

# SCATTERING FUNCTION IN DELAY AND DOPPLER INDUCED BY MOTION IN A DENSE MULTIPATH ENVIRONMENT

Israel Bar-David<sup>1</sup> and Rajeev Krishnamoorthy<sup>2</sup>

<sup>1</sup>Dept. of EE, Technion - Israel Inst. of Technology, Technion City, Haifa 32000, Israel

<sup>2</sup>Bell Labs, Lucent Technologies, 791 Holmdel-Keyport Rd., Holmdel, NJ 07733, USA

## Abstract

The Delay Spread Profile for a dense multipath environment, modeled as a Wide Sense Stationary Uncorrelated Scatterers (WSSUS) channel, has recently been derived [1] for stationary transmitter-receiver pairs. The result shows the dependence of the profile on transmitter-receiver distance. In this paper we extend the analysis to the case that this distance is changing at constant velocity, inducing different Doppler shifts depending on the relative location of the scatterers. The Scattering Function is the power spectral density of the received signal, as a function of the delay and Doppler coordinates. It is of interest for the analysis of mobile transmission of systems in which the bandwidth is comparable to, or larger than, the reciprocal delay spread of the communication channel. For evaluation of narrow-band systems the delay-independent power spectral density, known as the Jakes model, is appropriate. When the Delay variable is integrated out in the Scattering Function, the Jakes model is obtained. When the Doppler variable is integrated out, allowing for slight differences in the topological layouts of the two models, the Scattering Function reduces to the Delay Spread Profile mentioned above.

## 1. Introduction

Any linear time-varying system can be represented by the impulse response,  $h(\tau, t)$ , defined implicitly by the input-output ( $s(t)$  to  $r(t)$ ) relationship

$$r(t) = \int h(\tau, t) s(t - \tau) d\tau. \quad (1)$$

The spectral representation of particular interest for our application is obtained by the Fourier transform over the time-variable,  $t$ :

$$\psi(\tau, \nu) = \int h(\tau, t) e^{-j2\pi \nu t} dt, \quad (2)$$

of course assuming that the conditions for transformation exist. Further, the output signal is

$$r(t) = \int \int \psi(\tau, \nu) s(t - \tau) e^{j2\pi \nu t} d\nu d\tau, \quad (3)$$

where  $\psi(\tau, \nu)$  is the 2-dimensional complex weighting function of the delayed (by  $\tau$ ) and Doppler-shifted (by  $\nu$ ) input signal,  $s(t)$ .

If adjacent scatterers are uncorrelated at arbitrary closeness,  $\psi(\tau, \nu)$  can be modeled as a white process

$$\begin{aligned} E[\psi(\tau_1, \nu_1) \psi^*(\tau_2, \nu_2)] &= \\ &= S(\tau_1, \nu_1) \delta(\tau_1 - \tau_2) \delta(\nu_1 - \nu_2). \end{aligned} \quad (4)$$

In this case  $S(\tau, \nu)$  is called the Scattering Function.

Under Rayleigh statistics,  $\psi(\tau, \nu)$  is a complex Gaussian process and then knowledge of the dependence of  $S(\tau, \nu)$  on  $\tau$  and  $\nu$  enables the generation of sample-functions  $\psi(\tau, \nu)$  to be used in (3) for system performance simulation.

In the sequel,  $S(\tau, \nu)$  is derived for the model of relatively moving transmitter-receiver link in uniformly distributed uncorrelated scatterers, such as has been introduced in [1], without motion.

For evaluation of narrow-band systems the delay-independent power spectral density (PSD) known as the Jakes model [2], is appropriate. It is obtainable from  $S(\tau, \nu)$  when the delay variable,  $\tau$ , is integrated out, under a specific condition on the transmitter-receiver separation. The importance of  $S(\tau, \nu)$  is apparent for large bandwidth mobile communication systems, where delay-dependence is crucial. It should be noted that scattering functions have been investigated for iono- and tropo-scatter applications [3].



relation between  $\tau$  and  $r$  is easy ( $\tau = r/c$ ); we need the relation between  $x_i$  and  $v$ , and will do this via  $x_i$  and  $\alpha$ , since the relation between  $dx_i$  and  $dv$  is (see (5))

$$dv = -v_n \sin \alpha d\alpha. \quad (8)$$

So we focus on  $\frac{dx_i}{d\alpha}$  as shown in figure 2.

