

# OPTIMUM BEAMFORMING USING TRANSMIT ANTENNA ARRAYS

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**Abstract** - Transmit beamforming is a powerful means of increasing capacity in systems in which the transmitter is equipped with an antenna array, especially in systems in which receive beamforming is not available, as is typical in the base-to-mobile "downlink" in a cellular communication system. In this paper, the problem of transmit beamforming is formulated as one of minimizing the power radiated by the base station, subject to satisfying quality of service requirements at the mobiles. For a single cell system, a global minimum is shown to exist, and an iterative algorithm that converges to it is provided. The solution extends to accommodate receive beamforming at the mobiles. It also extends to provide a mechanism for space-time transmit filtering, which exploits the differences between the temporal as well as the spatial channels between the base station and different mobiles. Finally, a system with multiple source-destination pairs (e.g., in an ad hoc network) is considered. An iterative algorithm that outputs a convergent sequence of feasible points with monotonically decreasing costs is provided for this case.

## 1 Introduction

A fundamental limitation on the capacity of cellular and personal communication systems is the mutual interference among simultaneous users. Transmit and receive beamforming using antenna arrays can potentially circumvent this bottleneck by providing isolation among users at different locations. In typical cellular systems, it is reasonable to assume that the base station is equipped with an antenna array. However, the mobile units typically have only one antenna element. In such a setting, transmit beamforming from the base station to the mobiles provides a powerful method for increasing downlink (base-to-mobile) capacity. An implicit assumption is that the base station has feedback regarding the channel seen by each mobile, which in turn implies that these channels vary slowly. Thus, transmit beamforming is most effective in applications involving slowly moving mobile units. This paper provides a method for computing the optimum weights for transmit beamforming for the downlink in a single cell. The solution is then extended to several other situations: (a) optimum transmit and receive beamforming in a single cell; (b) opti-

imum space-time transmit filtering in a single cell, (c) transmit beamforming for multiple source-destination pairs (for which the algorithm outputs a sequence of feasible points with monotonically decreasing costs).

The basic problem posed is as follows: choose the beamforming weights so as to minimize the net power transmitted by the base station, subject to each mobile attaining a desired signal-to-interference ratio (SIR). The idea is to provide desired levels of isolations among mobiles in a cell, while minimizing the interference generated for other cells. This optimization problem was first considered, and solved in part, in [3]. The partial solution in [3] consists of an iterative algorithm that, assuming that there is a feasible solution, converges to a feasible (but not necessarily optimal) point. That is, it finds a set of beamforming weights that guarantee that the SIR constraint at each mobile is satisfied.

In this paper, we provide a complete solution to the problem addressed in [3]. In particular, it is shown that, for a transformed version of the original optimization problem, the algorithm in [3] converges to the globally optimum beamforming weights. Depending on the system parameters, the net transmitted power required with the optimum weights can be substantially smaller than that required with the feasible solution obtained in [3].

## 2 System Model

Consider a single cell. The base station has an  $M$ -element antenna array. Each mobile has a single antenna element. Let  $\mathbf{h}_i$ ,  $1 \leq i \leq K$ , denote the  $M \times 1$  channel from the base station to the mobile  $i$ , where  $K$  is the number of mobiles. The base station performs transmit beamforming and communicates simultaneously with all mobiles. The instantaneous transmitted signal can then be expressed as

$$\mathbf{x} = \sum_{i=1}^K b_i \mathbf{w}_i^* \quad (1)$$

where  $\mathbf{w}_i$  is an  $M$ -dimensional beamforming vector for the  $i$ -th user and  $b_i$  is the transmitted data symbol for the  $i$ -th user. At the  $i$ -th mobile, the instantaneous received signal can then be expressed as an inner product between the transmitted signal and the  $i$ -th channel plus noise:

$$r_i = \sum_{j=1}^K b_j \mathbf{w}_j^H \mathbf{h}_i + n_i \quad 1 \leq i \leq K \quad (2)$$

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where  $n_i$  is an additive white Gaussian noise of variance  $\sigma_i^2$ , which models other-cell interference and background noise present in the system. The term  $b_i \mathbf{w}_i^H \mathbf{h}_i$  represents the desired signal at the  $i$ -th mobile, while  $\sum_{j \neq i}^K b_j \mathbf{w}_j^H \mathbf{h}_i$  represents the multiple access interference. Assuming that the data symbols  $b_i$  are uncorrelated, the SIR at the  $i$ -th mobile,  $\gamma_i$ , can be expressed as follows

$$\gamma_i = \frac{|\mathbf{w}_i^H \mathbf{h}_i|^2}{\sum_{j \neq i}^K |\mathbf{w}_j^H \mathbf{h}_i|^2 + \sigma_i^2} \quad 1 \leq i \leq K \quad (3)$$

