Outage Probability of MRC With Arbitrary Power Cochannel Interferers in Nakagami Fading

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***Abstract*—We propose a new approach to outage probability analysis of predetection maximal ratio combining (MRC) diversity reception in Nakagami-m fading channels. We generalize prior work in that we consider L independent cochannel interferers with arbitrary powers and fading parameters as well as the effects of additive white Gaussian noise (AWGN). Our approach results in a general expression for outage probability under very broad assumptions. Moreover, our approach leads to a closed-form expression for outage probability in most cases of interest. We also provide numerical results that demonstrate the performance improvement obtained through MRC diversity combining in the presence of cochannel interferers.**

***Index Terms*—Cochannel interference (CCI), maximal ratio combining (MRC), Nakagami fading, outage probability.**

I. Introduction

DIVERSITY reception can significantly improve the performance of digital cellular radio systems with multipath fading and cochannel interference (CCI) [1], [2]. While maximal ratio combining (MRC) offers the best performance in additive white Gaussian noise (AWGN) channels, it offers suboptimal performance when CCI is present. However, MRC is often used in channels with CCI as it offers a good balance between practical complexity and satisfactory performance [3]–[8].

Outage probability ($P_{\rm out}$) is an important measure of wireless systems performance. Several recent works have investigated the outage probability of MRC diversity with CCI. Specifically, in [3] and [4], $P_{\rm out}$ is obtained when the signal of interest (SOI) experiences Rayleigh or Ricean fading while the interfering signals are subject to Rayleigh fading, with background noise neglected. A more general model is analyzed in [5], where both the SOI and interfering signals experience Nakagami fading and noise is not neglected. In [6], a closed-form expression for $P_{\rm out}$ is given considering Rayleigh fading on all signals as well as background noise, but these results apply to at most three unequal-power interferers. That work is extended in [7], where an arbitrary number of interferers is assumed with either the same or different receive powers. All of these results assume cochannel interferers with either the same or completely different receive powers, which precludes the situation in power-controlled cellular systems where in-cell interferers all have the same received power while out-of-cell interference signals have different powers. The case of arbitrary power CCI is studied in [8], where the outage probability of MRC is derived assuming Nakagami, Rice, or Rayleigh fading for the SOI and Rayleigh fading for the CCI, with noise neglected. In [3]–[8], $P_{\rm out}$ is obtained by calculating the probability density function (PDF) of the total interference. This method cannot be used to obtain a closed-form expression for Nakagami fading CCI with arbitrary powers and fading parameters, as there is no closed-form expression for the PDF of the total interference power in that case [9]. However, in [10], we introduced a new approach to obtain a closed-form expression for $P_{\rm out}$ assuming arbitrary signal powers and Rayleigh fading where the PDF of the total interference power is not needed. This approach is similar in spirit to the MGF approach for calculating average error probability, which also avoids calculating the PDF of the signal-to-interference-plus-noise ratio (SINR) [2].

In this letter, we expand our previous results in [10] to obtain analytical expressions for outage probability of MRC with CCI assuming Nakagami-m fading for the desired and the interfering signals. A general expression is derived where the number of interferers, interferers' powers, and the fading parameter m can take any value. We also derive a simpler form for $P_{\rm out}$ when AWGN is neglected. Moreover, for most cases of interest our expression for $P_{\rm out}$ with noise and CCI is in closed-form, and is thus, easily calculated.

The remainder of this letter is organized as follows. Section II introduces the system model. In Section III, we present our analysis for outage probability under general conditions. The closed-form solution for this outage probability is derived in Section IV. The results are summarized in Section V.

II. System Model

We assume a point-to-point channel with M-branch receiver diversity and L interference signals, as shown in Fig. 1. We assume a complex baseband equivalent model for all signals. Both the SOI, as well as the CCI undergo flat fading that is uncorrelated across the antennas. The signal at each of the M receive antennas is corrupted by AWGN with mean power σ^2. The output signal $y(t)$ is the weighted sum of the received signals at each diversity branch

$$y(t) = \mathbf{w}_{\rm mrc}^H \mathbf{x}(t) = \mathbf{w}_{\rm mrc}^H \left(\mathbf{h}_0 s_0(t) + \sum_{l=1}^{L} \mathbf{h}_l i_l(t) + \mathbf{n}(t) \right) \tag{1}$$

Paper approved by V. A. Aalo, the Editor for Diversity and Fading Channel Theory of the IEEE Communications Society. Manuscript received July 28, 2004; revised July 29, 2005. This work was supported in part by the Spanish MCYT under Project TIC2003-07953-C02-01, in part by LG Electronics Corporation, and in part by the Korea Electronics Technology Institute. This paper was presented in part at the 15th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, Barcelona, Spain, September 2004.