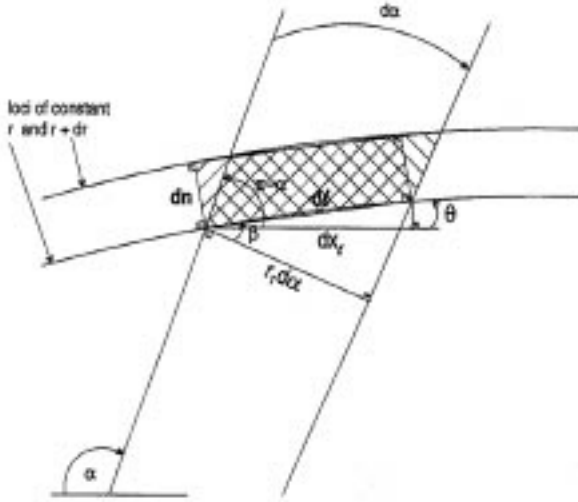


Figure 2. Elementary increments along the ellipse ( $dl$ ) and normal to it ( $dn$ )

First, we observe that for infinitesimal  $dr$  and  $da$ , the area  $dn dl$  (hashed //) is equal (to first order) the area of the parallelepiped (hashed \\\).

Define

$$\dot{y}_l = \frac{dy}{dx} = \tan \theta,$$

and

$$r_1 = \sqrt{\left(x + \frac{D}{2}\right)^2 + y^2}.$$

Then

$$\frac{dx_i}{dl} = \cos \theta. \quad (9)$$

$\beta$  is defined by

$$r_1 d\alpha = dl \cos \beta, \quad (10)$$

and so

$$\beta = \theta + \left(\alpha - \frac{\pi}{2}\right), \quad (11)$$

which results in

$$\cos \beta = \sin(\alpha + \theta),$$

$$dl = \frac{dx_i}{\cos \theta}.$$

Also,

$$dl = \frac{r_1 d\alpha}{\cos \beta}.$$

Finally,

$$\begin{aligned} \frac{dx_i}{dl} &= d\alpha \frac{r_1 \cos \theta}{\sin(\alpha + \theta)}, \\ &= d\alpha J_{x_i, \alpha}, \end{aligned} \quad (12)$$

which defines  $J_{x_i, \alpha}$ , the Jacobian of the new transformation. The final Jacobian,  $J_{final}$ , (using  $J_{10}$ , from equation (7a) above), is

$$dn dl = J_{10} J_{x_i, \alpha} d\alpha dr, \quad (13)$$

and using (8) we get

$$\begin{aligned} dn dl &= \left( J_{10} J_{x_i, \alpha} \frac{1}{v_n \sin \alpha} \right) dv dr \\ &= J_{final} dv dr. \end{aligned} \quad (14)$$

We now need to express  $J_{final}$  in terms of  $v$  and  $r$ . For this we need to solve the following two equations

$$\tan \alpha = \frac{y}{x + \frac{D}{2}}, \quad (15)$$

and, from (5) of [1],

$$x^2 + \frac{y^2 r^2}{r^2 - D^2} = \frac{r^2}{4}, \quad (16)$$

for  $x$  and in terms of  $r$  and  $\alpha$ , to substitute in  $J_{10}$ . Also,  $J_{x_i, \alpha}$  requires  $r_1$  and  $\dot{y}_l$  in terms of  $r$  and  $\alpha$ . From (4) of [1],

$$r_1 = r - \sqrt{\left(x - \frac{D}{2}\right)^2 + y^2}, \quad (17)$$

and

$$\dot{y}_l = \frac{dy}{dx},$$

which, from (5) of [1] is

$$2x dx + 2y \frac{r^2}{r^2 - D^2} dy = 0, \quad (18)$$

$$\dot{y}_l = \frac{dy}{dx} = -\frac{x}{y} \frac{r^2 - D^2}{r^2}. \quad (19)$$

Using  $x$  and  $y$  as a function of  $v$  and  $r$  in all of these yields

$$J_{final}(r, v) = \frac{1}{\sqrt{v_n^2 - v^2}} \left( \frac{1 - \frac{4x^2 D^2}{r^4}}{2\sqrt{1 - \frac{D^2}{r^2}} \sqrt{1 - \frac{4x^2}{r^2}}} \right) \times$$