### 3 Optimum Transmit Beamforming

The objective is to minimize the total transmitted power by the base station, subject to quality of service requirements at the mobiles, expressed as a minimum SIR constraint at each mobile, i.e.,  $SIR_i \geq \gamma_i$ . Since the total transmitted power at the base station can be expressed as  $\sum_{i=1}^K \|\mathbf{w}_i\|^2$ , this yields the following constrained optimization problem

$$\min_{\mathbf{w}_1, \dots, \mathbf{w}_K} \sum_{i=1}^K \|\mathbf{w}_i\|^2 \quad (4)$$

subject to

$$\gamma_i \left( \sum_{j \neq i}^K |\mathbf{w}_j^H \mathbf{h}_i|^2 + \sigma_i^2 \right) - |\mathbf{w}_i^H \mathbf{h}_i|^2 \leq 0 \quad \forall i \quad (5)$$

the constraints have been rewritten in terms of quadratic functions of the weight vectors. The following important observations can now be made.

1. *Existence of Global Minimum:* The cost function in (4) is continuous and coercive (i.e., its value converges to plus infinity along any sequence of its arguments such that the norm of the arguments also converges to plus infinity; see [1], page 540, for a formal definition). Furthermore, the constraint set is closed. Thus, if the constraint set is nonempty (i.e., a feasible solution exists), then, by the Weierstrass' Theorem (page 540 of [1]), a global minimum exists.
2. *Active Constraints:* At the global minimum the constraints (5) are active (i.e., they are satisfied with equality). This can be shown by contradiction: if there is a global minimum with constraint  $i$  in (5) inactive, then  $\|\mathbf{w}_i\|^2$  can be reduced without violating the constraints. Thus, we can obtain a feasible point with the cost function smaller than the global minimum, which is a contradiction.
3. *Normalized Problem:* The optimization problem depends only on the  $K$  pairs of parameters  $(\mathbf{h}_i, \sigma_i)$ ,  $1 \leq i \leq K$ . Furthermore, an equivalent optimization problem is obtained, if we rewrite (4) in

terms of the *normalized* parameter pairs  $(\tilde{\mathbf{h}}_i, 1)$ ,  $1 \leq i \leq K$ , where  $\tilde{\mathbf{h}}_i = \mathbf{h}_i/\sigma_i$ . We will refer to this equivalent optimization problem as the normalized downlink optimization problem.

The Lagrangian for the normalized downlink optimization problem is given by

$$\sum_{i=1}^K \|\mathbf{w}_i\|^2 + \sum_{i=1}^K \lambda_i \gamma_i \left( \sum_{j \neq i}^K |\mathbf{w}_j^H \tilde{\mathbf{h}}_i|^2 + 1 \right) - \lambda_i |\mathbf{w}_i^H \tilde{\mathbf{h}}_i|^2 \quad (6)$$

where  $\lambda_i$ ,  $1 \leq i \leq K$ , are the corresponding Lagrange multipliers, which can be shown to be nonnegative (p. 283 of [1]). Differentiating with respect to the  $\mathbf{w}_i$ , we obtain the following  $K$  equations as the necessary conditions for optimality.

$$\mathbf{w}_i - \lambda_i \tilde{\mathbf{h}}_i \tilde{\mathbf{h}}_i^H \mathbf{w}_i + \sum_{j \neq i}^K \lambda_j \gamma_j \tilde{\mathbf{h}}_j \tilde{\mathbf{h}}_j^H \mathbf{w}_i = 0 \quad \forall i \quad (7)$$

In addition, as noted earlier, the optimal solution must satisfy the  $K$  feasibility constraints in (5) with equality, so that

$$\gamma_i \left( \sum_{j \neq i}^K |\mathbf{w}_j^H \tilde{\mathbf{h}}_i|^2 + 1 \right) = |\mathbf{w}_i^H \tilde{\mathbf{h}}_i|^2 \quad \forall i \quad (8)$$

The key step in the development is to show that the optimum beamforming weights for the normalized downlink problem are the same, up to scaling, as those corresponding to optimum joint power control and receive beamforming for a *virtual normalized uplink* problem. The latter can be solved by an iterative algorithm in [2], and this algorithm can be modified to solve for the weights and scaling factors for the downlink problem of interest. The preceding approach was already considered in [3], where it was shown that the use of a virtual uplink as above yields a *feasible* solution to the downlink problem, and provided an iterative algorithm for computing such a solution. The somewhat surprising result obtained in this paper is that, when the algorithm in [3] is applied to the *normalized* downlink problem, it actually yields the *optimum* solution. Depending on the system parameters, this may be much better than the feasible solution obtained in [3].

