Homework (1)

1- Existence Theorem: Given an even positive function S(w), there is a stochastic process *x(t)* having *S(w)* as its spectrum.

2- Random sampling theorem: *x(t)* is a band-limited deterministic signal with Fourier transform *X(w)* and energy of $E = \int_{-\infty}^{\infty} x^2(t)dt$

Show that if t_i is a Poisson point process with average density λ and $|X(\omega)| >> \sqrt{\frac{E}{\lambda}}$

then

$$H(\omega) = \frac{1}{\lambda}\sum_i x(t_i)e^{-j\omega t_i}$$

an unbiased estimate of X(w).

3- Given two WSS, zero mean random processes a(t) and b(t) and a constant w show that the random process

$$x(t) = a(t)\cos\omega t - b(t)\sin\omega t$$

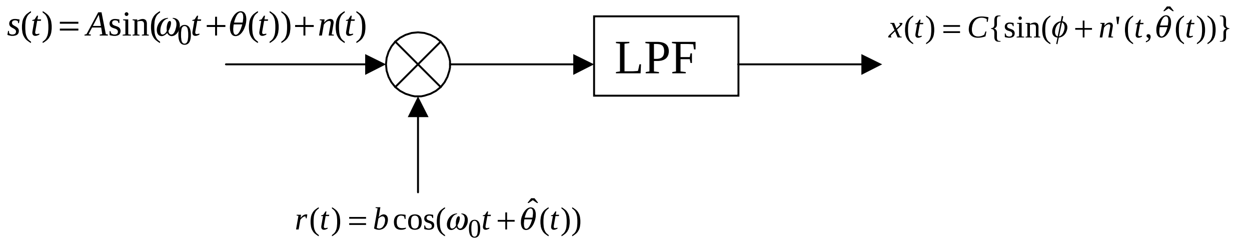
is WSS iff

$$R_{aa}(\tau) = R_{bb}(\tau)$$

$$R_{ab}(\tau) = -R_{ba}(\tau)$$

Homework (1)

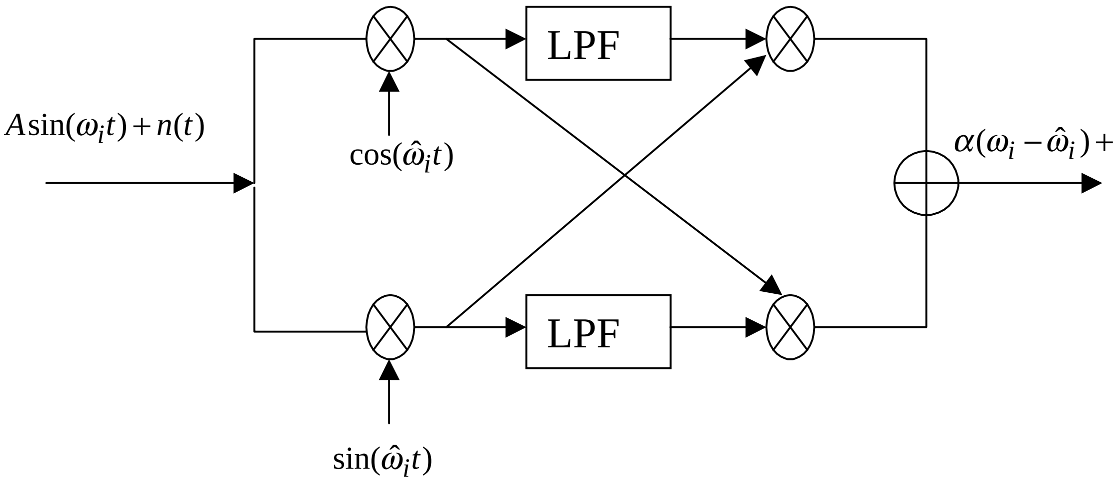
4- Find the $R(\tau)$ autocorrelation of *n'(t)* for $|\tau| < \tau_{corr,\hat{\theta}}$ in the following:



This can be interpreted as the autocorrelation of angular phase disturbance at the output of the filter for time periods smaller than VCO correlation time.

HomeWork (1)

5- Find the expected value of noise term *n(t)* for a Differentiator AFC



Hints for 4 and 5: Use The following properties for narrowband Gaussian noise:

$$n(t)=\sqrt{2}n_c(t)\cos(\omega_i t)+\sqrt{2}n_s(t)\sin(\omega_i t)$$

$$R_n(\tau)=2R_{n_c}(\tau)+j2R_{n_c n_s}(\tau)$$