lemma is a refined version of Lemma 1, where the sequences $B^n(l)$ are generated from two conditionally independent components $B_1^n(l_1)$ and $B_2^n(l_2)$.

Lemma 2: Let $(U, A, B_1, B_2, B, C) \sim p(u)p(a|u)p(b_1|u)$ $p(b_2|u)p(b|b_1, b_2)p(c|a, b, u)$, where $p(b|b_1, b_2)$ corresponds

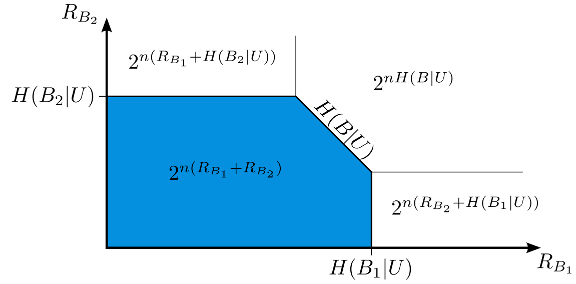

Fig. 3. Number of output sequences for deterministic MAC as a function of the number of input sequences. The capacity region, where output sequences map uniquely to pairs of input sequences, is shown in blue.

to a deterministic mapping $h : (b_1, b_2) \mapsto b$. Let $U^n \sim \prod_{i=1}^n p_U(u_i)$. For each $m \in [1 : 2^{nR_A}]$, let $A^n(m) \sim \prod_{i=1}^n p_{A|U}(a_i \,|\, u_i)$. For each $l_1 \in [1 : 2^{nR_{B_1}}]$, let $B_1^n(l_1) \sim \prod_{i=1}^n p_{B_1|U}(b_{1i} \,|\, u_i)$, conditionally independent of each $A^n(m)$ given U^n. Likewise, for each $l_2 \in [1 : 2^{nR_{B_2}}]$, let $B_2^n(l_2) \sim \prod_{i=1}^n p_{B_2|U}(b_{2i} \,|\, u_i)$, conditionally independent of each $A^n(m)$ and $B_1^n(l_1)$ given U^n. For each (l_1, l_2), let $B_i(l_1, l_2) = h(B_{1i}(l_1), B_{2i}(l_2))$ for $i \in [1 : n]$. Finally, let $C^n \sim \prod_{i=1}^n p_{C|U}(c_i \,|\, u_i)$, conditionally independent of each $A^n(m)$, $B_1^n(l_1)$, and $B_2^n(l_2)$ given U^n.

There exists a function $\delta(\varepsilon)$ with $\lim_{\varepsilon\to 0} \delta(\varepsilon) = 0$ such that if

$$R_A + \min\{R_{B_1} + R_{B_2}, R_{B_1} + H(B_2 \,|\, U), H(B_1 \,|\, U) + R_{B_2}, H(B \,|\, U)\} < I(A, B; C \,|\, U) - \delta(\varepsilon),$$

then $\mathrm{P}\{(U^n, A^n(m), B^n(l_1, l_2), C^n) \in \mathcal{T}_\varepsilon^{(n)}$ for some m, l_1, $l_2\} \to 0$ as $n \to \infty$, where typicality, entropies and mutual information are with respect to $p(u, a, b_1, b_2, b, c)$.

We defer the proof to extended version of this paper. The intuition is that B can be interpreted as the output of a deterministic multiple access channel with inputs B_1 and B_2 and input to output mapping h. Figure 3 shows the number of output sequences for different ranges of R_{B_1} and R_{B_2} when h is one-to-one in each argument. Note that when (R_{B_1}, R_{B_2}) is in the deterministic MAC capacity region, the number of output sequences is simply $2^{n(R_{B_1}+R_{B_2})}$. For (R_{B_1}, R_{B_2}) outside the capacity region, the number of output sequences saturates in one or both dimensions. The logarithm of the number of output sequences divided by n appears in the min expression of the lemma.