$$\times \left( \frac{\left( r - \sqrt{\left( x - \frac{D}{2} \right)^2 + y^2} \right)}{\sqrt{1 - \left( \frac{v}{v_m} \right)^2} - \frac{x}{y} \left( 1 - \frac{D^2}{r^2} \right) \frac{v}{v_m}} \right), \quad (20)$$

where (using equations (5), (6), (15), and (16))

$$\begin{aligned} y &= \left( x + \frac{D}{2} \right) \tan \alpha, \\ x &= \frac{-D \frac{r^2 \tan^2 \alpha}{r^2 - D^2} \pm r \sec \alpha}{2 \left( 1 + \frac{r^2 \tan^2 \alpha}{r^2 - D^2} \right)}, \\ \sec \alpha &= \frac{v_m}{v}, \\ \tan \alpha &= \sqrt{\left( \frac{v_m}{v} \right)^2 - 1}. \end{aligned} \quad (20a)$$

To check the derivation, the integral (for given  $\tau$ ) of  $J_{\text{total}}$  over  $v$  ( $0, \infty$ ) was compared to that of  $J_{10}$  over  $x$  ( $0, \tau/2$ ), both yielding  $dA(r)$ , the elementary area of an elliptical annulus, as in [1].

### 3. Results

#### 3.1 The Scattering Function, $S(\tau, v)$

Following equation (2) of [1],  $S(\tau, v)$  is defined by  $dP_{\text{sc}}(\tau, v) = S(\tau, v) d\tau dv$ , and therefore

$$S(\tau, v) = \frac{Q}{r_1^2(\tau, v) v_2^2(\tau, v)} \frac{1}{\sqrt{v_m^2 - v^2}} J_{\text{total}}(r, v). \quad (21)$$

where

$$r_1^2(\tau, v) = \left( x + \frac{D}{2} \right)^2 + y^2,$$

and

$$v_2^2(\tau, v) = \left( x - \frac{D}{2} \right)^2 + y^2,$$

and  $x, y, D$  and  $r$  can be obtained in terms of  $\tau$  and  $v$  from equation (20a) above. Distances are converted to time by dividing by the speed of light,  $c$ .  $Q$ , a function of the transmitted power, density and cross-section of scatterers, and antenna aperture, is a constant. Figure 3 shows  $S(\tau, v)$  for  $D = 100\lambda$ ,  $f_c = 900$  MHz, and  $V = 100$  km/h.

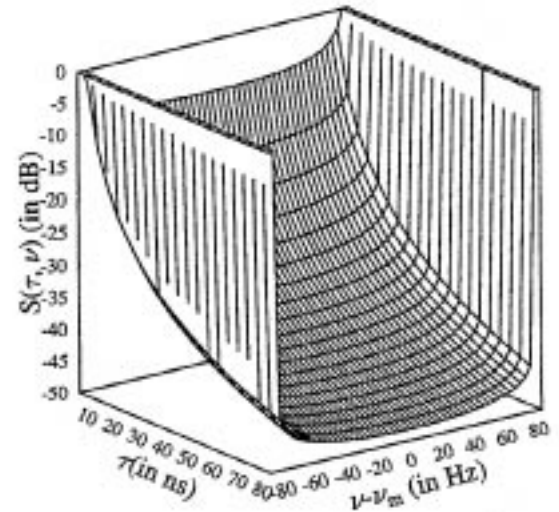


Figure 3. The Scattering Function,  $S(\tau, v)$

#### 3.2 The Stationary Case

As a further check on the derivation, figure 4 shows the Delay Spread Power Profile as a function of  $\tau$ , the time of arrival after the first arrival. This was obtained by integrating  $S(\tau, v)$  over all  $v$ . It closely matches the Delay Spread Power Profile,  $U(\tau; D)$  in Figure 2 of [1].

