Scalable Layered Space-Time Codes for Wireless Communications: Performance Analysis and Design Criteria

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Abstract — Dual antenna-array systems provide very high capacity compared to single antenna systems in a Rayleigh fading environment. If the transmitter does not have the channel state information, to utilize this high capacity, space-time codes must be employed. The diagonally-layered space-time (DLST) architecture is a structure that is capable of providing high data rate for a low decoding complexity. We analyze the performance of DLST codes with a hard decision-feedback decoder and a soft decision-feedback decoder with iterative decoding. We analyze the error probability performance and propose the criteria for designing the constituent codes.

I. INTRODUCTION

Recent studies have explored the ultimate limit of dual antenna-array systems from the information-theoretic point of view [1]-[3]. Consider a dual antenna-array system that has $n$ transmitting and $m$ receiving antennas. It has been shown that, if the narrowband slow fading channel can be modeled as an $n \times m$ matrix with i.i.d. complex Gaussian random entries, the average channel capacity of such a system is approximately $\min(n, m)$-times higher than that of a single antenna system for the same overall transmitting power.

In most applications, a major obstacle to utilizing this high throughput is that the transmitter cannot have the instantaneous information about the fading channel. The transmitter thus must employ a channel code that can guarantee good performance with the majority of possible channel realizations. Such a channel code is inherently multi-dimensional and thus is called a space-time code [4]-[5].

Aside from the consideration of combating channel uncertainty, another practical consideration for space-time codes is the decoding complexity. As stated above, the channel capacity of a dual antenna-array system is approximately proportional to $\min(n, m)$. This means that each channel usage on average can convey proportional to $\min(n, m)$ bits of information. The complexity of decoding such a high data-rate channel code using the maximal-likelihood (ML) criterion can be prohibitively high even if $\min(n, m)$ is just moderately large; thus, space-time codes that admit high performance, low complexity suboptimal decoding algorithms are desirable.

The layered space-time (LST) architecture proposed by Foschini in [6] is a framework of processing space-time signals. An LST code is a channel code that is designed and processed according to the LST architecture. An LST code is constructed by assembling 1-D constituent codes. At the receiver, these constituent codes can be separated and then decoded using conventional decoding algorithm developed for 1-D constituent codes, leading to a much lower decoding complexity compared to ML decoding.

In this paper, we consider two decoder structures for DLST codes. We first present the hard decision-feedback decoder. We discuss the actual decoding mechanism, analyze the error performance, and propose the design criteria based on the truncated multidimensional effective code length (TMEL) and the truncated multidimensional product distance (TMDP) of the constituent code. We then present the soft decision-feedback iterative decoder. We will present an interleaving mechanism that achieves a high performance. We analyze the error probability, and propose a design criterion based on the folded effective length of the constituent code.

The remainder of this paper is organized as follows. In Section II, the background of dual antenna-array systems and space-time codes is reviewed. In Section III, we introduce the basic LST architecture. The hard decision-feedback decoder is discussed. In Section IV, we present the soft decision-feedback iterative decoder. We give the summary in Section V.

II. BACKGROUND

In this paper, we focus on single-user to single-user communication using antenna arrays at both ends over narrowband flat-fading channels. We refer to a dual antenna-array system in which the transmitter has $n$ transmitting antennas and the receiver has $m$ receiving antennas as an $(n, m)$ system.

A general space-time code can be described as follows. The encoder first applies the space-time code to the input information bits to generate an $n$-row (possibly semi-infinite) matrix $C$. The matrix $C$ represents the signal that is to be transmitted by the transmitter. Specifically, the $k$th row, $t$th column element of $C$, denoted by $c_{kt}$, represents the signal to be transmitted by antenna $k$ at time slot $t$. We emphasize that there is no mechanism, such as time-, frequency-, or code-division multiplexing, employed to ensure that the signals transmitted by different transmitting antennas are orthogonal upon reception by the receiver.

For a narrowband flat-fading channel, the gain connecting transmitting antenna $k$ and receiving antenna $l$ at time $t$ can be denoted by a complex number $h_{lt}^{k}$. Let the signal received by the receiving antenna $l$ during the time slot $t$ is denoted by $r_{lt}^{k}$. This received signal $r_{lt}^{k}$ contains a superposition of transmitted signals $c_{kt}^{k}$, $k = 1, 2, ..., n$, and an AWGN component $v_{lt}$. Define the vectors $e_{t} = (c_{t1}^{1} c_{t2}^{2} ... c_{tn}^{n})^{T}$.  