B. Proof of Theorem 1

Fix a pmf $p(q)p(x_1|q)p(x_2|q)p(x_3|q)$.

Codebook generation. Randomly generate a sequence q^n according to $\prod_{i=1}^n p_Q(q_i)$. For each $k \in \{1, 2, 3\}$, randomly and conditionally independently generate sequences $x_k^n(m_k)$, $m_k \in [1 : 2^{nR_k}]$, each according to $\prod_{i=1}^n p_{X_k|Q}(x_{ki}|q_i)$. From the channel definition, this procedure induces intermediate sequences $x_{kl}^n(m_k)$ for $l \in \{1, 2, 3\}$, combined interference sequences $s_1^n(m_2, m_3)$, $s_2^n(m_1, m_3)$, $s_3^n(m_1, m_2)$, and output sequences $y_k^n(m_1, m_2, m_3)$.

Encoding. To send the message $m_k \in [1 : 2^{nR_k}]$, $k \in \{1, 2, 3\}$, encoder k transmits $x_k^n(m_k)$.

Decoding. Upon observing y_1^n, decoder 1 declares that $\hat{m}_1$ is sent if it is the unique message such that $(q^n, x_1^n(\hat{m}_1), s_1^n(\mathring{m}_2, \mathring{m}_3), x_{21}^n(\mathring{m}_2), x_{31}^n(\mathring{m}_3), y_1^n) \in \mathcal{T}_\varepsilon^{(n)}$ for some $\mathring{m}_2, \mathring{m}_3$, where $\mathcal{T}_\varepsilon^{(n)}$ is defined as in [8]. Decoding at the other receivers is performed similarly.

Analysis of the probability of error. Without loss of generality, assume that $m_k = 1$ for $k \in \{1, 2, 3\}$. Define $\mathcal{E}_{mlk} = (Q^n, X_1^n(m), S_1^n(l, k), X_{21}^n(l), X_{31}^n(k), Y_1^n(1, 1, 1)) \in \mathcal{T}_\varepsilon^{(n)}$, and the events

$$\begin{aligned}
\mathcal{E}_0 &= \mathcal{E}_{111}^c, \\
\mathcal{E}_1 &= \{\mathcal{E}_{m11} \text{ for some } m \neq 1\}, \\
\mathcal{E}_2 &= \{\mathcal{E}_{ml1} \text{ for some } m, l \neq 1\}, \\
\mathcal{E}_3 &= \{\mathcal{E}_{m1k} \text{ for some } m, k \neq 1\}, \\
\mathcal{E}_4 &= \{\mathcal{E}_{mlk} \text{ for some } m, l, k \neq 1\}.
\end{aligned}$$

Then the probability of decoding error at the first receiver averaged over codebooks is upper bounded as $\mathrm{P}(\mathcal{E}) = \mathrm{P}(\mathcal{E}_0 \cup \mathcal{E}_1 \cup \mathcal{E}_2 \cup \mathcal{E}_3 \cup \mathcal{E}_4) \leq \sum_{j=0}^4 \mathrm{P}(\mathcal{E}_j)$. We bound each term. First, by the law of large numbers, $\mathrm{P}(\mathcal{E}_0) \to 0$ as $n \to \infty$.

Next consider

$$\mathcal{E}_1 \subseteq \{(Q^n, X_1^n(m), S_1^n(1, 1), Y_1^n(1, 1, 1)) \in \mathcal{T}_\varepsilon^{(n)} \text{ for some } m \neq 1\}.$$

By Lemma 1 with $U^n = (Q^n, S_1^n(1, 1))$, $A^n = X_1^n$, $B^n = \emptyset$, and $C^n = Y_1^n(1, 1, 1)$, the probability of this event tends to zero as $n \to \infty$ if

$$R_1 < I(X_1; Y_1 \,|\, S_1, Q) = H(Y_1 \,|\, S_1, Q) = H(X_{11} \,|\, Q). \quad (1)$$

