Iterative and One-shot Conferencing in Relay Channels

Chris T. K. Ng*, Ivana Maric†, Andrea J. Goldsmith*, Shlomo Shamai (Shitz)‡ and Roy D. Yates†

*Dept. of Electrical Engineering, Stanford University, Stanford, CA 94305 US
†WINLAB, Rutgers University, Piscataway, NJ 08854 US
‡Dept. of Electrical Engineering, Technion - Israel Institute of Technology, Haifa 32000, Israel
Email: *{ngctk,andrea}@wsl.stanford.edu, †{ivanam,ryates}@winlab.rutgers.edu, ‡sshlomo@ee.technion.ac.il

***Abstract*— We compare the rates of one-shot and iterative conferencing in a cooperative Gaussian relay channel. The relay and receiver cooperate via a conference, as introduced by Willems, in which they exchange a series of communications over orthogonal links. Under one-shot conferencing, decode-and-forward (DF) is capacity-achieving when the relay has a strong channel. On the other hand, Wyner-Ziv compress-and-forward (CF) approaches the cut-set bound when the conference link capacity is large. To contrast with one-shot conferencing, we consider a two-round iterative conference scheme; it comprises CF in the first round, and DF in the second. When the relay has a weak channel, the iterative scheme is disadvantageous. However, when the relay channel is strong, iterative cooperation, with optimal allocation of conferencing resources, outperforms one-shot cooperation provided that the conference link capacity is large. When precise allocation of conferencing resources is not possible, we consider iterative cooperation with symmetric conference links, and show that the iterative scheme still surpasses one-shot cooperation, albeit under more restricted conditions.**

I. Introduction

We consider a relay network where the relay and receiver cooperate via orthogonal conference links with finite capacity. The conference cooperation model was introduced by Willems in [1], in which the capacity region was derived for a multiple-access channel (MAC) with conferencing encoders. The encoders use the links to exchange a series of communications, referred to as a *conference*, that allows for an iterative joint-encoding of a message. In that work, it was shown that an one-shot conference step was optimal, and iterative conferencing provides no advantage.

In [2] it was shown that receiver cooperation enlarges the broadcast channel (BC) rate region. The achievable rates of a BC with conferencing receivers were presented in [3]. When both receivers wish to decode a common message, the performance gain from iterative cooperation was illustrated in [4] on binary erasure channels with conferencing receivers. In particular, [3], [4] show that iterative cooperation outperforms one-shot cooperation in the case of a common message for the receivers.

In a relay network, however, when only one of the receivers wishes to decode the message, it is not clear if iterative cooperation is always advantageous. In this paper we characterize the relative performance of iterative and one-shot cooperation. We show that one-shot cooperation is sufficient when the relay has a weak channel, but iterative cooperation can be beneficial when the relay has a strong channel, provided that the conference link capacity is large. We also investigate conferencing rates in comparison to those of a full-duplex relay channel.

The rest of the paper is organized as follows. Section II defines the conferencing relay channel. The one-shot and iterative cooperation strategies are presented in Section III. Section IV contrasts the relative performance of one-shot and iterative cooperation, and Section V considers the iterative scheme with symmetric conference links. Then numerical examples of the cooperation rates are illustrated in Section VI, followed by conclusions in Section VII.

This work was supported by the US Army under MURI award W911NF-05-1-0246, and the ONR under award N00014-05-1-0168.

II. System Model

We consider a discrete-time memoryless channel where a transmitter is communicating with a relay and receiver, as illustrated in Fig. 1. The received signals are described by

$$y_1 = \sqrt{g}\,x + n_1, \qquad y_2 = x + n_2, \tag{1}$$

where $x, y_1, y_2, n_1, n_2 \in \mathbb{C}$, $g \in \mathbb{R}_+$: x is the signal sent by the transmitter, y_1, y_2 are the signals observed by the relay and receiver, respectively, and n_1, n_2 are independent zero-mean circularly symmetric complex Gaussian (ZMCSCG) white noise with normalized unit variance. The power gain g represents the relative channel strength of the relay and receiver. The transmitter is under an average power constraint P, i.e., $\mathrm{E}[\,|x|^2] \le P$, where the expectation is taken over repeated channel uses. We assume a static channel, and every node has perfect channel state information (CSI). There is no loss of generality in restricting the channel gains to be real, as the receivers can zero-phase the observed signals.