#### 3.1 Virtual Uplink Problem

For the uplink problem, the mobiles seek to minimize their transmitted powers to attain SIR constraints at the base station, taking into account the fact that the base station is equipped with a receive antenna array that can suppress interference, while maintaining unity gain in the desired direction. Thus, for  $1 \leq i \leq K$ , mobile  $i$  must determine its transmitted power  $P_i$ , and the base station must determine the beamforming vector  $\tilde{\mathbf{w}}_i$  used to receive the signal from mobile  $i$ . The problem can therefore be formulated as follows

$$\min_{\tilde{\mathbf{w}}_1, \dots, \tilde{\mathbf{w}}_K, P_1, \dots, P_K} \sum_{i=1}^K P_i \quad (9)$$

subject to

$$\frac{P_i}{\sum_{j \neq i}^K P_j |\tilde{\mathbf{w}}_i^H \tilde{\mathbf{h}}_j|^2 + \|\tilde{\mathbf{w}}_i\|^2} \geq \gamma_i \text{ and } \tilde{\mathbf{w}}_i^H \tilde{\mathbf{h}}_i = 1 \quad \forall i \quad (10)$$

where the channels  $\tilde{\mathbf{h}}_i$ , the unity noise variance, and the desired SIR values,  $\gamma_i$ , are all chosen to be identical to those for the “normalized” downlink problem. As in the downlink problem, it is easy to see that the optimum solution must satisfy the constraints (10) with equality. By considering the first order necessary conditions for optimality, it is easy to show that the global minimizer of (9) satisfies the following 2K equations

$$\tilde{\mathbf{w}}_i - \frac{P_i}{\gamma_i} \tilde{\mathbf{h}}_i \tilde{\mathbf{h}}_i^H \tilde{\mathbf{w}}_i + \sum_{j \neq i} P_j \tilde{\mathbf{h}}_j \tilde{\mathbf{h}}_j^H \tilde{\mathbf{w}}_i = 0 \quad \forall i \quad (11)$$

$$\tilde{\mathbf{w}}_i^H \tilde{\mathbf{h}}_i = 1 \quad \forall i \quad (12)$$

In [2], an iterative algorithm is proposed and is shown to converge, starting from any initial conditions, to the unique fixed point which is the global minimizer of (9). It is straightforward to show that (11) - (12) specify the unique fixed point of the algorithm, and, hence, for  $P_i$ ,  $1 \leq i \leq K$ , satisfying (11), we have  $\sum_{i=1}^K P_i = P_{uplink}^*$ , where  $P_{uplink}^*$  is the global minimum of (9). In [3], the algorithm of [2] is extended to obtain a feasible solution for the downlink problem by an appropriate scaling of the optimum virtual uplink beamforming vectors.

### 3.2 Optimization Algorithm

The algorithm for optimum transmit beamforming can be stated as follows.

1. Compute the normalized channels  $\tilde{\mathbf{h}}_i = \mathbf{h}_i / \sigma_i \quad \forall i$ .
2. Apply the algorithm in [3] to the “normalized” problem as follows  
Step 1: For  $1 \leq i \leq K$

$$\tilde{\mathbf{w}}_i^n = \arg \min_{\mathbf{w}_i} \sum_{j \neq i}^K P_j^n |\mathbf{w}_i^H \tilde{\mathbf{h}}_j|^2 + \|\mathbf{w}_i\|^2 \quad (13)$$

subject to  $\mathbf{w}_i^H \tilde{\mathbf{h}}_i = 1$ .

