Power and Bandwidth Allocation in Cooperative Dirty Paper Coding

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Introduction

- Wireless ad hoc network: nodes may cooperate by joint encoding or processing.
  - Cooperation consumes power and bandwidth.

- Benefits of cooperation:
  - Need to optimize resource allocation.

- Two cooperating transmitters:
  - Orthogonal cooperation channel.
  - Exchange messages, then perform dirty paper coding (DPC).
  - Interference channel (in the absence of the cooperation channel).
  - Investigate power and bandwidth allocation to characterize the cooperative DPC rate region.
Dirty Paper Coding (DPC)

- Gaussian channel
  - Transmitter has power $P$, unit-variance Gaussian noise $N$.
  - Without interference $S$, capacity = $\frac{1}{2} \log (1+P)$.
- Dirty paper coding (DPC) [Costa 83]
  - Interference non-causally known to the transmitter.
  - Capacity = $\frac{1}{2} \log (1+P)$, same as if no interference.
  - Codewords orthogonal to $S$, instead of attempting to cancel the interference.
Related Works

- Achievable rate regions
  - Two cooperative transmitters and one receiver [Sendonaris et al. 03, Yazdi et al. 03].

- Cooperative diversity
  - Forward parity bits [Hunter et al. 03]
  - Orthogonal protocols achieve full spatial diversity [Laneman et al. 04].

- Information theoretical bounds
  - Multiplexing gain, diversity [Host-Madsen et al. 05, 06].
  - Achievable rates and capacity upper bounds [Khojastepour et al. 04, Jindal et al. 04]
Channel Model

- Two cooperating transmitters
  - Close together ($G$ is large).
  - Cooperation channel: Orthogonal, static, full-duplex additive white Gaussian noise (AWGN).

- Channel state information (CSI) $h_1, \ldots, h_4$ known at all terminals.
Channel Model: Power and Bandwidth

- System-wide cost of cooperation:
  - Network power constraint: $P_1 + P_2 + P_t \leq P$ (short-term).

- Bandwidth assumptions:
  - 1) Separate bands: $B = B_t = 1$ (spatial reuse).
  - 2) Bandwidth allocation: $B + B_t = 1$ (no reuse).
Cooperative Dirty Paper Coding (DPC)

- Transmitter exchange messages through cooperation channel.
  - Network becomes equivalent to a broadcast channel (BC) with a two-antenna transmitter.

- Transmitters jointly encode using dirty paper coding.
  - Capacity-achieving for multi-antenna BC [Weingarten 04].

- Cooperative DPC rate region:
  - Convex hull of rates ($R_1, R_2$) achievable by cooperative DPC.
  - Optimize power and bandwidth allocation.

\[
C_{\text{coop-DPC}} = \bigcap_{0 \leq \mu \leq 1} \left\{ (R_1, R_2) \mid \mu R_1 + (1 - \mu) R_2 \leq d^*(\mu) \right\}
\]

\[
d^*(\mu) \triangleq \max_{(R_1, R_2)} \mu R_1 + (1 - \mu) R_2
\]
Duality: Broadcast Channel and Multiple Access Channel

- **Gaussian BC:**
  - Dual MAC: “reverse” channel inputs and outputs [Jindal et al. 04, Vishwanath et al. 03].

- **For every** \((R_1, R_2)\) **in BC region under power constraint** \(P\):
  - Dual MAC achieves same \((R_1, R_2)\) with a suitable power allocation \(P_1, P_2\) such that \(P_1 + P_2 = P\).
  - Allocation of \(P_1, P_2\) in MAC is convex; can be calculated efficiently.
Optimal MAC Sum Power Allocation

- **Multiple access channel:**
  - Two single-antenna transmitters and a two-antenna receiver, sum power constraint: \( P_1 + P_2 \leq P_s \).
  \[
  y = h_1 x_1 + h_2 x_2 + n
  \]

- **Capacity region:**
  \[
  R_1 \leq \log|I + h_1 P_1 h_1^H| \\
  R_2 \leq \log|I + h_2 P_2 h_2^H| \\
  R_1 + R_2 \leq \log|I + h_1 P_1 h_1^H + h_2 P_2 h_2^H|
  \]

- \( (R_1, R_2) \) on capacity region boundary:
  - Concave in \( P_1, P_2 \).
  - Found by applying Karush-Kuhn-Tucker (KKT) conditions on the Lagrangian.
MAC Decode Order

- Dual MAC capacity region corner points
  - Achieved by different decoder orders.

- BC capacity region has three segments; from dual MAC:
  - Decode order (1)
  - Decode order (2)
  - Time sharing
Cooperative DPC Rates

- Transmitters need to know each other’s codewords.
  - Cooperation channel (with power $P_t$) rates $\geq$ DPC rates.

\[
\max_{P_t, B_t, B} \mu R_1 + (1 - \mu) R_2
\]

such that: $(R_1, R_2) \in C_{BC}(P - P_t, B)$

$(R_1, R_2) \in C_{co}(P_t, B_t)$,

$P_1 + P_2 = P - P_t$

- Duality: $C_{BC}(P - P_t, B) = C_{MAC}(P - P_t, B)$

- Cooperation channel:
  - Capacity of an AWGN channel:

\[
(R_1, R_2) \in C_{co}(P_t, B_t) \text{ iff } (2^{R_1/B_t} - 1)B_t/G + (2^{R_2/B_t} - 1)B_t/G \leq P_t
\]
Numerical Allocation of Power and Bandwidth

- Rates monotonically increasing with power $P$ and bandwidth $B$.
  - Reject (R1,R2) in the interior of $C_{BC}$ or $C_{co}$.
  - Numerically compute the intersection of boundaries of $C_{co}$ and $C_{BC}$ (three segments).

- Bandwidth assumption 1) $B_t = B = 1$
  - One-dimensional numerical optimization to find $P_t$.

- Bandwidth assumption 2) $B_t + B = 1$
  - Local optimum: numerically search over $P_t$ and $B_t$.
  - Conjecture: DPC rates concave over $P_t$ and $B_t$.
  - Convexity not readily verified as objective function is numerically computed.
Numerical Results

- Channels $h_1, \ldots, h_4$: independent Rayleigh fading.
- Ergodic rate region:
  - 1000 randomly generated channel realizations.
  - Average over channel realizations.
- Network power constraint
  - $P = 10$ dB.
- Cooperation channel:
  - Weak: $G = 0$ dB
  - Strong: $G = 10$ dB
- Comparisons:
  - BC (transmitters co-located, $G = \infty$).
  - Non-cooperation: time-division (TD) between transmitters.
Cooperative DPC Rate Regions

- Cooperation better than TD only when $G$ is large.
- Cooperation more advantageous at sum-rate.
- Separate cooperation bands: close to BC when $G$ is large.
- Equal bandwidths $B_t = B = 0.5$ close to optimal allocation.
Conclusions

- Two cooperating transmitters:
  - Exchange messages over orthogonal cooperation channel.
  - Cooperative dirty paper coding.

- Power and bandwidth allocation.
  - Optimal sum power allocation in multi-antenna MAC.
  - BC-MAC capacity duality.

- Cooperative DPC improves capacity when cooperation channel is strong.
  - Cooperative DPC offers better performance near sum-rate.

- Only considered transmitter cooperation.
  - Further gains may be obtained if receivers as well as transmitters cooperate.