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Digital Object Identifier 10.1109/TCOMM.2007.900606

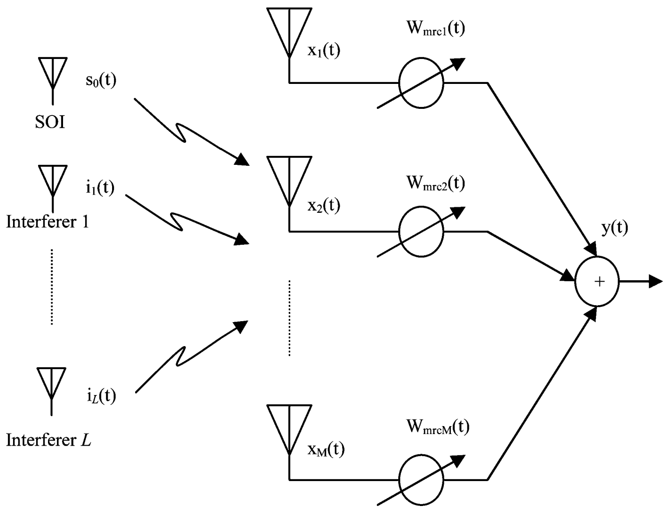

Fig. 1. System model: M-branch receive diversity and L interferers.

where $s_0(t)$ corresponds to the SOI (assumed to be power normalized, $\mathrm{E}[|s_o(t)|^2] = 1$); $i_l(t)$ is the lth interferer's signal, $\mathbf{w}_{\mathrm{mrc}}$ is the M-dimensional vector of receive antenna weights, $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T$ is the vector of equivalent baseband signals at the M receive antennas, $\mathbf{n}(t)$ is the noise vector, $\mathbf{h}_0 = [h_{01}, h_{02,} \ldots, h_{0M}]^T$ is the vector of flat fading gains for the SOI at each receive antenna, and $\mathbf{h}_l = [h_{l1}, h_{l2} \ldots, h_{l\mathrm{M}}]^T$ is the gain vector for the lth interference signal. We assume that all the signals undergo Nakagami-m fading with arbitrary fading parameter and power. We denote the fading parameter and mean received power of the SOI by m_0 and Ω_0, respectively. Similarly, m_l and Ω_l denote the fading parameter and the mean received power of the lth interferer, respectively.

The weight vector for MRC is $\mathbf{w}_{\mathrm{mrc}} = \mathbf{h}_0$. In [5], the SINR of MRC when CCI is present in Nakagami fading is obtained from an extension of the Rayleigh fading case, yielding

$$\gamma = \frac{y_0}{\sum_{l=1}^{L} z_l + \sigma^2} \tag{2}$$

where y_0 is the desired output signal power and z_l is the interference power of the lth interferer. Both z_l and y_0 are gamma random variables with PDFs given by

$$z_l \sim f_{z_l}(z) = \left(\frac{m_l}{\Omega_l}\right)^{m_l} \frac{z^{m_l-1}}{\Gamma(m_l)} \exp\left(-\frac{m_l}{\Omega_l} z\right), \quad 0 \le z \le \infty, \quad m_l \ge 1/2 \tag{3}$$

$$x_0 \sim f_{y_0}(y) = \left(\frac{m_0}{\Omega_0}\right)^{m_0 M} \frac{y^{m_0 M-1}}{\Gamma(m_0 M)} \exp\left(-\frac{m_0}{\Omega_0} y\right), \quad 0 \le y \le \infty, \quad m_0 \ge 1/2 \tag{4}$$

where $\Gamma(a) = \int_0^\infty t^{a-1} \exp(-t) dt$ is the gamma function.

III. Outage Probability Analysis

The outage probability is defined as the event that the received SINR is below a given threshold γ_T. Therefore, we can write

$$\begin{aligned} P_{\mathrm{out}} &= \Pr\left(\frac{y_0}{\sum_{l=1}^{L} z_l + \sigma^2} < \gamma_T\right) \\ &= 1 - \Pr\left(\frac{y_0}{\gamma_T} \ge \sum_{l=1}^{L} z_l + \sigma^2\right). \end{aligned} \tag{5}$$