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\[ r_t = (r_1^1, r_2^1, \ldots, r_n^1), \quad \nu_t = (\nu_1^1, \nu_2^1, \ldots, \nu_n^1), \] and the channel matrix \( H_{m \times n} \) for \( H \), \( H^{-1} = H^{\dagger -k} \). The discrete-time input-output relation of an \((n, m)\) multiple-antenna system over a narrowband flat-fading channel can be written in the following vector notation:

\[ r_t = H_{m \times n} c_t + \nu_t. \] (1)

We assume that the receiver has a perfect knowledge about the channel \( H \), while the transmitter has no knowledge about it.

The following terminology is used in this paper. The matrix \( C \), a coded matrix output of the transmitter encoder, is referred to as a space-time codeword matrix. An element of a space-time codeword matrix is referred to as a symbol.

III. THE LAYERED SPACE-TIME ARCHITECTURE WITH ONE-STAGE DECODING

Here we introduce the layered space-time architecture originally proposed by Foschini in [6]. In the next section, we will propose a new space-time structure which is a modification of this basic architecture. In both cases, the most important concept is the diagonal layering, or “cycling”, of the output symbols of the constituent codes (CCs).

A. Encoding

The encoding process is illustrated in Fig. 1. The input information bit sequence is first demultiplexed into \( n \) subsequences, and each subsequence is encoded by a 1-D encoder. These 1-D channel codes are referred to as the constituent codes. The output of the constituent coder \( k \) is a sequence of symbols. For example, in Fig. 1(a), three CCs \((\alpha, \beta, \gamma)\) are employed.

The next step is to form the multi-spatial-dimensional codeword matrix \( C \) by assigning these coded symbols to the slots of \( C \). In the diagonally layered space-time (DLST) architecture, the output symbols from a CC are assigned to the \( n \) rows of \( C \) in turn. In other words, the output symbols from a CC are transmitted by transmitting antennas \( 1, 2, \ldots, n \) in turn. This is illustrated in Fig. 1(b). In Fig. 1(b), we note that the codeword matrix consists of diagonal layers. The first \( n \) output symbols from constituent coder 1 are used to fill the leftmost NW-SE diagonal of \( C \), and the first \( n \) output symbols from constituent coder 2 are used to fill the next diagonal of \( C \), and so on.

If the data rate of the constituent code maintains constant regardless of \( n \), the data rate of a DLST code is obviously proportional to \( n \).

B. Hard Decision-Feedback Decoder

The corresponding decoder for the DLST code mentioned above is a diagonal-by-diagonal, hard decision-feedback decoder.

Consider a given instance in time, say \( \tau \). The transmitted \( n \)-tuple is \( c_\tau \), and the received \( m \)-tuple is \( r_\tau = H_{m \times n} c_\tau + \nu_\tau \). The received signal \( r_\tau \) is a superposition of transmitted coded symbols, i.e. \( c_\tau^1, c_\tau^2, \ldots, c_\tau^n \), scaled by the channel gain and corrupted by AWGN. An estimate of one of the transmitted symbols, say \( c_\tau^0 \), can be obtained using a linear combination of the components of \( r_\tau \). That is, the estimate of \( c_\tau^0 \) is \( \hat{c}_\tau^0 = w^\dagger r_\tau \), where \( w^\dagger \) is an \( m \)-dimensional row vector. We will refer to such an estimate as a decision variable.

A DLST codeword matrix is decoded diagonal by diagonal. To illustrate this, consider the example DLST codeword matrix in Fig. 1. The equalizer first generates the decision variables for the symbols belonging to the first diagonal of \( C \) using a linear operation. That is, it generates \( \hat{c}_0, \hat{c}_1, \) and \( \hat{c}_2 \). Based on these decision variables, the CC decoder decodes this diagonal. Once the hard decisions \( \hat{c}_0, \hat{c}_1, \) and \( \hat{c}_2 \) are available, they are fed back to the equalizer to remove the contributions of \( \alpha_0, \alpha_1, \) and \( \alpha_2 \) in \( r_0, r_1, \) and \( r_2 \). The receiver then repeats the process of decision variable generation, hard decision, and decision feedback to decode the next diagonal (\( \beta_1, \beta_2, \) and \( \beta_3 \)), and so on. The receiver diagram is shown in Fig. 2.

The complexity of the spatial processing described above is \( O(n^2 + mn) \) per transmitted \( n \)-tuple. Compared with ML decoding, the complexity of the hard decision-feedback decoder scales much more moderately in \( n \).

C. Analysis

We have mentioned that a decision variable is derived using a linear combination of the components of the interference-
adjusted received signal \( r_t \). That is,

\[ \hat{c}_t^k = w^1(r_t - \sum_{t = k + 1}^t h_t \hat{c}_t^k) \]  

(2)

for some \( w^1 \).