The relay and receiver cooperate by way of a conference, as defined in [1]. The conference links have finite capacity $\alpha\mathsf{C}$ and $(1-\alpha)\mathsf{C}$, as shown in Fig. 1, where $\mathsf{C} \in \mathbb{R}_+$ is the total conference link capacity available between the receivers, and $\alpha \in [0,1]$ represents the allocation of conferencing resources in each direction. In practice the conference links may be realized via orthogonal channelization with sufficiently long coding blocks. A conference is permissible if the total

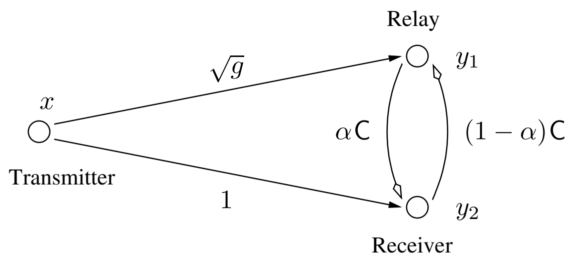

Fig. 1. Conferencing relay and receiver.

cardinality of the conference communications (possibly sent over multiple rounds) does not exceed that allowed by the capacity of the conference link. Formally, a K-round conference is given by two sets of K communicating functions $\{h_{t,1}, \ldots, h_{t,K}\}$, $t = 1, 2$. Each function $h_{t,k}$ causally maps the received signal y_t and the sequence of $k-1$ previously received communications from the other encoder into the kth communication $V_{t,k}$, where $V_{t,k}$ ranges over a finite alphabet $\mathcal{V}_{t,k}$. The conference is permissible if in N channel uses,

$$\frac{1}{N}\sum_{k=1}^{K}\log\big(|\mathcal{V}_{1,k}|\big) \le \alpha\mathsf{C}, \quad \frac{1}{N}\sum_{k=1}^{K}\log\big(|\mathcal{V}_{2,k}|\big) \le (1-\alpha)\mathsf{C}$$

for all N sufficiently large. The $\log$ is base 2, and C has units of bits per channel use.

This setting can be extended to a BC with cooperating receivers, where each of the receivers decodes its own independent message, possibly along with a common message both wish to decode. Our model is a special case in which only one receiver wishes to decode the message sent by the transmitter, while the other acts as a relay.

III. Cooperation Strategies

For the three-terminal cooperative network shown in Fig. 1, its capacity is not known in general, and the optimal cooperation strategy remains an open problem. To study the performance of the network, we consider the capacity upper bounds and achievable rates under one-shot and iterative conferencing. The cut-set bound described in [5, Thm. 14.10.1] provides a capacity upper bound for any general multiterminal network. For the achievable rates, we consider conferencing schemes based on the decode-and-forward (DF) [6, Thm. 1] and compress-and-forward (CF) [6, Thm. 6] coding strategies as they form the basis of the most common cooperative coding techniques for general multiuser channels.

A. One-shot cooperation

In one-shot cooperation, we require the receiver to decode the message sent by the transmitter after at most one round of conference communication (i.e., $K=1$). Since only the receiver wishes to decode the message, we naturally set $\alpha=1$. Hence the conferencing relay network reduces to a standard relay channel with an orthogonal link between the relay and the receiver. One-shot conferencing is a special case of the decentralized agents model investigated in [7], when one of the agents has an infinite capacity link to the destination.

