CDMA Multiuser Detection Based on State-space Estimation Techniques*

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Abstract

We propose a new space-time multiuser receiver for CDMA wireless systems. The receiver is based on the fixed-lag smoothed estimator which occurs in the well-researched area of state-space estimation. The receiver can work with long codes as opposed to most other multiuser detectors that require cyclic spreading codes. It is also naturally suited for adaptation and can be used with time-varying (doppler spread) multipath channels. All the desired users are decoded simultaneously with little decoding delay. The approach assumes that a reliable channel estimate is available which in practice can be obtained from training symbols. We also briefly study the computational complexity of the receiver.

1. Introduction

Multiuser detection of Direct Sequence Code Division Multiple Access (DS-CDMA) signals has been of much research interest due to the significant increase in capacity it promises. Conventional CDMA receiver uses matched filters for demodulation and combats channel fading by combining multipaths. It suffers from the well known near-far problem as it relies on the near-orthogonality of the users' signature waveforms (as seen by the receiver). In practice, due to multipath in the channel and due to the asynchronous nature of the users, often the received signature waveforms are not orthogonal even though the spreading codes of the users are orthogonal. The conventional receiver therefore cannot effectively suppress the interference from other users. However, the interference from other users has as rich a structure as the signal of interest and hence one can make use of this to suppress it rather than treat it as noise. Several multiuser receivers with different complexity and performance have been proposed. The optimum multiuser receiver [1] decodes all the users jointly and therefore its complexity grows exponentially with the number of users. Hence several suboptimal schemes have been proposed. Interference cancellation techniques [2] have been studied extensively due to their low implementational complexity and applicability to long codes. However, the successive version of it (which is more commonly used), suffers from a long decoding delay. Linear multiuser receivers [3] [4] are another important class of suboptimal receivers that suppress MAI (multiple access interference) efficiently. The channel and and interference statistics may vary with time in practice and hence adaptive versions of linear receivers have been studied [5]. With long codes (where the spreading code has a period much longer than the spreading factor) their adaptation is difficult or not possible. See [6] for a survey on multiuser detection.

In this paper, we consider a very general CDMA wireless system and propose a linear multiuser receiver based on state-space estimation techniques. The approach can be used for both the uplink and downlink (or multicode transmission), in frequency selective fading channels. In the development of our system model, we assume an uplink but with simple modifications, it can be applied to the downlink. State-space estimation techniques have been used in the past for CDMA multiuser detection in [7]. Due to the recursive nature of these techniques, they are expected to outperform FIR (finite impulse response) filtering techniques. In particular, we use the fixed-lag smoothed estimator [8] which gives a linear recursive MMSE (minimum mean square error) estimate of the desired input. Thus it is expected to perform better than the classical FIR MMSE multiuser detector [9], especially in asynchronous systems. Another important advantage of our scheme is that it can be used with time-varying channel or interference conditions. Our scheme can be used with long codes and unlike the successive interference canceling receiver, does not suffer long decoding delays.

We treat the out of cell interference as noise to make
the model simple. The effect of this interference up to its second order statistics (with some approximations) can be taken into account in our solution, if required. Full channel knowledge and knowledge of all the users' spreading codes is required. In this sense, it may be more applicable for the uplink of a CDMA cellular system (or a point-to-point wireless link employing CDMA).

The rest of this paper is organized as follows. In section (2), we describe the system model, introduce notations and state our assumptions. We describe the fixed-lag smoothed estimation method in section (3) and apply it to the CDMA model. We discuss a few interesting aspects of our scheme in section (4) and finally conclude in section (5).

2. System Model

We consider the CDMA uplink as stated before, but point out wherever appropriate, how the model for the downlink (or multicode transmission) differs. Let \( U \) be the number of users and \( P \), the spreading factor. Let \( n \) be the chip index and \( k \), the symbol index. The \( i \)-th user is assumed to transmit a symbol stream of i.i.d (independent identically distributed) symbols \( \{s_i(k)\}_{k=0}^{\infty} \) at the rate of \( 1/T_s \). Let the spreading code of user \( i \) be the sequence, \( \{c_i(k)\}_{k=0}^{\infty} \). The signal transmitted by the \( i \)-th user can be written as \( x_i(n) = A_i d_i(n) \), where \( A_i \) is the amplitude and \( d_i(n) \) is the modulated chip sequence given by \( d_i(n) = s_i(k)c_i(n) \), where \( k = [n/P] \). The users are symbol asynchronous in the uplink but one can assume that they are synchronous and absorb the relative delays in the symbol timings, into the propagation channels. (In the downlink, the users are anyway symbol synchronous.)

