Equalization (MSE)

SNR at the output of ZF-LE:

\[ \gamma = \left[ \frac{T^2 N_0}{2\pi} \int_{-\pi/T}^{\pi/T} \frac{d\omega}{\sum |H(\omega + \frac{2\pi n}{T})|^2} \right]^{-1} \]

Using MSE criterion can reduce the noise enhancement effect:

\[ \varepsilon = E|\varepsilon_k|^2 \]

\[ \varepsilon_k = a_k - \hat{a}_k \]

\[ \hat{a}_k = \sum_j w_j v_{k-j} \]

Using orthogonality principle:
\[ E(\varepsilon_k \nu_{k-l}) = 0 \quad -\infty < l < \infty \]

\[ E\left\{ a_k - \sum_j w_j \nu_{k-j} \nu_{k-l}^* \right\} = 0 \]

\[ \sum_j w_j E(\nu_{k-j} \nu_{k-l}^*) = E(a_k \nu_{k-l}^*) \]

Also:

\[ E(\nu_{k-j} \nu_{k-l}^*) = \begin{cases} 
 x_{l-j} + N_0 \delta_{lj} & |l-j| \leq L \\
 0 & \text{Otherwise} 
\end{cases} \]

and:

\[ E(a_k \nu_{k-l}^*) = \begin{cases} 
 g_{-l} & -L \leq l \leq 0 \\
 0 & \text{Otherwise} 
\end{cases} \]
So:

\[ W(z) = \frac{G^*(z^{-1})}{G(z)G^*(z^{-1}) + N_0} \]

for high SNR the ZF and MSE criteria result in are approximately equal filter.

The minimum error for MSE equalizer is:

\[ \varepsilon_{\text{min}} = E(\varepsilon_k a_k^*) = 1 - \sum_j w_j g_{-j} \]

Define:

\[ Q(z) = W(z)G(z) \]

Then:

\[ \varepsilon_{\text{min}} = 1 - q_0 \]

\[ = \int_{-\pi/T}^{\pi/T} \frac{N_0}{1/T \sum_n \left| H(\omega + \frac{2\pi n}{T}) \right|^2 + N_0} d\omega \]
Decision Feedback Equalizers

WMF equivalent channel model is monic and causal. Therefore, the symbol at the output has post-cursor interference only.

The optimal frontend filter for DFE is WMF which minimizes the noise variance at the input of slicer. The optimal postcursor equalizer is $G(z)^{-1}$.

DFE-ZF structure can be considered as a LE-ZF + a linear prediction error filter.

$$\hat{a}_k = \sum_{j=-K_1}^{0} w_j v_{k-j} + \sum_{j=1}^{K_2} w_j \tilde{a}_{k-j}$$