Zero-forcing (ZF) and minimum mean square error (MMSE) are two commonly used criteria for the selection of the linear combination coefficients \( w^1 \). In general, the MMSE criterion leads to easier implementation and higher performance. On the other hand, the ZF criterion is easier to analyze. Furthermore, at high SNR, these two criteria are asymptotically equivalent. In the following, we will examine both interference suppression criteria.

Note that due to the decoding algorithm specified above, when the receiver attempts to extract the decision variable for a symbol, say \( c_t^k \), the hard decisions on the symbols \( c_t^{k-1}, \ldots, c_t^0 \) are already available.

For the ZF criterion, let the QR decomposition of \( H_t \) be \( H_t = (U_R)R_t \), where \( U_R \) is a unitary matrix and \( R_t \) is an upper triangular matrix. The \( k \)th row of the matrix \( U_R \) is the linear combination coefficients used to generate \( \hat{z}_t^k \). To see this, left-multiply the received signal \( r_t \) by \( (U_R)^t \),

\[ x_t^k = (U_R)^t r_t = R_t c_t^k + v_t, \]  

(3)

where \( v_t = (U_R)^t v \) is an \( m \)-tuple of i.i.d. AWGN noise component. Because \( R_t \) is upper triangular,

\[ y_t^k = (R_t^t) c_t^k + v_t^k \]  

(4)

The interference term in (4) can be canceled using the available decisions \( \hat{c}_t^{k+1}, \hat{c}_t^{k+2}, \ldots, \hat{c}_t^m \) to obtain \( \hat{z}_t^k \). We can verify that in (4) the interferences from \( c_t^l, l < k \), are indeed completely suppressed. Assuming that these decisions are all correct, the decision variable \( \hat{z}_t^k \) is

\[ \hat{z}_t^k = (R_t^t) c_t^k + v_t^k, \quad k = 1, 2, \ldots, n. \]  

(5)

For the MMSE criterion, define \( \Psi_k = [h_1 \ldots h_k] [h_1 \ldots h_k]^t \) and \( A_k = \text{diag}(\rho_0, \ldots, \rho_k) \), where \( h_i \) is the \( i \)th column of \( H \) and \( \rho_i \) is the variance of \( c_t^i \). The MMSE linear combination coefficients for generating \( \hat{z}_t^k \) is the \( k \)th row of the matrix \( (\Psi_k + \sigma^2 A_k^{-1})^{-1} [h_1 \ldots h_k]^t \) [7].

Here we analyze the probability of a diagonal decision error. Consider the leftmost NW-SE diagonal of a DLST codeword matrix. On this diagonal, the transmitted symbols are \( c_t^1, \tau = 1, 2, \ldots, n \). The probability that, under the DLST decoding algorithm, the likelihood of a distinct diagonal \( e = \{ e_1 e_2 \ldots e_n \} \) is higher than that of the transmitted diagonal \( e = \{ c_1^1 c_2^2 \ldots c_n^n \} \), conditioned on the channel realization \( \hat{H}_t = \{ H_1, H_2, \ldots \} \), can be upper-bounded by [8]

\[ \text{Prob}(c \rightarrow e) \leq \prod_{\tau \in \eta(c, e)} \left( 1 + \frac{|c_t^\tau - e_t^\tau|^2}{E/4N_0 t} \right)^{(m - \tau)} \]  

(6a)

where \( \eta(c, e) = \{ \tau | c_t^\tau \neq e_t^\tau \} \) when the SNR is high and by

\[ \text{Prob}(c \rightarrow e) \leq \prod_{\tau \in \eta(c, e)} \left( 1 + \frac{E}{4N_0 t} \right)^{-1} \left( |c_t^\tau - e_t^\tau|^2 (m - \tau) \right) \]  

(6b)

when SNR is low. Equations (6a) and (6b) apply in both fast and slow fading environments.

D. Design Criteria for DLST Codes

Define the truncated multi-dimensional effective length (TMEL) and the truncated multi-dimensional product distance (TMDP) between two distinct diagonals \( c \) and \( e \) as

\[ \text{TMEL} = \sum_{\tau \in \eta(c, e)} m - \tau + 1 \]  

(7a)

\[ \text{TMDP} = \prod_{\tau \in \eta(c, e)} |c_t^\tau - e_t^\tau|^2 (m - \tau + 1). \]  

(7b)

At high SNR, the pairwise error probability between \( c \) and \( e \) is approximated by \( \text{Prob}(c \rightarrow e) = (\text{TMDP})^{-1} (E/4N_0)_{\text{TMEL}} \). The code design criterion is to select the CC with the largest minimum value of \( \text{Prob}(c \rightarrow e) = (\text{TMDP})^{-1} (E/4N_0)_{\text{TMEL}} \) over all pairs of distinct diagonals. If the exact operating SNR is not known but can be assumed to be reasonably high, an approximate design criterion is to maximize the minimum two-tuple (TMEL, TMDP) in dictionary order.