The event $\mathcal{E}_2$ can be treated similarly. Consider

$$\mathcal{E}_2 \subseteq \{(Q^n, X_1^n(m), X_{21}^n(l), X_{31}^n(1), Y_1^n(1, 1, 1)) \in \mathcal{T}_\varepsilon^{(n)} \text{ for some } m, l \neq 1\}.$$

Using Lemma 1 with $U^n = (Q^n, X_{31}^n(1))$, $A^n = X_1^n$, $B^n = X_{21}^n$, and $C^n = Y_1^n(1, 1, 1)$, we conclude that $\mathrm{P}(\mathcal{E}_2) \to 0$ if

$$\begin{aligned}
R_1 + \min\{R_2, H(X_{21} \,|\, X_{31}, Q)\} &< I(X_1, X_{21}; Y_1 \,|\, X_{31}, Q) \\
R_1 + \min\{R_2, H(X_{21} \,|\, Q)\} &< H(Y_1 \,|\, X_{31}, Q). \quad (2)
\end{aligned}$$

$\mathrm{P}(\mathcal{E}_3)$ can be analyzed in an identical fashion to $\mathrm{P}(\mathcal{E}_2)$, and we have $\mathrm{P}(\mathcal{E}_3) \to 0$ as $n \to \infty$ if

$$R_1 + \min\{R_3, H(X_{31} \,|\, Q)\} < H(Y_1 \,|\, X_{21}, Q). \quad (3)$$

Finally, the event $\mathcal{E}_4$ is augmented as follows

$$\mathcal{E}_4 \subseteq \Big\{(Q^n, X_1^n(m), S_1^n(l, k), Y_1^n(1, 1, 1)) \in \mathcal{T}_\varepsilon^{(n)} \text{ for some } m, l, k \neq 1\Big\}.$$

Lemma 2 with $U^n = Q^n$, $A^n = X_1^n$, $B_1^n = X_{21}^n$, $B_2^n = X_{31}^n$, $B^n = S_1^n$, $h = h_1$, and $C^n = Y_1^n(1, 1, 1)$ shows that $\mathrm{P}(\mathcal{E}_4) \to 0$ as $n \to \infty$ if

$$R_1 + \min\{R_2 + R_3, R_2 + H(X_{31} \,|\, Q), H(X_{21} \,|\, Q) + R_3, H(S_1 \,|\, Q)\} < H(Y_1 \,|\, Q), \quad (4)$$

where we have used $I(X_1, S_1; Y_1 \,|\, Q) = H(Y_1 \,|\, Q)$. Collecting (1) to (4) yields the conditions of $\mathscr{R}_1$. The probability of error at the second and third receiver can be bounded similarly, leading to the conditions of $\mathscr{R}_2$ and $\mathscr{R}_3$, which completes the proof.

C. Comparison to treating interference as noise

By treating interference as noise, we obtain the following inner bound to the the capacity region of the *general* discrete memoryless interference channel with three user pairs.

Theorem 2 (Inner bound by treating interference as noise): The set $\mathscr{R}_{\mathrm{TIN}}$ of rate triples (R_1, R_2, R_3) such that

$$R_k < I(X_k; Y_k \,|\, Q), \quad k \in \{1, 2, 3\}, \tag{5}$$

for some probability mass function $p(q)p(x_1|q)p(x_2|q)p(x_3|q)$ constitutes an inner bound to the capacity region of the general discrete memoryless three user pair interference channel.

This inner bound is achieved via randomly and independently generated codebooks as for the interference-decoding inner bound. Each receiver, however, decodes only its message. Thus, in contrast to interference decoding, a user pair does not need to know the codebooks of other user pairs. Note that this inner bound, with appropriate selections of the input pmfs, includes the interference alignment inner bounds in [2], [11], [12]. Maximum alignment is achieved when the number of combined interference sequences, e.g., S_1^n, is much smaller than the number of individual interference sequence pairs, e.g., (X_{21}^n, X_{31}^n). Since $I(X_k; Y_k \,|\, Q) = H(Y_k \,|\, Q) - H(S_k \,|\, Q)$, this occurs when $H(S_k \,|\, Q)$ is small, causing the number of S_k^n sequences to saturate as per the discussion after Lemma 1.