The cut-set (CS) bound was presented in [5], [6]. Under a Gaussian channel with conferencing relay and receiver the one-shot cooperation cut-set bound evaluates to

$$C_{\mathrm{os,CS}} = \min\big\{\log\big(1+(1+g)P\big),\ \log(1+P)+\mathsf{C}\big\}. \quad (2)$$

In the DF cooperation strategy [6], [8]–[10], transmission is done in blocks: the relay first fully decodes the transmitter's message in one block, then in the ensuing block the relay and the transmitter cooperatively send the message to the receiver. The achievable rate is

$$R_{\mathrm{os,DF}} = \min\big\{\log(1+gP),\ \log(1+P)+\mathsf{C}\big\}. \quad (3)$$

In the CF cooperation strategy [6], [9], [11], [12], the relay sends a compressed version of its observed signal to the receiver. The compression is realized using Wyner-Ziv source coding [13], which exploits the correlation between the received signal of the relay and that of the receiver. The receiver then performs maximal-ratio combining (MRC) of the compressed signal and its own observation; the following rate is achieved:

$$R_{\mathrm{os,CF}} = \log\big(1+P+gP/(1+\hat{N}_{\mathrm{os}})\big), \quad (4)$$

where $\hat{N}_{\mathrm{os}}$ is the variance of the Gaussian-distributed Wyner-Ziv compression noise given by

$$\hat{N}_{\mathrm{os}} = \frac{(1+g)P+1}{(2^{\mathsf{C}}-1)(P+1)}. \quad (5)$$

B. Iterative cooperation

Under iterative cooperation, we allow the relay and receiver to perform multiple rounds of conference communications (i.e., $K>1$) before decoding. Furthermore, the parameter α can be optimized to determine the best allocation of conferencing resources between the relay and receiver.

The cut-set bound is maximized at $\alpha=1$, thus the iterative cooperation cut-set bound is the same as that for one-shot cooperation: $C_{\mathrm{i,CS}} = C_{\mathrm{os,CS}}$. Intuitively, allowing feedback from a destination node to a source node does not increase the cut-set bound; therefore, it is optimal to simply maximize the capacity of the conference link to the destination. The DF and CF strategies have been applied to iterative cooperation [2]–[4]. With Wyner-Ziv source coding of Gaussian codewords, having the side information at the encoder does not improve the source coding rate [13]. Thus we consider the following two-round scheme: In the first round, the receiver sends its observation to the relay using Wyner-Ziv CF. Then the relay decodes the message, and provides feedback to the receiver in the second round via DF. The iteration achieves the minimum of the CF and DF rates:

$$\begin{aligned} R_{\mathrm{i}} = \max_{0\le\alpha\le1} \min\big\{ & \log\big(1+gP+P/(1+\hat{N}_{\mathrm{i}})\big), \\ & \log(1+P)+\alpha\mathsf{C}\big\}, \end{aligned} \quad (6)$$

where $\hat{N}_{\mathrm{i}}$ is the Wyner-Ziv compression noise variance in the first round:

$$\hat{N}_{\mathrm{i}} = \frac{(1+g)P+1}{(2^{(1-\alpha)\mathsf{C}}-1)(gP+1)}. \quad (7)$$

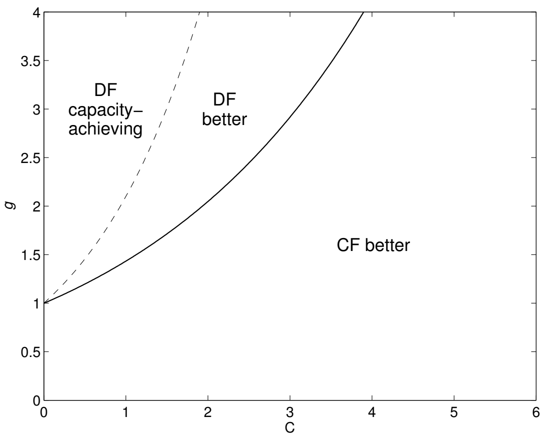

Fig. 2. The best one-shot cooperation strategy as a function of g and C.

IV. Iterative vs. One-shot Cooperation

In this section we first present the one-shot cooperation strategy, then we derive the optimal allocation of conferencing resources for the iterative scheme. Their relative performance is then characterized under different channel conditions.

A. Best one-shot cooperation strategy

We assume the conferencing relay and receiver can choose between the DF and CF strategy based on the channel condition and available conferencing resources. We note that DF is limited by the rate the relay can reliably decode, but CF improves with the conference link capacity C.