The spreading operation can be viewed as a filtering operation with a time-varying filter \( c_i(n, l) \) and input sequence \( f_i(n) \) as shown below:

\[
f_i(n) = \sum_{k=0}^{\infty} s_i(k) \delta(n - kP) \tag{1}
\]

\[
c_i(n, l) = \begin{cases} c_i(n - l) & \text{when } l = kP, \\ 0 & \text{otherwise} \end{cases}
\tag{2}
\]

Thus, \( d_i(n) = \sum_{l=0}^{n} c_i(n, l)f_i(n - l) \tag{3} \)

When the spreading codes are cyclic (i.e., periodic with period, \( P \)), this filter becomes time-invariant.

Let \( M \) be the number of receive antennas. Assume the received signal is sampled at the chip rate. The symbol \( n \) can now also be used to index the time (beginning from \( n = 0 \)) in terms of a chip duration. The pulse shaping filter used at the transmitters (of users) can be absorbed into the propagation channel. The base-band sampled signal received at the \( m \)-th receiver antenna (of the base station) is given by

\[
y_m(n) = \sum_{i=1}^{U} \sum_{l=0}^{n} h_{m}^{(i)}(n, l)d_i(n - l) + w_m(n) \tag{4}
\]

where \( h_{m}^{(i)} \) is the time-varying, chip rate sampled channel from the \( i \)-th user to \( m \)-th antenna of the base station and \( w_m \) is the additive noise (which may contain any out of cell interference) at the antenna. The system model is shown in Fig. 1 which depicts the signal received at the \( m \)-th receive antenna. With slight abuse of notation we stack the signals received at the receive antenna array (at chip index \( n \)), \( \{y_m(n)\}_{m=1}^{M} \) as a vector and denote it by \( y_n \). We follow the same notation for the noise signal \( \{w_m(n)\}_{m=1}^{M} \). (For the downlink, the superscript \( i \) for the channel does not appear and hence the filtering operation with the channel factors out of the summation over user index, \( i \)).

If the system is temporally oversampled at a rate that is a multiple of the chip rate, it can be represented in the same form as above by stacking the different time sample phases of a chip period. Using multiple receive antennas corresponds to spatial oversampling. For a temporally and spatially oversampled system, \( M \) would be equal to the number of receive antennas times the oversampling factor.

We now show how the above system can fit into a state-space model and apply state-space estimation techniques to estimate the transmitted symbols. Consider the following state-space model ((10))

\[
\begin{align*}
x_{n+1} &= F_n x_n + G_n u_n \\
y_n &= H_n x_n + w_n, \quad n \geq 0
\end{align*}
\tag{5}
\]

where

\[
\begin{bmatrix}
u_n \\
v_n^n \\
x_0
\end{bmatrix}
= \begin{bmatrix}
Q_n S_n & 0 \\
S_n^* R_n & 0 \\
0 & 0 & \Pi_0
\end{bmatrix}
\]

Here \( x_n \) is the zero-mean state vector and \( u_n \), the zero-mean input vector. The observation vector, \( y_n \), and the zero-mean noise vector \( w_n \) have already been described above.

Since state-space estimation techniques estimate the state vector \( x \) and our interest is in estimating the transmitted symbols, we incorporate these symbols in the state vector. Also note that since the propagation channels and the spreading codes are known, the only variables that determine the state of the system are the transmitted symbols. Although the state of the system changes only at the symbol rate, the received signal is at the chip rate and hence the state-space model has also been considered at the chip rate (and therefore time-indexed by \( n \)).

In particular, let the memory in the channel be \( L_k \) chips. Let \( q = [L_k/P] \). Then the state of the system involves
symbols over \( q + 1 \) symbol periods. And this state changes only once in a symbol period, with a new symbol coming in for each user (this is incorporated through the input vector \( u \)). Thus we have the following (\( k \) is an integer and denotes the symbol index again):

\[
    x_n = \begin{bmatrix} s^T(k) , \ldots , s^T(k-q) \end{bmatrix}^T, \quad \text{when } n = kP , \ldots , (k+1)P-1, \ k \geq 0
\]

\[
    u_n = s(k), \quad \text{when } n = kP - 1, \ k > 0
\]

\[
    G_n = \begin{bmatrix} I_U , 0_U , \ldots , 0_U \end{bmatrix}^T
\]

\[
    F_n = \begin{bmatrix} 0_U & \ldots & 0_U \\ I_U & \ldots & 0_U \\ \vdots & \ddots & \vdots \\ 0_U & \ldots & I_U, 0_U \end{bmatrix}
\]

\[
    \quad \text{when } n = kP - 1, \ k > 0
\]

where \( s(j) = [s_1(j), \ldots , s_U(j)]^T \) when \( j \) is a non-negative integer and \( s(j) = 0_U \times 1 \) otherwise. Also since \( x_0 = [s(0), 0_U, \ldots , 0_U] \), the value of \( I_0 \) is obtained immediately. Note that we have used 0 and 1 to denote the all-zero matrices and identity matrices, in the above equations.

