Adaptive Rate and Power DS/CDMA Communications in Fading Channels

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***Abstract*— We investigate the average information rates attained by adapting the transmit power and the information rate relative to channel variations in code division multiple access communication systems. Our results show that the rate adaptation provides a higher average information rate than the power adaptation for a given average transmit power, and the rate increase when using rate adaptation is more significant for channels with a faster decaying multipath intensity profile and weaker line-of-sight component.**

***Index Terms*—Adaptation, CDMA, fading.**

I. Introduction

THE RADIO link for either a portable or a vehicular unit can be characterized by time-varying multipath fading, which causes the transmission quality to vary with time. When the transmitter is provided with the channel state information, the transmission schemes can be adapted to it, allowing the channel to be used more efficiently. In general, the transmitter may vary its information rate and transmit power. During good channel conditions, more information is sent. As the channel condition becomes worse, lower information rates are applied in order to maintain adequate transmission quality. We may also consider adapting the transmit power relative to channel variations when the information rate is fixed. In current code-division multiple-access (CDMA) cellular systems, open- and closed-loop power control techniques are employed in adjusting the transmit power of each mobile [1].

In this letter, we investigate the average information rates attained by adapting the transmit power and the information rate relative to channel variations in code division multiple access communication systems. Our results show that the rate adaptation provides a higher average information rate than the power adaptation for a given average transmit power, and the rate increase when using rate adaptation is more significant for channels with a faster decaying multipath intensity profile and weaker line-of-sight component. The rate increase translates into a power gain, which results in a reduced interference to other cells in multiple cell systems, leading to a capacity increase.

II. System Model

We assume that the channel variation due to multipath fading is slow relative to the bit duration, and the multipath fading is characterized by the Nakagami-m probability density function. We look only at a single cell system. The implications of a multiple cell system can be accounted for by the out-of-cell interference coefficient [1].

The received signal $y(t)$ at the base station can be represented by

$$y(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} \sqrt{2G_{k,l}S_k} d_k(t-\tau_{k,l}) p_k(t-\tau_{k,l}) \cdot \cos\left[2\pi f_c(t-\tau_{k,l}) + \theta_{k,l}\right] + n(t) \tag{1}$$

where K is the number of users, $L = \lfloor T_m/T_c \rfloor + 1$ is the number of resolvable paths, where T_m is the maximum multipath delay time, and T_c is the chip time. S_k is the transmit power of user k, and $d_k(\cdot)$ and $p_k(\cdot)$ are the binary data sequence and the random binary spreading sequence for user k, respectively. $\tau_{k,l}$ and $\theta_{k,l}$ represent the path delay and the phase, respectively, for user k on the lth path, and they are assumed independent and uniformly distributed, the former over a bit interval and the latter over $[0, 2\pi]$. $n(t)$ represents the zero-mean white Gaussian noise with two-sided power spectral density $N_0/2$. $G_{k,l}$ is the gamma distributed random variable representing the channel power gain for user k on the lth path, and its probability density function is given by

$$P_{G_{k,l}}(g) = \left(\frac{m}{\Omega_l}\right)^m \frac{g^{m-1}}{\Gamma(m)} e^{-(m/\Omega_l)g}, \tag{2}$$

where m is the Nakagami fading parameter, and $\Gamma(m)$ is the gamma function defined as [2]

$$\Gamma(m) \triangleq \int_0^\infty t^{m-1} e^{-t}\, dt, \qquad m > 0. \tag{3}$$

We assume that $\{G_{k,l}\}$ are independent identically distributed (i.i.d.) random variables with

$$E[G_{k,l}] = \Omega_l = \Omega_o e^{-\delta(l-1)} \tag{4}$$

where δ reflects the rate at which decay occurs. We assume an L-branch RAKE combiner with maximal-ratio combining of the Nakagami fading paths.