At low SNR, the pairwise error probability is approximated...
by (6b). We define the exponent \( \sum_{c_k \in \mathbb{C}_k} |c_k^t - e_k^t|^2 (m - t + 1) \) to be the truncated multidimensional Euclidean distance (TMED) between \( c \) and \( e \). The code design criterion at low SNR is to maximize the minimum TMED between any pair of distinct diagonals.

IV. LAYERED SPACE-TIME CODES WITH ITERATIVE DECODING

Recently, the application of iterative decoding for joint equalization and decoding has attracted a lot of attention; e.g., [9]. This concept applies naturally to the processing of space-time signal processing when the transmitter does not have CSI. In the following, we will present the modified layered space-time structure that accommodates iterative decoding. We will also show how this new architecture evolves from the basic LST architecture with the (one-stage) hard decision-feedback decoder.

A. Soft Decision-Feedback Decoder

A soft decision-feedback decoder is obtained by using soft decisions instead of hard decisions in the feedback path in Fig. 2. Specifically, a CC decoder determines the sequence of marginal a posteriori probability mass functions (pmfs) of the decoded symbols. Here, the convenient expression \( p(c_k^t) \) is used to denote the a posteriori pmf of a decoded symbol \( c_k^t \). With soft decision-feedback, ideally the CC decoder sends this pmf to the equalizer.

Given the a posteriori pmfs of \( c_k^{t+1}, \ldots, c_k^T \), the decision variable for \( c_k^t \) has the following pmf:

\[
\text{Prob}\left( \hat{c}_k^t = w_k^t \left( r_e - \sum_{l = k+1}^{n} h_l c_l^t \right) \right) = \prod_{l=k+1}^{n} p(c_l^t) \tag{8}
\]

The product term is due to the fact that, under the layered space-time architecture, the symbols transmitted at time \( \tau \) from different antennas are independent of each other. The equalizer can compute certain useful statistical quantities about \( c_k^t \) from the pmf in (8) and sends them to the CC decoders. Unfortunately, the complexity of computing the pmf in (8) is exponential in \( n - k \). Therefore, algorithms based on equation (8) do not scale up well with \( n \).

For a scalable method of computing the decision variable, one possible choice is to obtain \( \hat{c}_k^t \) using

\[
\hat{c}_k^t = w_k^t \left( r_e - \sum_{l = k+1}^{n} h_l c_l^t \right)
\]

for some \( w_k \) and some soft-decisions \( \tilde{c}_k^{t+1}, \ldots, \tilde{c}_k^T \). If the goal is to minimize the mean square error contained in \( \hat{c}_k^t \), it can be easily shown that the soft decision \( \tilde{c}_k^t \) should be chosen as the expected value of \( \tilde{c}_k^t \), i.e., \( \hat{c}_k^t = E(c_k^t) \). For the coefficients \( w_k \) that minimizes the mean square error, denote the variance of the soft decoded symbol \( c_k^t \) as \( \sigma_k^2 \). Then

\[
w_k = \text{the } k\text{th row of } (H^TH + \sigma_k^2 A_k^{-1})^{-1} H^T \tag{9}
\]

where \( A_k = \text{diag}(\rho_0, \ldots, \rho_k, \sigma_2^2, \ldots, \sigma_n^2) \). According to this algorithm, the feedback information contains a posteriori expectation and variance.

By replacing hard decisions with soft decisions, one can often expect that the effect of error propagation is reduced. Our preliminary result shows that this is indeed the case. The reason is that, if the soft decision on some symbol, say \( c_k^t \), is deemed less reliable by the CC decoder, the a posteriori expectation of \( c_k^t \) will be smaller while its a posteriori variance will be higher. When generating the decision variables for \( c_k^t, t < k \), the MMSE decision-feedback equalizer will automatically place a higher emphasis on suppressing the interference from \( c_k^t \) using an appropriate linear projection and a lower emphasis using a direct cancellation.

