The Role of SNR in Achieving MIMO Rates in Cooperative Systems

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***Abstract*— We compare the rate of a multiple-antenna relay channel to the capacity of multiple-antenna systems to characterize the cooperative capacity in different SNR regions. While it is known that in the asymptotic regime, at a high SNR or with a large number of cooperating nodes, cooperative systems lack full multiplexing gain, in this paper we consider cooperative capacity gain at moderate SNR with a fixed number of cooperating antennas. We show that up to a lower bound to an SNR threshold, a cooperative system performs at least as well as a MIMO system with isotropic inputs; whereas beyond an upper bound to the SNR threshold, the cooperative system is limited by its coordination costs, and the capacity is strictly less than that of a MIMO orthogonal channel. The SNR threshold depends on the network geometry (the power gain g between the source and relay) and the number of cooperating antennas M; when the relay is close to the source ($g \gg 1$), the SNR threshold lower and upper bounds are approximately equal. As the cooperating nodes are closer, i.e., as g increases, the MIMO-gain region extends to a higher SNR. Whereas for a populous cluster, i.e., when M is large, the coordination-limited region sets in at a lower SNR.**

I. Introduction

In wireless networks, node cooperation has been proposed as a strategy to improve communication reliability and system capacity. The idea of cooperative diversity was pioneered in [1], [2], where the transmitters cooperate to achieve a larger rate region by repeating detected symbols from each other. In [3] the transmitters forward parity bits to achieve cooperation diversity. Cooperative diversity and outage behavior was studied in [4], where it was shown that orthogonal cooperative protocols can achieve full spatial diversity order. Multiple-antenna systems and cooperative ad-hoc networks were compared in [5], [6]; capacity gain was characterized to demonstrate that cooperation is most beneficial when the cooperating nodes are close together. Information-theoretic achievable rate regions and bounds were derived in [7]–[12] for channels with transmitter and/or receiver cooperation. For relay networks it was shown in [13] that decode-and-forward is close to optimal when the relay is near the source, whereas compress-and-forward is more beneficial when the relay is near the destination. Capacity gain from transmitter and receiver cooperation were compared in [14], [15] in a relay channel, under different channel state information (CSI) and power allocation assumptions. It was shown that transmitter cooperation is more favorable with CSI at the transmitter (CSIT), while receiver cooperation is superior under optimal power allocation without CSIT.

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The capacity of cooperative systems has been studied in the asymptotic regime. In [16], it was shown that at asymptotically high signal-to-noise ratio (SNR), the multiplexing gain of an N-source-destination-pair network is upper bounded by $N/2$. Thus cooperative systems do not enjoy full multiplexing gain. In fact, the $N/2$ upper bound was believed to be loose and the multiplexing gain was conjectured to be one. From another perspective, the capacity of large Gaussian relay networks was considered in [17]. In an N-node network, for asymptotically large N, capacity scales as $\Theta(\log N)$. Hence cooperative systems fail to achieve the linear capacity scaling order (in the number of antennas) found in multiple-input-multiple-output (MIMO) systems. Under what conditions, then, is cooperation of most value? In this paper, we consider cooperative capacity gain at moderate SNR with a fixed number of cooperating antennas. We show that up to an SNR threshold, which depends on the network geometry and the number of antennas, a cooperative system performs at least as well as a MIMO system with isotropic inputs.

The rest of the paper is organized as follows. Section II motivates and defines the multiple-antenna relay channel model. Bounds on the cooperative capacity in different SNR regions are derived in Section III. Then Section IV illustrates the numerical results with an example of a 2×2 cluster network, followed by conclusions in Section V.

II. System Model

A. Motivation

We consider a multiple-antenna Gaussian relay channel, where the source has a single antenna, the relay has $M-1$ antennas, and the destination has M antennas, as illustrated in Fig. 1. The relay is assumed to be close to the source, and together they form the transmit cluster. The destination is far from the transmitter cluster, and thus approximately equidistant from the source and the relay. The multiple-antenna relay channel serves as an optimistic model for cooperative