A comparison of (3) and (4) yields that one-shot DF outperforms one-shot CF when

$$g > \frac{\sqrt{2^{\mathsf{C}}\, 4P(P+1)+1}-1}{2P}. \tag{8}$$

In particular, DF meets the cut-set bound when

$$g \geq (2^{\mathsf{C}}(P+1)-1)/P. \tag{9}$$

Therefore, for large g relative to 2^{C}, the one-shot DF strategy is capacity-achieving. On the other hand, outside of the region defined by (8), CF achieves a higher rate. Fig. 2 shows the best one-shot cooperation strategy for different values of g and C. We set $P = 10$ in the examples in this paper.

B. Weak relay channel

When the relay has a weak channel (i.e., $g \leq 1$), we show that iterative cooperation is not necessary; in particular, one-shot CF outperforms the iterative scheme. To prove $R_{\mathrm{os,CF}} \geq R_{\mathrm{i}}$ for $g \leq 1$, it suffices to show that for all $\alpha \in [0,1]$,

$$R_{\mathrm{os,CF}} \geq \log\bigl(1 + gP + P/(1+\hat{N}_{\mathrm{i}})\bigr). \tag{10}$$

We rewrite the condition (10) as

$$\hat{N}_{\mathrm{i}} \geq g\hat{N}_{\mathrm{os}}/(1+\hat{N}_{\mathrm{os}}(1-g)), \tag{11}$$

which follows from

$$\hat{N}_{\mathrm{i}} \geq \hat{N}_{\mathrm{os}} \geq g\hat{N}_{\mathrm{os}}/(1+\hat{N}_{\mathrm{os}}(1-g)) \tag{12}$$

when $g \leq 1$. Thus the iterative scheme provides no advantage when the relay has a weak channel.

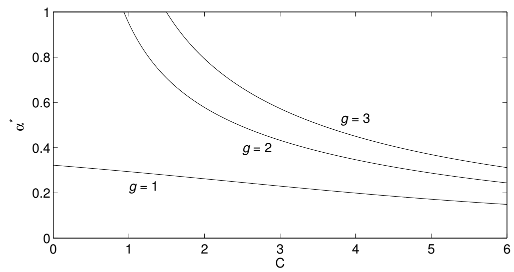

Fig. 3. Optimal conference link capacity allocation α^* for the two-round iterative cooperation scheme.

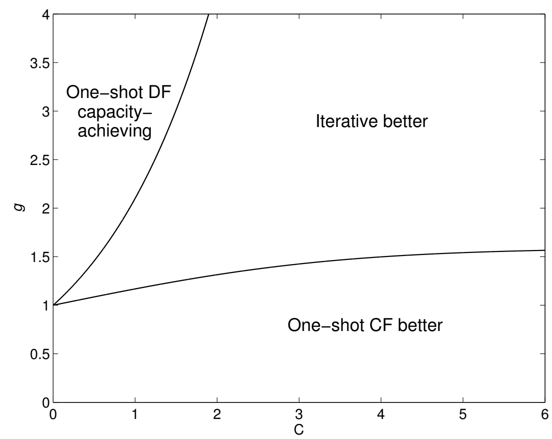

Fig. 4. The best cooperative strategy as a function of g and C.

C. Strong relay channel

When the relay has a strong channel (i.e., $g > 1$), the best cooperation strategy depends on the channel condition and the available conferencing resources. First we derive the optimal conference capacity allocation α^* for the iterative scheme. In region (9) where DF is optimal, the iterative strategy reduces to a one-shot cooperation scheme (the receiver does not communicate in the first round; the relay performs DF in the second round), hence $\alpha^* = 1$. Outside of the region in (9), we observe that the first expression inside of the $\min\{\cdot\}$ in (6) is monotonically decreasing in α while the second is increasing. The optimal α^* is obtained by equating the two expressions:

$$\alpha^* = \frac{1}{\mathsf{C}} \log\left(\frac{\sqrt{A(A+B)}-A}{2P(1+P)}\right), \tag{13}$$

where $A \triangleq 2^{\mathsf{C}}(P+1)(gP+1)$ and $B \triangleq 4P(gP+P+1)$. The optimal conference capacity link allocation α^* is illustrated in Fig. 3 for selected values of g.