It is clear from (2) - (4) that each user's transmitted symbol stream undergoes two filtering operations (both filters being time-varying in general) as shown in Fig. 1. The received signal is the sum of the outputs of these filters for all the users (with additive noise and output of cell interference). Thus the received signal can be written as in (5) using the common method of representing time-varying filters in state-space form. The matrix \( H_n \) is formed from the spreading codes, the propagation channels of the users and their amplitude of transmission.

The transmitted symbols which determine \( u_n \) and the receiver noise \( w_n \) are clearly uncorrelated (actually independent). Hence \( S_n = 0 \). The out of cell interference can be both temporally and spatially correlated, though the additive Gaussian noise is temporally and spatially white. Hence if we consider out of cell interference, \( w_n \) is temporally correlated and we need to make an approximation that it is temporally white in order to apply our technique. Spatial correlation does not pose a problem as it is incorporated into \( R_n \). Also, since \( R_n \) can change with \( n \), the time variation of out of cell interference (which is typically the case), can also be incorporated.

Given this state-space model, it is now possible to obtain an estimate of the state vector and thus the transmitted symbols.

### 3. Detection Algorithm

In this section, we describe recursive algorithms for computing the filtered and fixed-lag smoothing state estimators, \( \hat{x}_{n|n} \) and \( \hat{x}_{n|n+L} \), that we use for the multiuser detection in the CDMA system modeled by (5). The celebrated Kalman filter provides us with a recursive algorithm for computing the filtered and predicted, \( \hat{x}_{n|n} \) and \( \hat{x}_{n+1|n} \), estimators of the state vector in (5), which then may be used for symbol detection. However, if we allow for a delay in the estimation, the additional measurements may improve the estimate. This leads to higher-order so-called fixed-lag smoothing estimators (see, e.g., [8]). (We should note that, in a limit as the delay goes to infinity, the estimator converges to the non-causal (so-called smoothing) solution.) The higher accuracy of the estimate obtained by allowing for the delay in the estimation is paid by an increase in the computational complexity of the estimation algorithm.

Here we consider the fixed-lag smoothing state estimator, \( \hat{x}_{n|n+L} \), for detection. The value of the lag \( L \) is determined from the delay spread of the channel and the spreading factor. The fixed-lag estimator allows for fast, computationally efficient, implementation.

For the standard state-space model of (5), the Kalman filter recursions for the predicted and filtered state estimators, \( \hat{x}_{n|n} \) and \( \hat{x}_{n+1|n} \), \( n \geq 0 \), written in so-called measurement and time-update form (see, e.g., [10]) is

\[
    \hat{x}_{n|n} = \hat{x}_{n-1|n} + K_f,\epsilon_n,
\]

\[
    \hat{x}_{n+1|n} = F_n,\hat{x}_{n|n}.
\]

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where
\[ e_n = y_n - H_n \hat{x}_{n|n-1}, \]
\[ K_{f,n} = F_n P_{n|f,n} R_n^{-1}, \]
\[ P_{f,n} = F_n - K_{f,n} H_n, \]
\[ K_{p,n} = F_n P_{n|p,n} R_n^{-1}, \]
\[ R_{e,n} = R_n + H_n P_{n|p,n} H_n^* \]
\[ P_{n+1} = F_n P_{n|p,n} F_n^* + G_n Q_n G_n^* - K_{p,n} R_{e,n} K_{p,n}^*, \]
and where \( \hat{x}_{0|0} = 0, e_0 = y_0, \) and \( P_0 = \Pi_0. \) Note that \( e_n \) is the so-called innovations of the observation process \( y_n. \) The number of computations required for each iteration of the algorithm (12) is \( O((q + 1)^3 U^3). \)

The fixed-lag smoother with an estimation lag \( L \) yields the smoothed estimate \( \hat{x}_{n+L|n}. \) The straightforward way of implementing the fixed-lag smoother is by means of augmentation of the basic state-space model in (5). In particular, using (5), we can write

\[
\begin{align*}
\{ z_{n+1} &= A_n z_n + B_n u_n \\
y_n &= C_n z_n + w_n
\end{align*}
\tag{13}
\]

where
\[ z_n = \begin{bmatrix} x_n & x_{n-1} & x_{n-2} & \cdots & x_{n-L-1} \end{bmatrix}^T, \]
\[ A_n = \begin{bmatrix} F_n & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}, \]
\[ B_n = \begin{bmatrix} G_n \\ \vdots \\ 0 \end{bmatrix}, \]
\[ C_n = \begin{bmatrix} H_n & 0 & 0 & \cdots & 0 \end{bmatrix}. \]