The Nakagami-m distribution spans a range of fading environments from one-sided Gaussian fading ($m = 1/2$, which corresponds to worst-case fading) to nonfading ($m = \infty$). It is well known that m = 1 corresponds to Rayleigh fading, and the Rician and lognormal distributions can be closely approximated by the Nakagami distribution with $m > 1$.

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III. Signal-to-Interference Ratio

A coherent correlation receiver recovering the signal of user i on the lth path forms a decision statistic $Z_{i,l}$, given by

$$\begin{aligned} Z_{i,l} &= \sqrt{\frac{2}{T_i}} \int_{\tau_{i,l}}^{\tau_{i,l}+T_i} y(t) p_i(t-\tau_{i,l}) \\ &\quad \cdot \cos\left[2\pi f_c(t-\tau_{i,l})+\theta_{i,l}\right] dt \\ &= \sqrt{G_{i,l} S_i T_i} d_i + I_S + I_M + \eta \end{aligned} \tag{5}$$

where T_i is the bit duration, and d_i is the information bit taking values $+1$ and -1 with equal probability, all for user i. The first term in (5) is the desired signal term. The second term I_S is the self-interference of the user of interest, the third term I_M is the multiple-access interference term induced by the other $K-1$ users, and η is the white Gaussian noise of mean zero and variance $N_0/2$. I_S and I_M are independent random variables with mean zero and variances

$$E[I_S^2] = \sum_{m=1,\, m\neq l}^{L} G_{i,m} S_i T_c/3 \tag{6}$$

$$E[I_M^2] = \sum_{k=1,\, k\neq i}^{K} \sum_{m=1}^{L} G_{k,m} S_k T_c/3. \tag{7}$$

The RAKE receiver for user i having perfect knowledge of fading magnitude on each finger forms a decision statistic Z_i

$$Z_i = \sum_{l=1}^{L} \sqrt{G_{i,l}} Z_{i,l}. \tag{8}$$

Typically, the self-interference does not have a significant effect on performance, especially for large K. This is due to the fact that I_M has $(K-1)L$ components, whereas I_S has $(L-1)$ components which are relatively small for large K. Hence, I_M is actually the dominant interference. Therefore, for simplicity of analysis, we ignore the self-interference term in (5). Then, the bit energy E_b and the equivalent noise spectral density $N_e/2$ at the RAKE receiver output are $E_b = (\sum_{l=1}^{L} G_{i,l})^2 S_i T_i$ and $N_e/2 = \sum_{l=1}^{L} G_{i,l} [\sum_{k=1,\, k\neq i}^{K} \sum_{m=1}^{L} G_{k,m} S_k T_c/3 + N_o/2]$. Consequently, the bit energy-to-equivalent noise spectral density ratio E_b/N_e is

$$E_b/N_e = \frac{G_i S_i T_i}{\sum_{k=1, k\neq i}^{K} 2 G_k S_k T_c/3 + N_o} \tag{9}$$

where

$$G_i \triangleq \sum_{l=1}^{L} G_{i,l}. \tag{10}$$

The probability density function (pdf) of G_i is given by [2]

$$P_{G_i}(g) = \left(\frac{m_g}{\Omega_g}\right)^{m_g} \frac{g^{m_g-1}}{\Gamma(m_g)} e^{-(m_g/\Omega_g)g} \tag{11}$$

where

$$\Omega_g = \Omega_o \sum_{l=1}^{L} e^{-\delta(l-1)} \tag{12}$$

$$m_g = \frac{m\left(\sum_{l=1}^{L} e^{-\delta(l-1)}\right)^2}{\sum_{l=1}^{L} \left(e^{-\delta(l-1)}\right)^2}. \tag{13}$$

In (12) we assumed an exponential delay profile model, where the signal-to-noise ratios of the different resolved signal paths decrease exponentially with increasing delays.