Under optimal conference link capacity allocation, the iterative scheme outperforms one-shot cooperation for large g and C. Fig. 4 shows the best one-shot or iterative cooperation strategy in different regions of g and C. The boundary between the iterative and the one-shot CF region is numerically computed, as the expression for the iterative rate with the optimal α^* is rather tedious.

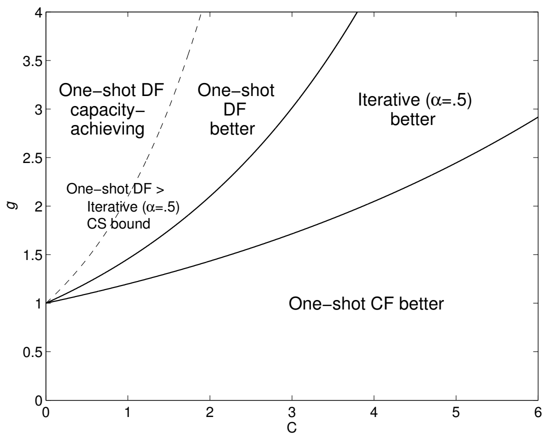

Fig. 5. Iterative ($\alpha = .5$) vs. one-shot cooperation. The best strategy is labeled for each region.

V. Symmetric Conference Links

In Section IV-C we show that the optimal conference link capacity allocation α^* depends on the system SNR P, the relative channel condition g, and the total conferencing resources C. In the case when capacity allocation is not possible, we consider if iterative cooperation can still outperform one-shot cooperation. Specifically, we now assume that the cooperating receivers have symmetric conference links, i.e., the conference links have equal capacity ($\alpha = .5$). Under this condition the cut-set bound becomes

$$C_{\text{i,CS}}|_{\alpha=.5} = \min\big\{ \log\big(1 + (1+g)P\big), \log(1+P) + \mathsf{C}/2 \big\}. \tag{14}$$

Similarly, the two-round iterative scheme achieves the rate

$$R_{\text{i}}|_{\alpha=.5} = \min\big\{ \log\big(1 + gP + P/(1 + \hat{N}_{\text{i}}|_{\alpha=.5})\big), \log(1+P) + \mathsf{C}/2 \big\}. \tag{15}$$

Comparing (15) to the DF rate (3), we note that now the iterative scheme overtakes one-shot DF only when

$$g \geq (2^{\mathsf{C}/2}(P+1) - 1)/P, \tag{16}$$

which represents a smaller region over which iterative cooperation is beneficial. Furthermore, outside of region (16) the one-shot DF rate is strictly higher than the symmetric conference link cut-set bound (14). Hence for such g and C, one-shot cooperation conclusively outperforms any iterative scheme with symmetric conference links.

When we compare the iterative scheme to the CF rate (4), we note that the iterative scheme becomes superior when

$$g \leq \frac{\sqrt{2^{\mathsf{C}/2}\, 4P(P+1) + 1} - 1}{2P}. \tag{17}$$

The best one-shot or iterative cooperation strategy for different values of g and C is presented in Fig. 5. As expected, without optimal allocation of α the region where iteration is beneficial shrinks. Nevertheless, the iterative scheme still surpasses one-shot cooperation for large g and C.

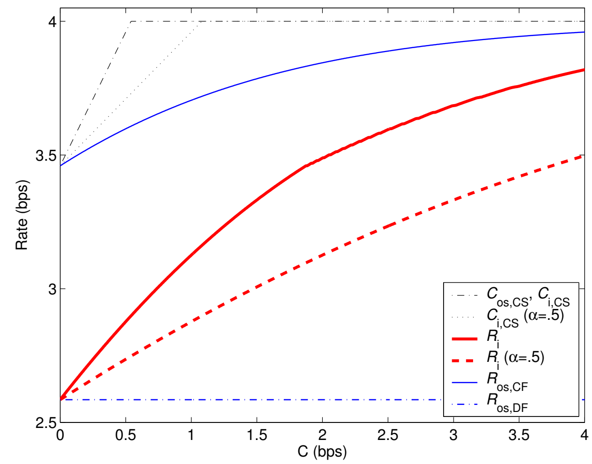

Fig. 6. Weak relay channel ($g = 0.5$). One-shot CF outperforms the iterative cooperation scheme.