Step 2: For  $1 \leq i \leq K$

$$P_i^{n+1} = \gamma_i \sum_{j \neq i}^K P_j^n |(\tilde{\mathbf{w}}_i^n)^H \tilde{\mathbf{h}}_j|^2 + \gamma_i \|\tilde{\mathbf{w}}_i^n\|^2 \quad (14)$$

Step 3: For  $1 \leq i \leq K$

$$\tilde{P}_i^{n+1} = \gamma_i \sum_{j \neq i}^K \tilde{P}_j^n |(\tilde{\mathbf{w}}_j^n)^H \tilde{\mathbf{h}}_i|^2 + \gamma_i \quad (15)$$

**Proposition 1:** *The preceding algorithm converges to the global minimum of the downlink problem if there is a feasible solution.*

The proposition implies that the following quantities converge:  $P_i^n \rightarrow P_i^\infty$ ,  $\tilde{P}_i^n \rightarrow \tilde{P}_i^\infty$ ,  $\tilde{\mathbf{w}}_i^n \rightarrow \tilde{\mathbf{w}}_i^\infty \quad \forall i$ . The optimum transmit beamforming vectors are given by  $\mathbf{w}_i = \sqrt{\tilde{P}_i^\infty} \tilde{\mathbf{w}}_i^\infty$ ,  $1 \leq i \leq K$ .

### 3.3 Proof of Optimality

We first state and prove the following key lemma before proving Proposition 1.

**Lemma:** *Any beamforming vectors satisfying the 2K equations (7) - (8) achieve the global minimum of the optimization problem in (4) - (5).*

**Proof of the Lemma:** Let  $(\lambda_i, \mathbf{w}_i)$ ,  $1 \leq i \leq K$ , be any  $K$  beamforming vector and eigenvalue pairs satisfying (7) - (8). Some simple manipulations yield the following relationship between the cost and the corresponding Lagrange multipliers

$$\sum_{i=1}^K \lambda_i \gamma_i = \sum_{i=1}^K \|\mathbf{w}_i\|^2 \quad (16)$$

From (8), observe that  $\mathbf{w}_i^H \tilde{\mathbf{h}}_i > 0 \quad \forall i$ . Thus, a set of complex scalars  $\{\alpha_i, 1 \leq i \leq K\}$  can be found such that, setting  $\tilde{\mathbf{w}}_i = \alpha_i \mathbf{w}_i$  gives

$$\tilde{\mathbf{w}}_i - \lambda_i \tilde{\mathbf{h}}_i \tilde{\mathbf{h}}_i^H \tilde{\mathbf{w}}_i + \sum_{j \neq i} \lambda_j \gamma_j \tilde{\mathbf{h}}_j \tilde{\mathbf{h}}_j^H \tilde{\mathbf{w}}_i = 0 \quad \forall i \quad (17)$$

$$\tilde{\mathbf{w}}_i^H \tilde{\mathbf{h}}_i = 1 \quad \forall i \quad (18)$$

Identifying  $P_i = \lambda_i \gamma_i$  and comparing (17) - (18) with (11) - (12), it becomes clear that  $\lambda_i \gamma_i$  play the role of the uplink powers  $P_i$  satisfying the fixed point equation for the virtual uplink problem. This gives  $\sum_{i=1}^K \lambda_i \gamma_i = P_{uplink}^*$ , which, together with (16), implies that  $\sum_{i=1}^K \|\mathbf{w}_i\|^2 = P_{uplink}^*$ . This enables us to conclude that, for any feasible point for the downlink problem satisfying (7) - (8), the downlink cost function equals the minimum  $P_{uplink}^*$  of the uplink cost function. Since the global minimizer of the downlink problem necessarily satisfies (7) - (8), we conclude that

1.  $P_{uplink}^* = P_{downlink}^*$ , where  $P_{downlink}^*$  is the global minimum of the transmit beamforming optimization problem.
2. Any feasible point satisfying (7) - (8) is a global minimizer of the transmit beamforming optimization problem.

This establishes the Lemma. ■

**Proof of Proposition 1:** Let  $(P_i^\infty, \tilde{\mathbf{w}}_i^\infty, \tilde{P}_i^\infty)$ ,  $1 \leq i \leq K$  denote the limits of the iterative algorithm, which, by the results in [3], is guaranteed to converge if there is a feasible solution to the downlink problem. We will establish that  $\sqrt{\tilde{P}_i^\infty} \tilde{\mathbf{w}}_i^\infty$ ,  $1 \leq i \leq K$ , satisfies (7) - (8),

and hence, by the Lemma, is the global minimizer of the transmit beamforming problem.

The transmit beamforming set  $\sqrt{\tilde{P}_i^\infty} \tilde{\mathbf{w}}_i^\infty$ ,  $1 \leq i \leq K$ , satisfies (8) by the result in [3], where it is shown that  $\sqrt{\tilde{P}_i^\infty} \tilde{\mathbf{w}}_i^\infty$ ,  $1 \leq i \leq K$ , is a feasible point of (4) satisfying the constraints with equality.