Then, one can easily find \( \hat{x}_{n|n} \) (and, thus, \( \hat{x}_{n-d|n} \) for all the delays \( d, 0 \leq d \leq L \) by directly employing recursion (12) on the augmented model of (13). However, one can extract the fixed-lag smoothing equations for a particular delay as (see [8])

\[ \hat{x}_{n-d|n} = \hat{x}_{n-d|n-1} + K_{n}^{(d+1)}(y_n - H_n \hat{x}_{n|n-1}), \]
where
\[ K_{n}^{(d+1)} = \frac{P_{n|n-1}^{(d)}}{H_n^* (H_n P_{n|n-1} H_n^* + R_n)^{-1}}, \]
\[ P_{n+1|n}^{(d)} = P_{n|n-1}^{(d)} F_n - K_n^{(d)} H_n, \]
\[ P_{n|n-1}^{(d+1)} = P_{n|n-1}^{(d)} - P_{n|n-1}^{(d)} H_n [K_k^{(d+1)}] \]
and where \( \hat{x}_{-d|n} = 0, P_{n|n-1}^{(0)} = P_{n|n-1}. \) The computational complexity of such an algorithm would be \( O(L(q + 1)^3 U^3). \) However, efficient algorithms (e.g., the fast recursions) that reduce the calculation cost to \( O(L(q + 1)^2 U^2) \) can be implemented to relieve the computational burden. We use the algorithm proposed in [8]:

\[ \hat{x}_{n-L|n} = \hat{x}_{n-L|n-L} + P_{n-L|n-L-1} \sum_{d=1}^{L} e_n^{(d+1)} \]

where
\[ e_n^{(d+1)} = [F_n - K_{p,n} H_n] e_n^{(d)}, \]
\[ e_n^{(1)} = H_n^* [H_n P_{n|n-1} H_n^* + R_n] (y_n - H_n \hat{x}_{n|n-1}), \]
\[ P_{n+1|n} = P_{n|n-d}^{(d+1)} H_n [F_t - K_t H_t]^*, \]
and where \( \hat{x}_{n+1|n}, \hat{x}_{n|n}, K_{p,n} \) and \( P_{n|n-1} \) are obtained from the standard Kalman filter in (12).

4. Discussion and directions for future work

We briefly discuss a few interesting aspects of our scheme and point a few directions for future work, in this section.

The effect of the symbol vector \( s(k) \) on the received signal (i.e., measurement or observation) exists for up to \( q \) symbol periods beyond the symbol period \( k, \) due to the delay spread in the channel. We therefore used the fixed-lag smoothed estimation technique. By choosing the lag equal to the delay spread in the channel, the multipath diversity in the channel can be captured. We believe this approach will have a considerable performance gain over a filtered or predicted estimation technique that does not tap this diversity.

As stated before the estimation algorithm gives a recursive MMSE estimate of the state vector. In other words, the MMSE estimate of the symbols of all the users over a few \( (q + 1) \) symbol periods is obtained simultaneously. The receiver outperforms the FIR MMSE detector (due to its recursive nature). Note that the structure of the interference due to the other users, in a sense, is incorporated well in the detection process of any one user, unlike common FIR MMSE techniques (91) which incorporate only the second order effects of the interference. Also, while the adaptation of linear multiuser receivers when long codes are used, is complicated, our scheme is clearly, naturally suited for adaptation.

The complexity of the algorithm has a quadratic dependence on the number of users, \( U \) being decoded. It is possible to reduce this complexity by breaking the users into groups and decoding each group separately using this approach. The interference due to the other groups while decoding any one group can be incorporated into \( R_n. \) Alternatively one may subtract the interference from a group that has already been decoded before decoding the next group. Thus, it can be combined in various ways with parallel or successive interference canceling ideas. The tradeoffs in complexity, decoding delay and the performance improvements by using such techniques can be a subject of future study. Another interesting possibility is to make use of the finite alphabet property of the symbols and using their hard estimates in the fixed-lag smoothed estimation process.

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An interesting future study is to investigate if the complexity of the filtering process can be reduced by making use of the fact that state vector needs to be estimated only once every symbol period and not every chip period. Also, the observability property of the system is linked to the linear independence of the signature waveforms (combination of spatio-temporal propagation channel and the spreading codes) of the users. Investigation of this can also be an interesting future work.

5. Conclusions

We proposed a new space-time multiuser receiver for a CDMA system by making use of well researched state-space estimation techniques. Our scheme is expected to outperform the FIR MMSE receiver and is applicable in systems that use long codes (e.g., IS-95, Wideband CDMA [11]). The computational complexity of the receiver was briefly studied and possible solutions to reduce it were suggested. We discussed several interesting aspects of the receiver and suggested directions for future work.

References