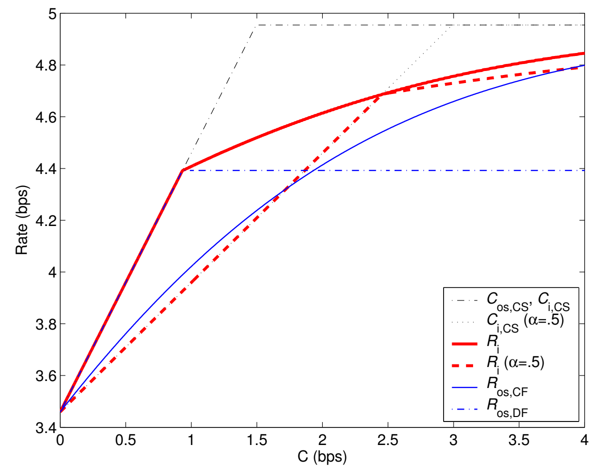

Fig. 7. Strong relay channel ($g = 2$). The best cooperation strategy depends on g and C.

VI. Achievable Rates

In this section we present numerical examples to illustrate the rates achievable by the one-shot and iterative cooperation strategies. We assume the channel has unit bandwidth, and $P = 10$. In Fig. 6 we consider the case when the relay has a weak channel ($g = 0.5$). Under one-shot cooperation, DF fails to take advantage of the conference link, since it is limited by the rate at which the relay can decode. Moreover, as one-shot CF achieves a higher rate than the iterative scheme, iterative cooperation is disadvantageous.

When, on the other hand, the relay has a strong channel ($g = 2$) as shown in Fig. 7, CF has the worst performance. For small C, the DF rate meets the cut-set bound; however, as C increases the DF rate plateaus but the iterative cooperation rate continues to improve. When we restrict the iterative scheme to symmetric conference links, the iterative performance degrades as expected. Nevertheless, the symmetric conference link iterative scheme still manages to surpass one-shot cooperation as C becomes large.

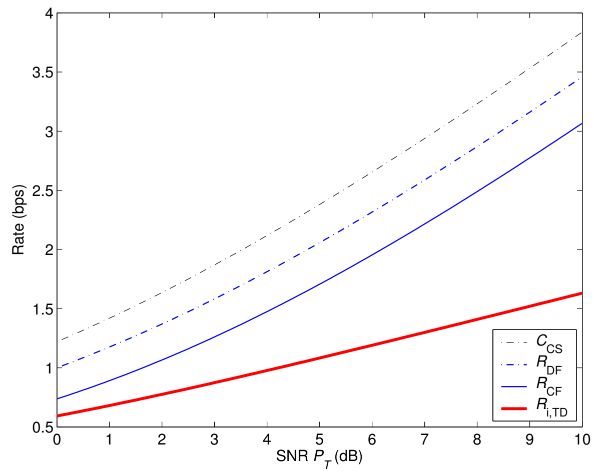

Fig. 8. Performance loss from orthogonal channelization.

We note that when g and C are very large, the performance gap between the different cooperation strategies vanishes, as they all approach the performance limit of the cut-set bound. In addition, for large C the cut-set bound (2) evaluates to the single-input-multiple-output (SIMO) channel capacity $\log(1+(1+g)P)$, which is a capacity upper bound on the sum-rate of a BC with cooperating receivers. Consequently, electing the proper cooperation strategy is most relevant at moderate values of g and C, where efficient application of system power and conferencing resources is significant.