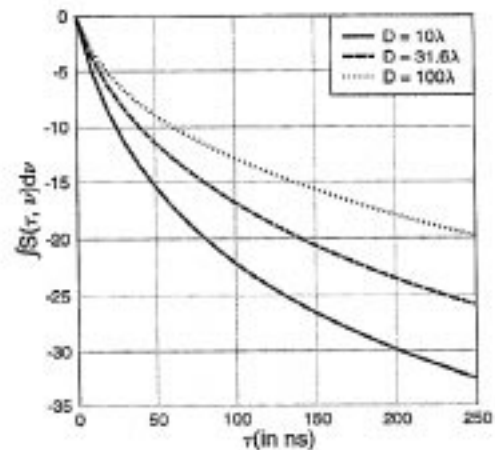


Figure 4. The Power Delay Profile as a function of the time-delay after the first arrival

### 3.3 Narrow BW Case, the Jakes Power Density Spectrum

Integrating  $S(\tau, \nu)$  (equation (21)) over all  $\tau$ , gives a result close to Jakes' PDS,  $dP(\nu)$ , for  $D \rightarrow 0$  when properly normalized. The figure below shows  $dP(\nu)$  for a mobile velocity of 100 km/h (Doppler of about 80 Hz). It shows good agreement with Figure 1.7-5 in [2].

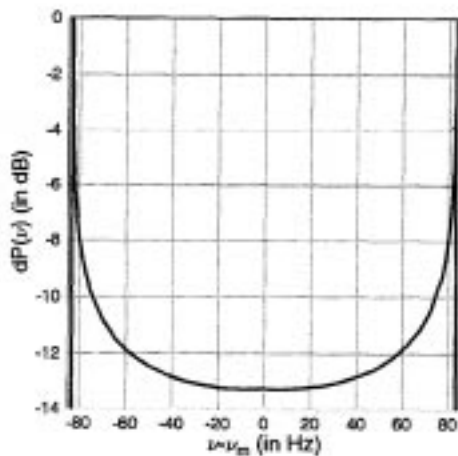


Figure 5.  $dP(\nu)$  for Velocity = 100 km/hr

### 4. Simulation of wideband fading channels

Modeling the temporal dynamics of channel impulse responses is important for accurately simulating the performance of mobile communication systems. For example, the North American TDMA cellular/PCS standards [4], specify the minimum required performance of receivers for a given set of vehicle velocities and Delay Spread profiles. The specified channels are simulated using the Jakes model. In order to realistically simulate the performance of these

communication systems in the field, it is critical to accurately model the mobile channel impulse response.

Figure 6 shows an example of the in-phase (I) component of a channel impulse response generated using  $S(\tau, \nu)$ , for a vehicle velocity of 100 km/h, and a transmitter-receiver separation of  $D = 100\lambda$ . The temporal evolution of the first five samples of the channel impulse response are shown.

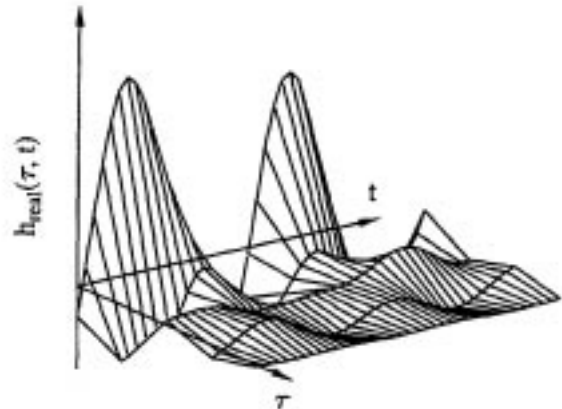


Figure 6: Evolution of the in-phase (I) component of the channel impulse response generated using  $S(\tau, \nu)$

### References

- [1] I. Bar-David and R. Krishnamoorthy, "Delay Spread Profiles and Receiver Performance in a Dense Multipath Environment," accepted for publication in *IEE Proceedings-Communications*
- [2] W. C. Jakes (Ed.), "Microwave Mobile Communications," New York, NY, IEEE Press, 1993
- [3] "HF Ionospheric Channel Simulators," *CCIR Report 549-3*, 1990
- [4] "TDMA Cellular/PCS - Radio Interface - Minimum Performance Standard for Mobile Stations," *PN3605 (IS137-A)*