Equalization (overview)

Channel model:

\[ r_k = \sum_{n=0}^{L} h_n a_{n-k} + z_k \]

At the output of matched filter:

\[ y_k = \sum_{n} a_n x_{k-n} + v_k \]

\[ x_n = \int_{-\infty}^{\infty} h^*(t)h(t+nT)dt \]

The term \( \frac{1}{x_0} \sum_{n \neq k} a_n x_{k-n} \) represents ISI. Correlation of noise term at the output of MF is:

\[ 1/2E\{v_k^* v_j\} = \begin{cases} N_0 x_{k-j} & |k - j| \leq L \\ 0 & \text{otherwise} \end{cases} \]
\[ X(z) = G(z)G^*(z^{-1}) \]

Introducing a noise whitening filter: \( 1/G^*(z^{-1}) \) the output is

\[ u_k = \sum_{n=0}^{L} g_n a_{n-k} + \eta_k \]

Where the noise is white.

Linear equalizer:

The equalizer (with 2K taps) output:

\[ \hat{a}_k = \sum_{j=-K}^{K} w_j u_{k-j} \]
Single filter $q$ representing the cascade of channel and equalizer:

$$\hat{a}_k = q_0 a_k + \sum_{n \neq k} a_n q_{k-n} + \sum_j w_j \eta_{k-j}$$

Zero-forcing equalizer:

$$D = \sum_{n \neq 0} |q_n|$$

$$= \sum_{n \neq 0} \sum_j |w_j g_{n-j}|$$

$D=0$ implies that

$$W(z) = \frac{1}{G(z)}$$

Noise PSD at the output of equalizer:

$$\Phi(\omega) = \frac{N_0}{X(e^{j\omega T})} \quad |\omega| \leq \frac{\pi}{T}$$
SNR for zero forcing equalizer with infinite taps:

\[
\gamma = \frac{1}{\sigma^2}
\]

\[
= \left[ \frac{TN_0}{2\pi} \int_{-\pi/T}^{\pi/T} \frac{d\omega}{X(e^{j\omega})} \right]^{-1}
\]

\[
X(e^{j\omega}) = \frac{1}{T} \sum \left| H(\omega + \frac{2\pi n}{T}) \right|^2 \quad |\omega| \leq \frac{\pi}{T}
\]

So the SNR can be expressed as following

\[
\gamma = \left[ \frac{T^2 N_0}{2\pi} \int_{-\pi/T}^{\pi/T} \frac{d\omega}{\sum \left| H(\omega + \frac{2\pi n}{T}) \right|^2} \right]^{-1}
\]