B. Iterative Decoder

The input/output relation (1) can be interpreted that, after applying the "outer" space-time code on the information bits, the resulting space-time codeword is further encoded by a memoryless "inner" code \( H \). The output of the inner code then is corrupted by AWGN before being received by the receiver antenna array. From this viewpoint, the space-time code and the transform \( H \) are serially concatenated codes. At the receiver, the received signal is passed to the decoder (equalizer) for the inner code, whose output is subsequently passed to the decoder for the outer code. The two decoders can exchange soft information to hopefully improve the reliability of the decisions of both decoders. The block diagram is shown in Fig. 3.

Note that now it is no longer necessary to perform decoding in a diagonal-by-diagonal fashion. The equalizer passes the sequences of decision variables to the CC decoders. The CC decoders in turn sends the a posteriori symbol expected values and variances to the equalizer.

C. Interleaver Design

To fully utilize the potential of iterative decoding with convolutional codes, the 1-D CC coded symbol sequences must be individually interleaved, or permuted in time, before they are combined to form the DLST codeword matrix. The block diagram is shown in Fig. 3. An Interleaver is placed at the output of each CC. At the receiver, corresponding deinterleavers and interleavers are placed on the feedforward and feedback paths, respectively.

Due to the nature of convolutional codes, concentrated impulse noises tend to create much more decision errors compared to distributed impulse noises. Therefore, it is desirable that the decision errors made by one CC, after passing through the feedback interleaver, equalizer, and feedforward deinterleaver, appears to be spread out to the other CC decoders.

The detailed behavior of the interleaver is specified as the following. The input symbol sequence is first written, column by column into an \( n \)-row table. Each row of this table is then permuted; the permutation order for each row is different from
the other. Then the symbols are read out column-by-column.

D. Performance Analysis

Here we analyze the performance assuming ML decoding. Given that a codeword matrix $C$ is transmitted, it is shown that, in a slow fading environment, the average probability that the likelihood of another codeword matrix $E$ is higher than that of $C$ is upper bounded by \cite{4}

$$\text{Prob}(C \rightarrow E) \leq \prod_{i=1}^{r} \left(1 + \lambda_i \frac{E_i}{4\sigma_i^2}\right)^{-m}, \quad (10)$$

where $\lambda_1, \lambda_2, ..., \lambda_r$ are the nonzero eigenvalues of the matrix $(C - E)(C - E)^{T}$ and $E_i$ is the power of a symbol. In the framework of DLST codes, again suppose that the transmitted codeword matrix is $C$. Consider another codeword matrix $E$ such that the differences between $C$ and $E$ are limited to the slots designated to a certain CC. Such error events involve decision errors residing in only one CC coded symbol sequence and they dominate the error performance. Because no more than one element in each column of $C - E$ can be nonzero, (10) can be further simplified as

$$\text{Prob}(C \rightarrow E) \leq \prod_{k=1}^{r} \left(1 + \|C^k - E^k\|^2 \frac{E_k}{4\sigma_k^2}\right)^{-m} \prod_{k \in \eta(C, E)} \left(\frac{E_k}{4\sigma_k^2}\right)^{-m}, \quad (11)$$

where $C^k$ and $E^k$ are the $k$th rows of $C$ and $E$, respectively, $\eta(C, E) = \{ i \mid (|C^i - E^i| > 0) \}$, and $r = |\eta(C, E)|$.

According to (11), the design criteria for DLST codes with iterative decoding is to select the CC such that over all pairs of distinct codeword matrices $(C, E)$, the minimum value of $|\eta(C, E)|$ is maximized. This is achieved by selecting the CC such that, over any pairs of codewords $c = \{c_1, c_2, \ldots\}$ and $e = \{e_1, e_2, \ldots\}$, the minimum value of the following quantity is maximized:

$$\sum_{k=1}^{n} \frac{1}{r} \left|\sum_{i=0}^{n} [e_{k+n} - c_{k+n}]\right|^2.$$

In (12), the function $1(x)$ is one if $x$ is nonzero. The minimum of the quantity in (12) can be appropriately named as the folded effective length of the constituent code.

V. SUMMARY

In this paper we considered diagonally layered space-time codes. We showed that, if the wireless channel is i.i.d. Rayleigh fading, an $(n, m)$ dual antenna-array system employing a DLST code can achieve a throughput $\min(n, m)$ times higher than that of a single-antenna system, even when the transmitter does not have the instantaneous CSI.

We proposed both hard decision-feedback decoder and iterative decoder. The decoding complexity of both decoders is only quadratic in the number of antennas, making DLST codes suitable for systems that have a large number of antennas. We analyzed the performance and proposed the corresponding design criteria for DLST codes. In particular, for DLST codes with iterative decoder, the design criterion is to choose the CC that has the highest folded effective length.

VI. REFERENCES