We now establish that (7) is satisfied. As shown in [2],  $(P_i^\infty, \hat{\mathbf{w}}_i^\infty)$ ,  $1 \leq i \leq K$ , is the global minimizer of (9) and  $(P_i^\infty, \hat{\mathbf{w}}_i^\infty)$ ,  $1 \leq i \leq K$ , satisfies (11). Again comparing (7) and (11), we observe that, by making the substitution  $P_i^\infty/\gamma_i = \lambda_i$ , (7) can be recovered from (11). Thus, we conclude that, with  $P_i^\infty/\gamma_i$  playing the role of the Lagrange multiplier  $\lambda_i$  in (7), for an arbitrary power scaling  $A_i$ ,  $\mathbf{w}_i = A_i \hat{\mathbf{w}}_i^\infty$ ,  $1 \leq i \leq K$ , satisfies (7). In particular,  $\sqrt{\tilde{P}_i^\infty} \tilde{\mathbf{w}}_i^\infty$ ,  $1 \leq i \leq K$ , satisfies (7). ■

## 4 Joint Transmit and Receive Beamforming

It is shown that the solution for single-cell transmit beamforming given in the preceding section extends immediately to a situation where the mobiles may also be equipped with antenna arrays. However, several local minima may exist for beamforming performed by multiple source-destination pairs, as might be encountered in ad hoc topologies. In this case, convergence of the algorithm to the global minimum cannot be guaranteed.

### 4.1 A Single Cell

Assume that the base station is equipped with an  $M$ -element antenna array, while each mobile is equipped with an  $N$ -element antenna array. While the proposed solution applies equally well to the downlink and to the uplink communication, consider downlink communication for concreteness. The channel from the base station to the  $i$ -th mobile is described by an  $N \times M$  matrix denoted as  $\mathbf{H}_i$ . Assume that the array response at a receive antenna from each transmit element at the base station is the same (except for complex scaling). This assumption is true if the scattering environment surrounding the receive antenna is not excessively rich. In this case, the channel matrix  $\mathbf{H}_i$  can be assumed to be rank one, and can be decomposed into an outer product as follows:

$$\mathbf{H}_i = \mathbf{h}_i \tilde{\mathbf{h}}_i^T \quad 1 \leq i \leq K \quad (19)$$

where  $\mathbf{h}_i$  is an  $N \times 1$  vector denoting the array response at the  $i$ -th mobile to the base station and  $\tilde{\mathbf{h}}_i$  is an  $M \times 1$  vector denoting the array response at the base station to the  $i$ -th mobile. Let  $\{\mathbf{w}_i, 1 \leq i \leq K\}$  denote the transmit beamforming weights used at the base station and  $\{\mathbf{v}_i, 1 \leq i \leq K\}$  denote the receive beamforming weights used at the mobiles. Upon performing receive beamforming, the  $i$ -th mobile observes the following sig-

nal

$$r_i = \sum_{j=1}^K b_j \mathbf{v}_i^H \mathbf{h}_i \tilde{\mathbf{h}}_j^T \mathbf{w}_j^* + \mathbf{v}_i^H \mathbf{n}_i \quad \forall i \quad (20)$$

where  $\mathbf{n}_i$  is an  $N \times 1$  vector containing uncorrelated samples of additive white Gaussian noise of variance  $\sigma_i^2$  present at each antenna element. Assuming that the data symbols  $b_i$  are uncorrelated, the SIR at the  $i$ -th mobile,  $\gamma_i$ , can be expressed as follows

$$\gamma_i = \frac{|\mathbf{v}_i^H \mathbf{h}_i \tilde{\mathbf{h}}_i^T \mathbf{w}_i^*|^2}{\sum_{j \neq i}^K |\mathbf{v}_i^H \mathbf{h}_i \tilde{\mathbf{h}}_j^T \mathbf{w}_j^*|^2 + \sigma_i^2 \|\mathbf{v}_i\|^2} \quad \forall i \quad (21)$$

The joint transmit and receive optimization problem is formulated as minimizing the total transmitted power by the base station subject to SIR constraints at each of the mobiles. To avoid nulling out the desired user, the receive beamformer is required to have unity gain in the desired direction, i.e.,  $\mathbf{v}_i^H \mathbf{h}_i = 1 \quad \forall i$ .