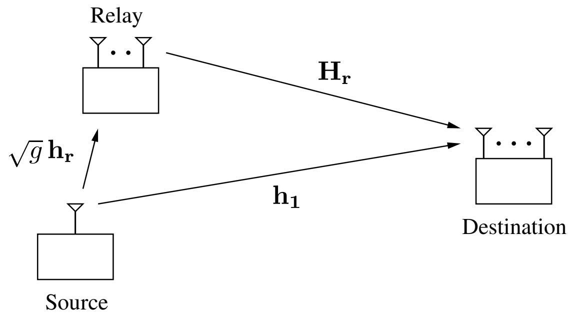

Fig. 1. Cooperation system model: the source has a single antenna; the relay is close to the source and it has $M-1$ antennas; the destination has M antennas.

systems that comprise transmit and receive clusters of single-antenna nodes. In particular, the multiple-antenna relay channel performance represents an upper bound for the case where the source utilizes multiple single-antenna nodes clustered together that coordinate to form a relay.

In general, the capacity of a relay channel remains an open problem; however, when the relay is close to the source, the decode-and-forward coding scheme is close to capacity-achieving [13]. Therefore, the capacity of the cooperative system in Fig. 1 is well-characterized. In this paper, we will study the cooperative capacity in different SNR regions, based on the given geometry (proximity between the source and relay) and the number of cooperating antennas in each cluster.

B. Channel model

Consider a discrete-time frequency-flat block fading wireless channel where the received signals are corrupted by additive white Gaussian noise (AWGN). The relay network in Fig. 1 is described by

$$\mathbf{y_r} = \sqrt{gP}\,\mathbf{h_r} x_1 + \mathbf{n_r} \tag{1}$$

$$\mathbf{y} = \sqrt{P/M}\,\begin{bmatrix}\mathbf{h_1}\,\mathbf{H_r}\end{bmatrix}\begin{bmatrix}x_1\\ \mathbf{x_r}\end{bmatrix} + \mathbf{n}, \tag{2}$$

where $x_1 \in \mathbb{C}$ is the signal sent by the source, $\mathbf{y} \in \mathbb{C}^M$ is the signal vector received by the destination, $\mathbf{x_r}, \mathbf{y_r} \in \mathbb{C}^{M-1}$ are the transmitted and received signal vectors of the relay, respectively, and $\mathbf{n_r} \in \mathbb{C}^{M-1}$, $\mathbf{n} \in \mathbb{C}^M$ are independent zero-mean circularly symmetric complex Gaussian (ZMCSCG) white noise with normalized covariance matrices: $\mathrm{E}[\,\mathbf{n_r}\mathbf{n_r}^H] = \mathbf{I}_{M-1}$, $\mathrm{E}[\,\mathbf{n}\mathbf{n}^H] = \mathbf{I}_M$.

The random vectors $\mathbf{h_r}$, $\mathbf{h_1}$ and $\mathbf{H_r}$ are the fading complex channel gains between the nodes. In this paper, to isolate the effect of geometry on cooperation, we consider the phase fading model [13], in which each entry in $\mathbf{h_r}$, $\mathbf{h_1}$ and $\mathbf{H_r}$ has a constant magnitude of unity, and an independent identically distributed (iid) random phase uniform between 0 and 2π in each channel realization. The phase fading model is applicable when the channel has a strong line-of-sight (LOS) component with limited scattering. We assume all nodes have perfect CSI, and the transmitters are able to adapt to the instantaneous channel realizations in each fading block. For compact presentation, we let S represent the the CSI (i.e., $\mathbf{h_r}$, $\mathbf{h_1}$, $\mathbf{H_r}$), and s designate a particular channel state realization. The channel power gain between the source and relay is denoted by $g \in \mathbb{R}_+$; as the cooperating nodes are assumed to be close together, the scenario of interest is when g is large.

The output of the relay depends causally on its past inputs. Further, there is a per-node average transmit power constraint on the system. To facilitate comparison with clusters that consist of single-antenna nodes, we equalize the average transmit power per antenna: $\mathrm{E}_S[\,|x_1|^2] \le 1$, and $\mathrm{Tr}(\mathbf{R}_{\mathbf{x_r}}) \le M-1$, where $\mathrm{Tr}(\cdot)$ is the trace of the matrix, and $\mathbf{R}_{\mathbf{x_r}} \triangleq \mathrm{E}_S[\,\mathbf{x_r}\mathbf{x_r}^H]$. Since the transmitters can perform power adaptation based on the CSI, the expectation $\mathrm{E}_S[\,\cdot\,]$ is taken over repeated channel uses over all channel states. The normalization factor $P \in \mathbb{R}_+$ represents the power available at the transmit cluster; in this paper it is also referred to as the SNR of the system.

