Diversity (time-frequency-space)

Assumptions:
- Nonselective fading for each branch
- Independent branches
- Slow fading

\[ r_k(t) = \alpha_k e^{-j\varphi_k} x_k(t) + n_k(t), \quad k = 1, \cdots, L \]

Detection variables for maximum ratio combiner:

\[
U = 2E \sum_{k=1}^{L} \alpha_k^2 + \sum_{k=1}^{L} \alpha_k N_k
\]

\[
N_k = e^{j\varphi} \int_{0}^{T} n_k(t)x_k^*(t)dt
\]
Diversity (time-frequency-space), Detection

Decision variable $U$ is Gaussian with

$$E(U) = 2E \sum_{k=1}^{L} \alpha_k^2, \quad \sigma^2 = 2EN_0 \sum_{k=1}^{L} \alpha_k^2$$

So

$$P(\gamma) = \frac{1}{2} \text{erf} (\sqrt{\gamma})$$

$$\gamma = \frac{E}{N_0} \sum_{k=1}^{L} \alpha_k^2 = \sum_{k=1}^{L} \gamma_k$$

SNR has a chi-squared distribution with $2L$ degrees of freedom with PDF

$$f(\gamma) = \frac{1}{(L-1)!(\bar{\gamma}_c)} \gamma^{L-1} e^{-\gamma/\bar{\gamma}_c}$$

$\bar{\gamma}_c$ is the average SNR per channel
Detection with diversity

\[ P = \int_{0}^{\infty} P(\gamma) f(\gamma)d\gamma \]

\[ \approx \left( \frac{1}{4\bar{\gamma}_c} \right)^L \binom{2L-1}{L} \]

The error decreases with the \( L \)th power of SNR.

The performance of a rake receiver with perfect channel estimate is equivalent to a \( L \)th order MRC.
Estimation

Parameter space, Observation space, probabilistic mapping, Estimation rule

Cost function:

\[ C(a, \hat{a}) = f(\hat{a}(R) - a) \]

Bay’s estimate: Minimizing the cost function

Random parameters \( a \): with a priori probability density \( P_a(A) \)

\[
E\{C(a, \hat{a}(R))\} = \int dA \int C[A - \hat{a}(R)] P(A, R) dR
\]
For mean square error:

\[ C(a - \hat{a}(R)) = (a - \hat{a}(R))^2 \]

Using

\[ P(A, R) = P(R)P(A \mid R) \]

\[ E_{ms} \{ C(a, \hat{a}(R)) \} = \int dRP(R) \int dA[A - \hat{a}(R)]^2 P(A \mid R)dR \]

Then

\[ \hat{a}_{ms}(R) = \int AP(A \mid R)dA \]

Which is the mean of a posteriori density
For maximum a posteriori estimate we use a different cost function.

MAP estimate is usually shown as:

\[
\frac{\partial \ln P(A \mid R)}{\partial A} \bigg|_{A=\hat{a}(R)} = 0
\]

But

\[
\ln P(A \mid R) = \ln P(R \mid A) + \ln P(A) - \ln P(R)
\]

So the above condition is equivalent to

\[
\frac{\partial \ln P(R \mid A)}{\partial A} \bigg|_{A=\hat{a}(R)} + \frac{\partial \ln P(A)}{\partial A} = 0
\]
Estimation of deterministic parameters:

Estimator mean:

\[ E(\hat{a}(R)) = \int \hat{a}(R) P(R \mid A) dR \]

Unbiased, minimum variance, consistent estimate

Maximum likelihood estimate: is the value of variable at which the likelihood function is maximum:

\[ \left. \frac{\partial \ln P(R \mid A)}{\partial A} \right|_{A=\hat{a}(R)} = 0 \]

Maximum likelihood estimate is the limiting case of max a posteriori estimate in which the a priori knowledge is zero
Detection/Estimation in fading

- Different receiver components such as gain control and frequency control circuit should track the channel by using pilot signal or data-aided techniques.
- Detection techniques for flat fading channels are an extension of detection in AWGN channels.
- Detection in frequency selective channels:
  - Equalizers
  - Spread spectrum
  - OFDM