Since the signal destined for any mobile originates from a single source, the base station, it is easy to see that the optimum receive beamformers are simply spatial matched filters, given by  $\mathbf{v}_i = \mathbf{h}_i / \|\mathbf{h}_i\|^2 \quad \forall i$ . With this choice of receive beamforming vectors, the effective channels from the base station to the mobiles and the noise variances at the mobiles become respectively  $\mathbf{v}_i^H \mathbf{h}_i \tilde{\mathbf{h}}_i = \tilde{\mathbf{h}}_i \quad \forall i$ ,  $\sigma_i^2 \|\mathbf{v}_i\|^2 = \sigma_i^2 / \|\mathbf{h}_i\|^2 \quad \forall i$ . The optimal transmit beamforming vectors can now be obtained by applying (13) - (15) to the transmit beamforming with parameters  $(\tilde{\mathbf{h}}_i, \sigma_i^2 / \|\mathbf{h}_i\|^2)$ ,  $1 \leq i \leq K$ .

### 4.2 Multiple Source-Destination Pairs

Assume that the system consists of  $K$  source-destination pairs which interfere with each other. This scenario may apply either to an ad hoc network, or to a multiple cell system in a cellular network. Denote by  $\mathbf{H}_{ji}$  the  $N \times M$  channel from the source of the  $j$ -th pair to the destination of the  $i$ -th pair, where, as above,  $\mathbf{H}_{ji}$  can be decomposed as an outer product of two column vectors. Unlike in the case of a single source (or base station), the spatial signatures of interference and of the desired transmission do not lie in the same complex subspace. Hence, the matched filtering solution for the receive beamforming vectors is suboptimum. The problem has a large number of fixed points, each corresponding to a different way of suppressing interference. Roughly speaking, each receiver has the choice of nulling out one or more of the signals from interfering transmitters, while each transmitter has the choice of nulling out one or more of the receivers. The following algorithm, which iterates between (locally) optimal transmit and receive beamforming, can be shown to converge to one such fixed point with the convergent sequence having monotonically decreasing costs.

Step 1: Perform receive beamforming for  $1 \leq i \leq K$

$$\mathbf{v}_i^{n+1} = \arg \min_{\mathbf{v}_i} \sum_{j \neq i}^K |\mathbf{v}_i^H \mathbf{h}_i \tilde{\mathbf{h}}_j^T (\mathbf{w}_j^n)^*|^2 + \sigma_i^2 \|\mathbf{v}_i\|^2 \quad (22)$$

subject to  $\mathbf{v}_i^H \mathbf{h}_{ii} = 1$ .

Step 2: Perform transmit beamforming for  $1 \leq i \leq K$

$$\{\mathbf{w}_i^{n+1}, 1 \leq i \leq K\} = \arg \min_{\mathbf{w}_1, \dots, \mathbf{w}_K} \sum_{i=1}^K \|\mathbf{w}_i\|^2 \quad (23)$$

subject to

$$\gamma_i \left( \sum_{j \neq i}^K |(\mathbf{v}_i^{n+1})^H \mathbf{h}_{ji} \tilde{\mathbf{h}}_{ji}^T \mathbf{w}_j^*|^2 + \sigma_i^2 \|\mathbf{v}_i^{n+1}\|^2 \right) - |\tilde{\mathbf{h}}_{ii}^T \mathbf{w}_i^*|^2 \leq 0 \quad \forall i \quad (24)$$

Note that the preceding algorithm, if applied to the problem of joint transmit and receive beamforming in a single cell, converges to the global minimum in one iteration. In general, it can be shown that the algorithm produces a convergent sequence of transmit beamforming weights which are feasible and have non-increasing costs. Convergence to the global minimum is guaranteed if, and only if, there exists a unique fixed point of the algorithm. However, multiple fixed points were observed even for the case of two transmit-receive pairs in our experiments.