To simplify the notation we define a new random variable $u_0 \equiv y_0/\gamma_T$, which is also gamma-distributed. We also define the normalized powers $P_0 \equiv \Omega_0/(\gamma_T m_0)$ and $P_l \equiv \Omega_l/m_l, l = 1, \ldots, L$. Additionally, we define the total interference power $z \equiv \sum_{l=1}^{L} z_l$. As in [5], we assume that z_l are independent. According to these definitions, from (5) we have $P_{\mathrm{out}} = 1 - P$, where

$$P = \int_0^\infty f_z(z) \int_{(z+\sigma^2)}^\infty f_{u_0}(u) du\, dz \tag{6}$$

with $f_{u_0}(u)$ as the PDF of u_0. We can solve for the inner integral in (5) as

$$\begin{aligned} &\int_{(z+\sigma^2)}^\infty f_{u_0}(u) du \\ &= \left(\frac{1}{P_0}\right)^{m_0 M} \frac{1}{\Gamma(m_0 M)} \int_{(z+\sigma^2)}^\infty u^{m_0 M-1} \exp\left(-\frac{1}{P_0} u\right) du \\ &= \frac{\Gamma(m_0 M, (z+\sigma^2)/P_0)}{\Gamma(m_0 M)} \end{aligned} \tag{7}$$

where $\Gamma(a, x) = \int_x^\infty t^{a-1} \exp(-t)\, dt$ is the complementary incomplete gamma function. Unfortunately, there is no closed-form general solution to (6) for the inner integral given in (7). Numerical solutions are typically obtained using different approaches. For example, the MGF approach or the Gil–Pelaez Theorem used in [11] to calculate the outage probability in a system without diversity can be used here. However, both of these methods result in integrals that must be numerically solved. We propose a different approach based on the Moschopoulos theorem ([9], [12]), which allows for a numerical solution to (6). From [9] and considering (3) we have

$$f_z(z) = \prod_{l=1}^{L} \left(\frac{P_1}{P_l}\right)^{m_l} \sum_{k=0}^{\infty} \frac{\delta_k e^{-z\frac{1}{P_1}} z^{k-1+\sum_{l=1}^{L} m_l}}{(P_1)^{\left(k+\sum_{l=1}^{L} m_l\right)} \Gamma\left(k + \sum_{l=1}^{L} m_l\right)} \tag{8}$$

where $P_l = \min_l\{P_l\}$ and the coefficients δ_k are obtained recursively by

$$\begin{aligned} \delta_0 &= 1 \\ \delta_{k+1} &= \frac{1}{k+1} \sum_{i=1}^{k+1} \left[\delta_{k+1-i} \sum_{l=1}^{L} m_l \left(1 - \frac{P_1}{P_l}\right)^i\right], \\ &\qquad k = 0, 1, 2, \ldots \end{aligned} \tag{9}$$

Hence, (6) becomes

$$\begin{aligned} P &= \int_0^\infty f_z(z)\frac{\Gamma(m_0M,(z+\sigma^2)/P_0)}{\Gamma(m_0M)}dz \\ &= \prod_{l=1}^{L}\left(\frac{P_1}{P_l}\right)^{m_l} \\ &\quad\times\sum_{k=0}^{\infty}\frac{\delta_k}{(P_1)^{\left(k+\sum_{l=1}^{L}m_l\right)}\Gamma\left(k+\sum_{l=1}^{L}m_l\right)\Gamma(m_0M)} \\ &\quad\times I(M,P_0,P_1,m_0,\ldots,m_L,\sigma^2) \end{aligned} \tag{10}$$

where

$$I(M,P_0,P_1,m_0,\ldots,m_L,\sigma^2)=\int_0^\infty e^{-z\frac{1}{P_1}}z^{\sum_{l=1}^{L}m_l+k-1} \times\Gamma(m_0M,(z+\sigma^2)/P_0)dz. \tag{11}$$

By properties of the gamma function it is clear that $0\le[\Gamma(m_0M,(z+\sigma^2)/P_0)/\Gamma(m_0M)]\le 1$, and therefore, the integral in (10) converges. The integral (11) is adequate for numerical integration using the Gauss–Laguerre quadrature formula [13, eq. (25.4.45)]. Making the change of variables $(z/P_1)=t$ in (11), we have

$$\begin{aligned} &I(M,P_0,P_1,m_0,\ldots,m_L,\sigma^2) \\ &=(P_1)^{\sum_{l=1}^{L}m_l+k}\int_0^\infty e^{-t}t^{\sum_{l=1}^{L}m_l+k-1} \\ &\quad\times\Gamma(m_0M,(P_1t+\sigma^2)/P_0)dt \\ &=(P_1)^{\sum_{l=1}^{L}m_l+k} \\ &\quad\times\left\{\sum_{i=1}^{n}w_it_i^{\sum_{l=1}^{L}m_q+k-1}\Gamma\left(m_0M,(P_1t+\sigma^2)/P_0\right)+R_n\right\} \end{aligned} \tag{12}$$

where t_i and w_i are the ith zero and weight of the Laguerre polynomial, respectively; R_n is the error of the approximation with n terms, as defined in [13, eq. (25.4.45)].