It is not difficult to show that the inner bound in Theorem 2 is a special case of Theorem 1 for the deterministic case. The conditions of region $\mathscr{R}_1$ in Theorem 1 can be made more stringent by replacing the min expression with any one of its argument terms. For example, $(R_1, R_2, R_3) \in \mathscr{R}_1$ is implied by

$$\begin{aligned} R_1 &< H(X_{11} \,|\, Q), \\ R_1 + H(X_{21} \,|\, Q) &< H(Y_1 \,|\, X_{31}, Q), \\ R_1 + H(X_{31} \,|\, Q) &< H(Y_1 \,|\, X_{21}, Q), \\ R_1 + H(S_1 \,|\, Q) &< H(Y_1 \,|\, Q), \end{aligned}$$

or, equivalently,

$$\begin{aligned} R_1 < \min\big\{&H(X_{11}|Q), H(Y_1|X_{31}, Q) - H(X_{21}|Q), \\ &H(Y_1|X_{21}, Q) - H(X_{31}|Q), H(Y_1|Q) - H(S_1|Q)\big\}. \end{aligned} \tag{6}$$

To simplify this expression, consider

$$\begin{aligned} H(X_{11} \,|\, Q) &\geq I(X_{11}; Y_1 \,|\, Q) \\ &= H(Y_1 \,|\, Q) - H(Y_1 \,|\, X_{11}, Q) \\ &= H(Y_1 \,|\, Q) - H(S_1 \,|\, Q), \end{aligned}$$

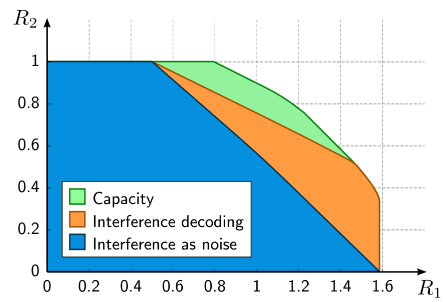

Fig. 4. Capacity region and inner bounds for 2-DIC example.

as well as

$$\begin{aligned} &\big[H(Y_1 \,|\, X_{21}, Q) - H(X_{31} \,|\, Q)\big] - \big[H(Y_1 \,|\, Q) - H(S_1 \,|\, Q)\big] \\ &= H(Y_1, X_{21} \,|\, Q) - H(X_{21} \,|\, Q) - \underbrace{H(X_{31} \,|\, Q)}_{H(S_1 \,|\, X_{21}, Q)} \\ &\quad - H(Y_1 \,|\, Q) + H(S_1 \,|\, Q) \\ &= H(X_{21} \,|\, Y_1, Q) - H(X_{21} \,|\, S_1, Q) \\ &\geq H(X_{21} \,|\, Y_1, S_1, Q) - H(X_{21} \,|\, S_1, Q) \\ &= 0, \end{aligned}$$

and, by symmetry,

$$\big[H(Y_1|X_{31}, Q) - H(X_{21}|Q)\big] - \big[H(Y_1|Q) - H(S_1|Q)\big] \geq 0.$$

Thus, the $\min$ in (6) is always achieved by the last term, and (6) simplifies to

$$\begin{aligned} R_1 &< H(Y_1 \,|\, Q) - H(S_1 \,|\, Q) \\ &= I(X_1; Y_1 \,|\, Q). \end{aligned}$$

Using a similar argument, it follows that the conditions for $\mathscr{R}_2$ and $\mathscr{R}_3$ in Theorem 1 are implied by (5).