IV. Rate and Power Adaptations

It follows from (9) that in order to maintain adequate transmission quality which depends on E_b/N_e, the information rate $R_i \triangleq 1/T_i$ b/s and the transmit power S_i of user i should be given by

$$R_i = \frac{3R_c}{2(E_b/N_e)_o} \cdot \frac{G_i S_i}{\sum_{k=1,\, k\neq i}^{K} G_k S_k + 3N_o R_c/2} \tag{14}$$

where $(E_b/N_e)_o$ is the value required for adequate performance of the modem and decoder, and $R_c \triangleq 1/T_c$ is the chip rate. Typically, $(E_b/N_e)_o$ depends on its implementation, use of error correcting coding, channel impairments such as fading, and error rate requirements.

We first consider the case where the transmit power of each user is fixed at S_T (i.e., $S_i = S_T$ for all i), and the information rate is adapted. We call this the rate adaptation. Then, it follows from (14) and the fact that $E[G_i] = \Omega_g$ for all i that the average information rate $\overline{R}_i$ with the rate adaptation is

$$\overline{R}_i = \frac{3R_c}{2(E_b/N_e)_o} \cdot E\left[\frac{1}{I}\right] \tag{15}$$

where

$$I \triangleq \frac{1}{\Omega_g} \sum_{\substack{k=1 \\ k\neq i}}^{K} G_k + \frac{3R_c N_0}{2\Omega_g S_T}. \tag{16}$$

Since all G_k's are assumed to be i.i.d. Gamma random variables, the pdf of I is given by [2]

$$P_I(x) = \frac{m_g^{m_g(K-1)}}{\Gamma(m_g(K-1))} (x-b)^{m_g(K-1)-1} e^{-m_g(x-b)}, \quad x \geq b \tag{17}$$

where

$$b \triangleq \frac{3R_c N_0}{2\Omega_g S_T}. \tag{18}$$

It follows from (15) and (17) that

$$\overline{R}_i = \frac{3R_c}{2(E_b/N_e)_o} \frac{m_g^{m_g(K-1)}}{\Gamma(m_g(K-1))} \int_0^\infty \frac{t^{m_g(K-1)-1}}{t+b} e^{-m_g t} dt. \tag{19}$$

Since $1/I$ in (15) is *convex* $\cup$, a lower bound on $\overline{R}_i$ can be obtained by using *Jensen's inequality* [3]

$$\begin{aligned} \overline{R}_i &\geq \frac{3R_c}{2(E_b/N_e)_o} \cdot \frac{1}{E[I]} \\ &= \frac{1}{(E_b/N_e)_o [2(K-1)/(3R_c) + N_0/(\Omega_g S_T)]}. \end{aligned} \tag{20}$$

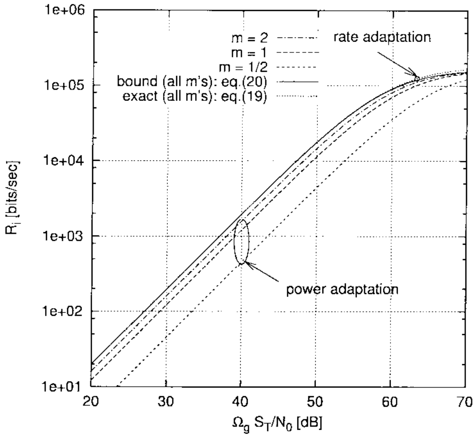

Fig. 1. The average information rate $\overline{R}_i$ versus $\Omega_g S_T/N_o$; $K = 10$, $R_c = 5M$, $L = 3$, $(E_b/N_e)_o = 7$ dB, and $\delta = 0.5$.

In fact, (20) represents the average information rate when user rates are adapted based on average interference. Fig. 1 shows that the average information rates obtained by (19) and (20) are virtually identical.