In an interference limited environment, thermal noise can be neglected. In this case, (11) can be written in closed form with the help of [14, eq. (6.455.1)] yielding

$$\begin{aligned} &I(M,P_0,P_1,m_0,\ldots,m_L,0) \\ &=\frac{\left(\frac{1}{P_0}\right)^{m_0M}\Gamma\left(\sum_{l=1}^{L}m_l+k+m_0M\right)}{\left(\sum_{l=1}^{L}m_l+k\right)\left(\frac{1}{P_0}+\frac{1}{P_1}\right)^{\sum_{l=1}^{L}m_l+k+m_0M}} \\ &\quad\times{}_2F_1\left(1,\left(\sum_{l=1}^{L}m_l+k+m_0M\right);\left(\sum_{l=1}^{L}m_l+k+1\right);\right. \\ &\quad\left.\times\ \frac{1/P_1}{1/P_0+1/P_1}\right) \end{aligned} \tag{13}$$

where ${}_2\mathrm{F}_1$ is the Gaussian hypergeometric function, defined as

$${}_2F_1(a,b;c;x)=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\frac{x^n}{n!},\text{with }(a)_n=\frac{\Gamma(a+n)}{\Gamma(a)}.$$

Note that [14, eq. (6.455.1)] can also be used in [5, eq. (30)] when background noise is neglected, thereby, yielding a closed-form expression for equal power interferers without any restriction on the fading parameters of either the SOI or the CCI. In Section IV, we show how a closed-form expression for P_{out} can be obtained without neglecting the background noise by imposing a mild restriction on the fading parameter of the SOI.

IV. Closed-Form Results For Outage Probability

In this section, we show that if m_0M is an integer, a closed-form expression for P_{out} with no restrictions on the powers or fading parameters of interferers can be obtained. For m_0M an integer, using [14, eq. (8.352.2)], (7) becomes an integral that may be solved in closed form as

$$\begin{aligned} \int_{(z+\sigma^2)}^{\infty}f_{u_0}(u)du &= \frac{\Gamma(m_0M,(z+\sigma^2)/P_0)}{\Gamma(m_0M)} \\ &= e^{-\frac{(z+\sigma^2)}{P_0}}\sum_{k=0}^{Mm_0-1}\frac{1}{k!}\left(\frac{1}{P_0}\right)^k(z+\sigma^2)^k \\ &= e^{-\frac{\sigma^2}{P_0}}\left(\prod_{l=1}^{L}e^{-\frac{z_l}{P_0}}\right)\sum_{k=0}^{Mm_0-1}\frac{1}{k!}\left(\frac{1}{P_0}\right)^k\left(\sigma^2+\sum_{l=1}^{L}z_l\right)^k. \end{aligned} \tag{14}$$

Using the multinomial theorem [15, eq. (3.16)], we have

$$\left(\sigma^2+\sum_{l=1}^{L}z_l\right)^k=\sum_{\sum_{l=0}^{L}k_l=k}\frac{k!}{k_0!k_1!k_2!\ldots k_L!}\sigma^{2k_0}z_1^{k_1}z_2^{k_2}\ldots z_L^{k_L}, \tag{15}$$

where the summation in (15) is taken over all combinations such that $\sum_{i=0}^{L}k_i=k, k_i\in\mathrm{N}$. The multinomial decomposition in (15) allows for a reformulation of (6) as

$$\begin{aligned} P=\int_0^\infty\int_0^\infty\cdots\int_0^\infty & f_{z_1}(z_1)f_{z_2}(z_2)\cdots f_{z_L}(z_L) \\ &\times\int_{(z+\sigma^2)}^{\infty}f_{u_0}(u)dudz_1dz_2\cdots dz_L. \end{aligned} \tag{16}$$

Using (3) and (14)–(16), after some manipulation we obtain

$$\begin{aligned} P=\exp&\left(-\frac{\sigma^2}{P_0}\right)\sum_{k=0}^{Mm_0-1}\left(\frac{1}{P_0}\right)^k \\ &\times\sum_{\sum_{l=0}^{L}k_l=k}\frac{\sigma^{2k_0}}{k_0!}\prod_{l=1}^{L}\frac{1}{k_l!P_l^{m_l}\Gamma(m_l)} \\ &\times\int_0^\infty e^{-z_l\left[\frac{1}{P_0}+\frac{1}{P_l}\right]}z_l^{k_l+m_l-1}dz_l\ . \end{aligned} \tag{17}$$