In the system model we assume the conference channel capacity does not affect system power or bandwidth. However, if the conference links are realized via channel orthogonalization, the capacity gain from iterative cooperation could be erased by suboptimal spectral efficiency. Fig. 8 shows the cut-set bound, DF, and CF rates [9] of a full-duplex relay channel, where $g=2$, with unit channel gain between the relay and receiver, and the transmitter and relay each under power constraint $P_T/2$. In the iterative scheme, the conference links are realized through equal-duration time-division (TD) among the transmitter, relay and receiver, where each node is under power constraint P_T during its time slot. We observe that the orthogonal scheme suffers a considerable performance loss, which is aggravated as the system SNR P_T increases. This large performance loss was exhibited over a wide range of channel gains and SNRs. These results indicate that using system power and bandwidth to perform conferencing does not provide capacity benefit.

VII. Summary

We compare iterative and one-shot conferencing in relay channels. Under one-shot conferencing we show that DF outperforms CF when the relay has a strong channel. In particular, when g is sufficiently large in comparison to 2^{C}, DF is capacity-achieving. By contrast, when 2^{C} is large relative to g, CF is superior; expressly, the CF rate approaches the capacity cut-set bound as C increases.

For the iterative conferencing scheme, we show that it underperforms one-shot CF when the relay has a weak channel (i.e., $g \leq 1$). Under a strong relay channel (i.e., $g > 1$), however, the iterative scheme outperforms one-shot cooperation when C is large. In addition, we consider iterative cooperation with symmetric conference links when precise allocation of conferencing resources is not possible. With symmetric conference links (i.e, $\alpha = .5$), the performance of the iterative scheme degrades as expected. Specifically, when g is large relative to 2^{C} we show that one-shot DF exceeds the cut-set bound of symmetric conference link iterative cooperation. Notwithstanding, the symmetric conference link iterative scheme still surpasses one-shot cooperation as C becomes large.

We consider a two-round iterative conferencing scheme that comprises CF in the first round, and DF in the second. Further work includes allowing partial decoding under multiple rounds ($K > 2$) of iteration, and investigating the optimal number of rounds of iteration. Moreover, conferencing can be applied as a bounding tool in the more general setting of a BC with cooperating receivers. Letting the conferencing capacity go to infinity would yield the capacity of a multiple antenna system, while letting it go to zero would reduce the system to a standard BC. With finite conference link capacity, iterative conferencing can be applied as a cooperation strategy by the receivers.

Acknowledgment

The authors would like to thank Stark Draper for helpful discussions on iterative cooperation schemes.

References

[1] F. M. J. Willems, "The discrete memoryless multiple access channel with partially cooperating encoders," *IEEE Trans. Inform. Theory*, vol. 29, no. 3, pp. 441–445, May 1983.

[2] Y. Liang and V. V. Veeravalli, "Cooperative relay broadcast channels," *IEEE Trans. Inform. Theory*, submitted.

[3] R. Dabora and S. D. Servetto, "Broadcast channels with cooperating decoders," *IEEE Trans. Inform. Theory*, submitted.

[4] S. C. Draper, B. J. Frey, and F. R. Kschischang, "Iterative decoding of a broadcast message," in *Proc. Allerton Conf. Commun., Contr., Comput.*, 2003.

[5] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley-Interscience, 1991.

[6] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. 25, no. 5, pp. 572–584, Sept. 1979.

[7] A. Sanderovich, S. Shamai, Y. Steinberg, and G. Kramer, "Communication via decentralized processing," in *Proc. IEEE Int. Symp. Inform. Theory*, 2005, pp. 1201–1205.

[8] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[9] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, no. 9, pp. 3037–3063, Sept. 2005.

[10] A. Host-Madsen, "Capacity bounds for cooperative diversity," *IEEE Trans. Inform. Theory*, submitted.

[11] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channel," *IEEE Trans. Inform. Theory*, vol. 51, no. 6, pp. 2020–2040, June 2005.

[12] M. A. Khojastepour, A. Sabharwal, and B. Aazhang, "Improved achievable rates for user cooperation and relay channels," in *Proc. IEEE Int. Symp. Inform. Theory*, 2004, p. 4.

[13] A. D. Wyner and J. Ziv, "The rate-distortion function for source coding with side information at the decoder," *IEEE Trans. Inform. Theory*, vol. 22, no. 1, pp. 1–10, Jan. 1976.