### 4.3 Optimum Space-Time Transmit Filtering

For temporally dispersive channels, temporal filtering in addition to beamforming may be used to obtain interference suppression. For a single cell, the optimum transmit beamforming solution provided in Section 3 can be generalized to allow transmit filtering in both space and time. Due to space limitations, attention is restricted to a simple example to illustrate the ideas involved. Consider a single base station equipped with an  $M$ -dimensional antenna array, and two mobiles, each equipped with a single antenna element. Assume that the channel to each mobile consists of two multipath rays with a delay spread of one time unit. Each space-time channel can be described in the  $z$ -transform domain as follows  $\mathbf{H}_i(z) = \mathbf{a}_1^0 + \mathbf{a}_1^1 z^{-1}$ ,  $i = 1, 2$ , where  $\mathbf{a}_1^j$  is the  $M$ -dimensional spatial channel to the  $i$ -th mobile along the  $j$ -th multipath. The transmitter performs space-time filtering of the outgoing bit sequence. For simplicity of illustration, restricting the temporal span of the beamforming filters to two, the transmit beamforming filters can be expressed as follows:  $\mathbf{W}_i(z) = \mathbf{v}_i^0 + \mathbf{v}_i^1 z^{-1}$ ,  $i = 1, 2$ , where  $\mathbf{v}_i^j$  is the  $j$ -th  $M$ -dimensional temporal component of the  $i$ -th filter. The  $z$ -transform of the received vector at the  $i$ -th mobile is as follows  $R_i(z) = \mathbf{H}_i(z) (B_1(z) \mathbf{W}_1^H(z) + B_2(z) \mathbf{W}_2^H(z)) + N_i(z)$ , where  $B_i(z)$  is the  $z$ -transform of  $i$ -th user's bit stream. The following expression can now be obtained for the received sample at the first mobile at  $t = 1$ :

$$\begin{aligned} r_1[1] = & [\mathbf{a}_1^1; \mathbf{a}_1^0]^H (b_1[0][\mathbf{v}_1^1; \mathbf{0}] + b_1[1][\mathbf{v}_1^0; \mathbf{v}_1^1] + \\ & b_1[2][\mathbf{0}; \mathbf{v}_1^0] + b_2[0][\mathbf{v}_2^1; \mathbf{0}] + \\ & b_2[1][\mathbf{v}_2^0; \mathbf{v}_2^1] + b_2[2][\mathbf{0}; \mathbf{v}_2^0]) + n_1[1] \end{aligned} \quad (25)$$

where  $[\mathbf{a}_1^1; \mathbf{a}_1^0]$  denotes a column vector with  $\mathbf{a}_1^1$  stacked on top of  $\mathbf{a}_1^0$ . This expression can be rewritten as a

correlation between  $[\mathbf{v}_1^0; \mathbf{v}_1^1]$  and  $[\mathbf{v}_2^0; \mathbf{v}_2^1]$ , which play the role of effective transmit beamforming vectors, and the acyclic shifts of  $[\mathbf{a}_1^1; \mathbf{a}_1^0]$ , which play the role of effective channels. Assuming, without loss of generality, that the sample  $r_1[1]$  is used as a decision statistic for the bit  $b_1[1]$ , we obtain SIR expressions identical in form to (3). The optimization problem of minimizing the sum of squared Euclidean norms of  $[\mathbf{v}_i^0; \mathbf{v}_i^1]$ ,  $i = 1, 2$ , subject to the SIR constraints can now be solved by applying (13) - (15).

## 5 Numerical Results

In this section, we present numerical results obtained for a system with a five-element linear antenna array and six mobile users, distributed at distinct angles and distances throughout the cell. In a cellular system, the noise variance at a given mobile is mainly due to interference from adjacent base stations (not explicitly modeled), and can therefore depend on the location of the mobile within its cell. To illustrate such location effects, we employ the following model. Let mobile  $i$  be at an angle  $\theta_i$  and distance  $d_{i,1}$  from the desired base station and, distance  $d_{i,2}$  from an interfering base station. Assume that the strength of the channel and noise variance at the mobiles are determined by the path loss from the desired and interfering base stations, respectively. Assuming  $1/d^4$  law for dependence of path loss on distance and  $\lambda/4$  antenna spacing, we have:  $\mathbf{h}_i(k) = \frac{1}{\sqrt{5}d_{i,1}^2} \exp(j\frac{\pi}{2}(k-1)\sin\theta_i)$ ,  $\sigma_i^2 = 1/d_{i,2}^4$ ,  $1 \leq k \leq 5$ ,  $1 \leq i \leq 6$ , where  $\mathbf{h}_i(k)$  is the  $k$ -th component of the  $i$ -th channel vector and  $\lambda$  is the wavelength. To generate Figure 1, it was assumed that  $d_{i,1} + d_{i,2} = 1 \forall i$  and the following numerical values were used  $d_{1,1} = 0.2$ ,  $d_{2,1} = 0.4$ ,  $d_{3,1} = 0.3$ ,  $d_{4,1} = 0.25$ ,  $d_{5,1} = 0.45$ ,  $d_{6,1} = 0.45$ . We now apply the algorithm in (13)-(15) to the transmit beamforming problem with these parameters.