III. Examples

Example 1: 2-DIC

To compare the inner bound to the capacity region, we need to resort to a two user example where the capacity region is known. To this end, consider the 2-DIC with input alphabets $\mathcal{X}_1 = \{0, 1, 2\}$, $\mathcal{X}_2 = \{0, 1\}$, loss functions $g_{12} = \{0 \mapsto 0, 1 \mapsto 0, 2 \mapsto 1\}$ and $g_{11} = g_{22} = g_{21} = \mathrm{Id}$, and combining functions $f_1 = f_2$ being addition (h_1 and h_2 are not relevant in this case). Figure 4 compares the inner bounds and capacity region of this channel. As expected, interference decoding is superior to treating interference as noise, but does not achieve the full capacity. To achieve capacity, Han–Kobayashi [3] rate splitting and superposition coding is needed.

Example 2: Additive 3-DIC

Consider a cyclically symmetric 3-DIC with $\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{X}_3 = \{0, 1, 2\}$, and $\mathcal{Y}_1 = \mathcal{Y}_2 = \mathcal{Y}_3 = \{0, 1, 2, 3, 4\}$, where $g_{11} = g_{22} = g_{33} = g$, $g_{12} = g_{23} = g_{31} = g_+$ and $g_{21} = g_{32} = g_{13} = g_-$ as well as $h_1 = h_2 = h_3 = h$ and $f_1 = f_2 = f_3 = f$. The direct path loss functions are the identity mapping, $g = \mathrm{Id}$,

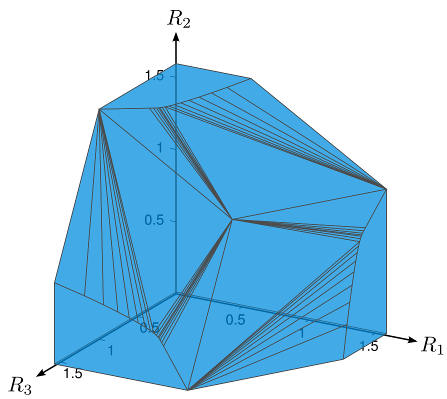

Fig. 5. Inner bound by treating interference as noise for Example 2.

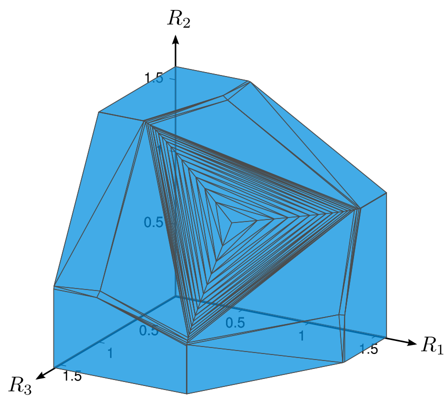

Fig. 6. Interference-decoding inner bound for Example 2.

while the cross path loss functions are given by $g_- = \{0 \mapsto 0, 1 \mapsto 1, 2 \mapsto 0\}$ and $g_+ = \{0 \mapsto 0, 1 \mapsto 1, 2 \mapsto 1\}$ (similar to the Blackwell broadcast channel [15]). Finally, the combining functions h and receiver functions f are taken to be addition.

For this channel, interference decoding achieves a larger rate region than treating interference as noise. Approximations of the inner bounds are shown in Figures 5 and 6. The same maximum symmetric rate $R_{\text{sum}} = 3$ is achieved by both schemes. However, as we move to more asymmetric rate triples, treating the interference as noise quickly declines in sum rate, while interference decoding can maintain sum rates close to optimum for a range of asymmetric rate triples.

Example 3: Finite-field deterministic model

Consider the three user pair cyclically symmetric finite field deterministic model investigated in [12], which is a special case of the channel considered in this paper. The input and output alphabets for this channel are $\mathbb{F}_2^N$ and $\mathbb{F}_2^{2N}$, respectively, the loss functions g_{kl} are vector shifting operations, where the amount of shift is parameterized by $(\alpha, \beta) \in [1, 2] \times [0, 1]$, and the interference combining functions h_k and the receiver functions f_k are componentwise additions over $\mathbb{F}_2$.