If we normalize $\overline{R}_i$ by the total bandwidth R_c, then we get the average spectral efficiency in bits/second/hertz. It should be noted that the instantaneous information rate R_i in (14) should not exceed R_c in order to keep the bandwidth constant. Therefore, when $R_i > R_c$, i.e.,

$$G_i S_i > \left(\frac{E_b}{N_e}\right)_o \cdot \left[\frac{2}{3}\sum_{\substack{k=1\\k\neq 1}}^{K} G_k S_k + N_o R_c\right]$$

R_i should be limited by R_c. However, since the probability of $R_i > R_c$ is negligibly small for parameter values of practical interests, (15) yields the actual average information rate for the rate adaptation.

When the information rate of each user is fixed at $1/T$, i.e., $T_i = T$, the transmit power S_i may be adapted such that a target transmission quality is maintained. That is, the transmit power S_i of user i is adjusted to compensate for fading such that $G_i S_i$ is equal to S_R, where S_R is the target received signal power that meets $(E_b/N_e)_o$ requirement. It follows from (9) that S_R is given by

$$S_R = \frac{N_0}{T} \cdot \frac{(E_b/N_e)_o}{1 - 2(K-1)(E_b/N_e)_o T_c/(3T)}. \tag{21}$$

This power adaptation ensures that all mobile signals are received with the same power S_R. Then, the average transmit power S_T at the mobile unit is

$$S_T = E[S_i] = E[S_R/G_i] = \frac{m_g S_R}{(m_g - 1)\Omega_g}. \tag{22}$$

Therefore, it follows from (14) and (22) that the (average) data rate $\overline{R}_i = 1/T$ with the power adaptation is

$$\overline{R}_i = \frac{1}{(E_b/N_e)_o} \frac{1}{2(K-1)/(3R_c) + N_0/[\Omega_g S_T (m_g - 1)/m_g]}. \tag{23}$$

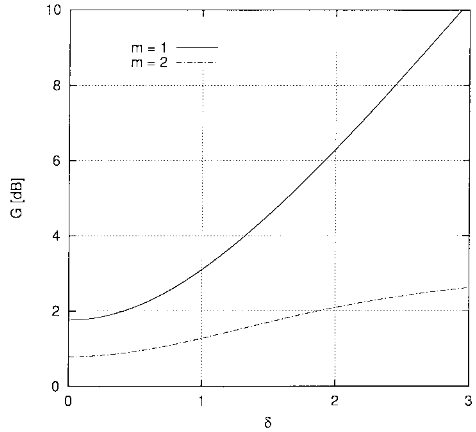

Fig. 2. The power gain G versus δ; $L = 3$.

The average information rates versus $\Omega_g S_T/N_o$ are shown in Fig. 1 for different Nakagami m parameters. As the channel becomes worse (less m), we find that the rate adaptation provides a higher average information rate than the power adaptation. Comparison of (20) and (23) indicates that the rate adaptation provides a power gain G

$$G = m_g/(m_g - 1) \tag{24}$$

over the power adaptation. Fig. 2 is a plot of the power gain G versus δ for several values of m. We find that the power gain is higher for channels with larger δ (i.e., faster decaying multipath intensity profile) and smaller m (i.e., weaker line-of-sight component). The power loss with the power adaptation is due to compensating for the deep fades. Since the transmit power at the mobile unit can be reduced by a factor of the power gain when using rate adaptation, the battery life can be prolonged by the same factor, and interference to other cells can be reduced in multiple cell systems, leading to a capacity increase. However, the rate adaptation provides a variable bit rate whereas the power adaptation provides a constant bit rate.

It should be noted that either adaptation process can introduce degradations. For example, the power adaptation requires a complicated hardware to accurately compensate for the fading. In particular, practical power control schemes have a maximum transmit power limit, which results in only a partial compensation for the deep fades and thereby induces a performance degradation. Similarly, the rate adaptation requires matching the possible bit rates at both transmitter and receiver and buffering input data for channels with low average channel power gain. Also, the rate quantization for the rate adaptation leads to a performance degradation.

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