With the change of variable $t=z_l\left[1/P_0+1/P_l\right]$, the integral in (17) becomes a gamma function with parameter (k_l+m_l) multiplied by $\left[1/P_0+1/P_l\right]^{-k_l-m_l}$. Therefore, the outage

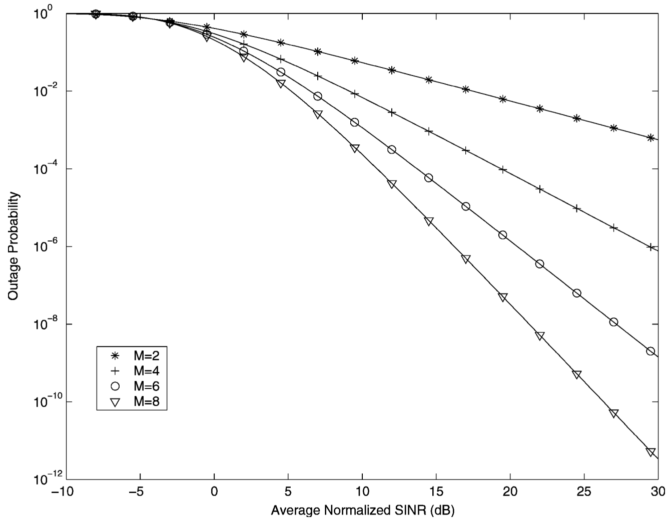

Fig. 2. P_{out} versus average normalized SINR for MRC with six interferers with average powers 1, 0.2, 0.2, 1.5, 0.8, and 0.8. Respective fading parameters are 0.6, 0.55, 0.98, 0.98, 1.33, and 1. $\sigma^2 = 0\,\text{dB}$.

probability becomes

$$P_{\text{out}} = 1 - e^{-\frac{\sigma^2}{P_0}} \sum_{k=0}^{Mm_0-1} \left(\frac{1}{P_0}\right)^k \sum_{\sum_{l=0}^{L} k_l = k} \frac{\sigma^{2k_0}}{k_0!} \times \prod_{l=1}^{L} \frac{1}{k_l! P_l^{m_l}} \left[\frac{1}{P_0} + \frac{1}{P_l}\right]^{-k_l - m_l} \frac{\Gamma(k_l + m_l)}{\Gamma(m_l)}. \quad (18)$$

This expression applies for any set of interference powers and fading parameters with the restriction that $m_0 M$ must be an integer. Since M, the number of receive antennas, is an integer, (18) is valid whenever m_0 is an integer, which includes most cases of practical interest.

In Fig. 2, we plot numerical results obtained using (18). This figure shows outage probability as function of the long-term average SINR, normalized by the outage threshold $\gamma_T (\text{SINR}/\gamma_T = \Omega_0 / (\sum_{l=1}^{L} \Omega_l + \sigma^2))$. Six interferers ($L = 6$) are considered. For the SOI, a fading parameter of $m_0 = 0.5$ is assumed. Each curve corresponds to a different number of antennas in the array ($M = 2, 4, 6, 8$). From (18), it is clear that diversity effectively increases the fading parameter m_0 of the SOI by a factor of M. It is well known that MRC has this effect on performance in the absence of CCI. Our results indicate that the same performance benefits are obtained in the presence of CCI with arbitrary powers.

If the SOI and all the interferers exhibit Rayleigh fading ($m_i = 1, i = 0, 1, \ldots, L$), (18) reduces to

$$P_{\text{out}} = 1 - e^{-\frac{\sigma^2}{P_0'}} \sum_{k=0}^{M-1} \left(\frac{1}{P_0}\right)^k \times \sum_{\sum_{l=0}^{L} k_l = k} \frac{\sigma^{2k_0}}{k_0!} \prod_{l=1}^{L} \frac{1}{P_l} \left[\frac{1}{P_0} + \frac{1}{P_l}\right]^{-k_l - 1} \quad (19)$$

which agrees with [10, eq. (14)].

V. Summary

We have studied the outage probability of MRC diversity systems in the presence of white Gaussian noise and non correlated cochannel interfererence. Both the SOI and the CCI exhibit Nakagami fading with arbitrary powers and fading parameters. With these assumptions we obtain a general expression for outage probability, which is in closed-form for most cases of interest. Our results indicate that MRC diversity effectively increases the fading parameter of the SOI by a factor equal to the number of diversity branches.

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