The sum capacity of this channel is computed in [12] for a large range of (α, β), and achievability is established by constructing linear encoding and decoding matrices for every (α, β). As it turns out, this scheme can be interpreted as treating the combined interference as noise, and thus Theorem 2 subsumes the achievability results in [12]. In fact, the necessary input distributions are the ones implicitly stated there. It would be interesting to investigate whether interference decoding can achieve higher sum rates than treating interference as noise in the (α, β) range where we do not know the sum capacity. Moreover, even in the range where we know the sum capacity, interference decoding may achieve higher asymmetric rates than treating interference as noise, as in Example 2. The main challenge in settling these questions is the prohibitively large space of possible input distributions in Theorem 1.

IV. Conclusion

Interference decoding is a promising middle ground between treating interference as noise and more sophisticated schemes that employ rate splitting and superposition coding. As in treating interference as noise, simple point-to-point codes can be used. However, more sophisticated coding schemes are needed to achieve capacity.

References

[1] M. A. Maddah-Ali, A. S. Motahari, and A. K. Khandani, “Communication over MIMO X channels: Interference alignment, decomposition, and performance analysis,” *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3457–3470, Aug. 2008.

[2] V. R. Cadambe and S. A. Jafar, “Interference alignment and degrees of freedom of the K-user interference channel,” *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[3] T. S. Han and K. Kobayashi, “A new achievable rate region for the interference channel,” *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.

[4] G. Bresler, A. Parekh, and D. Tse, “The approximate capacity of the many-to-one and one-to-many Gaussian interference channels,” *IEEE Trans. Inf. Theory*, Sep. 2008, submitted for publication, arXiv:0804.4489.

[5] B. Nazer and M. Gastpar, “Compute-and-Forward: Harnessing interference through structured codes,” *IEEE Trans. Inf. Theory*, Aug. 2009, submitted for publication, arXiv:0908.2119v2.

[6] A. A. El Gamal and M. H. M. Costa, “The capacity region of a class of deterministic interference channels,” *IEEE Trans. Inf. Theory*, vol. 29, no. 2, pp. 343–346, Mar. 1982.

[7] T. Gou and S. A. Jafar, “Capacity of a class of symmetric SIMO Gaussian interference channels within O(1),” *IEEE Trans. Inf. Theory*, submitted for publication, arXiv:0905.1745.

[8] A. El Gamal and Y.-H. Kim, “Lectures on Network Information Theory,” 2010, arXiv:1001.3404v1.

[9] A. S. Avestimehr, S. N. Diggavi, and D. N. C. Tse, “A deterministic approach to wireless relay networks,” in *Proc. Allerton*, Sep. 2007.

[10] G. Bresler and D. Tse, “The two-user Gaussian interference channel: a deterministic view,” *Euro. Trans. Telecomm.*, vol. 19, no. 4, pp. 333–354, Jun. 2008, arXiv:0807.3222.

[11] S. A. Jafar and S. Vishwanath, “Generalized degrees of freedom of the symmetric K user Gaussian interference channel,” arXiv:0804.4489.

[12] B. Bandemer, G. Vazquez-Vilar, and A. El Gamal, “On the sum capacity of a class of cyclically symmetric deterministic interference channels,” in *Proceedings of ISIT 2009*, Seoul, Korea, Jun. 2009.

[13] B. Nazer and M. Gastpar, “The case for structured random codes in network capacity theorems,” *Euro. Trans. Telecomm., Special Issue on New Directions in Information Theory*, vol. 19, no. 4, pp. 455–474, Jun. 2008.

[14] H.-F. Chong, M. Motani, H. K. Garg, and H. El Gamal, “On the Han-Kobayashi region for the interference channel,” *IEEE Trans. Inf. Theory*, vol. 54, no. 7, pp. 3188–3195, Jul. 2008.

[15] E. van der Meulen, “A survey of multi-way channels in information theory: 1961-1976,” *IEEE Trans. Inf. Theory*, vol. 23, no. 1, pp. 1–37, Jan. 1977.