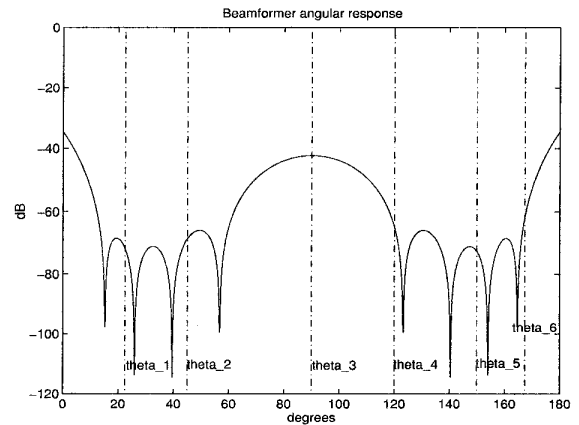


Figure 1: Plot of  $W_3(\theta)$  vs  $\theta$ .

Figure 1 displays, as a function of  $\theta$ , the magnitude squared of the normalized correlation between the 3-rd beamforming vector and the linear array response,

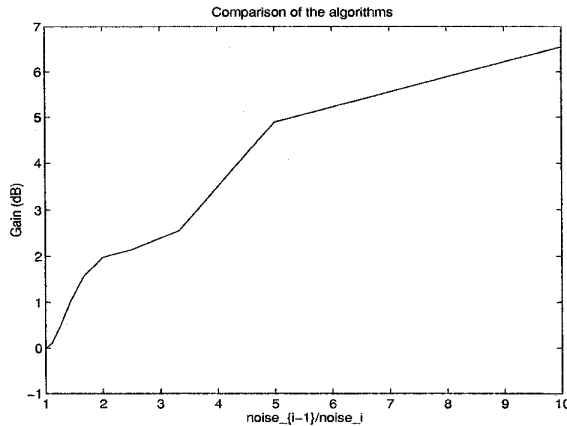


Figure 2: Plot of  $P_{feas}/P_{min}$  vs  $\sigma_{i-1}^2/\sigma_i^2$ .

which we term the beamformer angular response and denote by  $W_i(\theta)$  for the  $i$ -th mobile. Then

$$W_i(\theta) = \frac{|(\hat{\mathbf{w}}_i^\infty)^H \mathbf{h}(\theta)|^2}{\|\hat{\mathbf{w}}_i^\infty\|^2 \|\mathbf{h}(\theta)\|^2} \quad (26)$$

where  $\mathbf{h}(\theta)(k) = \frac{1}{\sqrt{5}} \exp(j\frac{\pi}{2}(k-1)\sin\theta)$ ,  $1 \leq k \leq 5$ . The angles of arrival for the six users are shown using dashed vertical lines. From Figure 1, note that the beamformer places a lobe in the angular direction corresponding to the desired mobile and suppresses other users. Note that the lobe placed in the direction of the desired mobile is wide and is robust, to a certain degree, against channel estimation errors.

Figure 2 displays a comparison between the global minimum of the transmit optimization problem,  $P_{min}$ , and a feasible point,  $P_{feas}$ , obtained by applying the original iterative algorithm in [3] (without normalization). The parameters are slightly different from those in Figure 1. In particular, the noise variances are assumed to form a decreasing sequence, with  $\frac{\sigma_{i-1}^2}{\sigma_i^2}$  fixed at some number larger than one, for  $i = 2, \dots, 6$ , and  $\sigma_1^2 = 1/d_{1,2}^2$ . All other parameters are unchanged. Figure 2 displays  $P_{feas}/P_{min}$  as a function of  $\sigma_{i-1}^2/\sigma_i^2$ ,  $2 \leq i \leq 6$ . Note that, if  $\sigma_{i-1}^2/\sigma_i^2 = 1$ ,  $2 \leq i \leq 6$ , i.e. the noise variances are the same at all mobiles, no normalization is necessary, and the performance of the two algorithms is identical. As the noise variances at the mobiles become more disparate (as would be typical for mobiles distributed uniformly within a cell), observe that  $P_{feas}/P_{min}$  increases significantly, which implies that significant savings for total transmitted power are obtained by the simple modification of the algorithm in [3] proposed here.

Numerical results for the extensions in Sections 4 and 5 will be presented at the conference.

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