For notational convenience, we define $\mathbf{x} \triangleq [\,x_1\;\mathbf{x_r}]^T$, and $\mathbf{H} \triangleq [\,\mathbf{h_1}\;\mathbf{H_r}]$. When the relay is colocated with the source, and the individual node power constraint relaxed to a sum power constraint, the cooperative system in Fig. 1 becomes a multiple-antenna channel:

$$\mathbf{y} = \sqrt{P/M}\,\mathbf{H}\mathbf{x} + \mathbf{n}, \qquad \mathrm{Tr}(\mathbf{R_x}) \le M. \tag{3}$$

In this paper we refer to (3) as the corresponding MIMO channel when drawing comparisons between a cooperative system and a multiple-antenna system.

III. Cooperative Capacity Gain

The three-terminal network shown in Fig. 1 is a relay channel [18], [19], and its capacity is not known in general. To study the capacity of the cooperative system, Section III-A and III-B consider the cut-set capacity upper bound and decode-and-forward achievable rate [19], [20], and compare them against MIMO channel capacity under different input distribution and channel gain assumptions. Then in Section III-C an SNR threshold is derived to delineate the cooperative capacity gain in different SNR regions, based on the network geometry and the number of cooperating antennas.

A. Cut-set bound

In [19], [20], a capacity upper bound was derived for the relay channel. It is commonly referred to as the cut-set bound, and under block fading with full CSI it is given by

$$R_{\mathrm{CS}} = \mathrm{E}_S\Big[\max_{p(x_1,\,\mathbf{x_r}|s)} \min\big\{\mathcal{I}_{\mathrm{SIMO}}, \mathcal{I}_{\mathrm{MIMO}}\big\}\Big], \tag{4}$$

where $\mathcal{I}_{\mathrm{SIMO}} \triangleq I(x_1;\mathbf{y_r}\mathbf{y}\,|\,\mathbf{x_r}, S\!=\!s)$ and $\mathcal{I}_{\mathrm{MIMO}} \triangleq I(\mathbf{x}\,;\mathbf{y}\,|\,S\!=\!s)$ are the mutual information conditioned on the channel state realization, and $p(x_1,\,\mathbf{x_r}\,|\,s)$ denotes all conditional joint input distributions that satisfy the per-node power constraints on x_1, $\mathbf{x_r}$.

When the additive channel noise is ZMCSCG, it was established in [21] that ZMCSCG inputs maximize the mutual information. Hence for a Gaussian relay channel, specifying the optimal input distribution $p(\mathbf{x})$ is tantamount to finding the maximizing input covariance matrix $\mathbf{R_x}$. However, as reported in [22], the optimization problem is non-convex and computing

$\mathbf{R_x}$ for even modest M remains a computationally arduous task. To obtain a more tractable upper bound to the capacity of the cooperative system, we relax the cut-set bound as

$$R_{\mathrm{CS}} \le \mathrm{E}_S\Big[\min\Big\{\max_{p(x_1,\,\mathbf{x_r}|s)} \mathcal{I}_{\mathrm{SIMO}},\, \max_{p(x_1,\,\mathbf{x_r}|s)} \mathcal{I}_{\mathrm{MIMO}}\Big\}\Big] \quad (5)$$

$$\le \mathrm{E}_S\Big[\min\Big\{\max_{p(x_1,\,\mathbf{x_r}|s)} \mathcal{I}_{\mathrm{SIMO}},\, \max_{p(\mathbf{x}|s)} \mathcal{I}_{\mathrm{MIMO}}\Big\}\Big], \quad (6)$$

where the relaxation in (5) is from optimizing the input distribution for each mutual information term independently, and in (6) the notation $p(\mathbf{x}\,|\,s)$ is overloaded to signify the per-node power constraints being replaced by the sum power constraint on $\mathbf{x}$: $\mathrm{Tr}(\mathbf{R_x}) \le M$.

We recognize the first mutual information term as the capacity of a $1 \times (2M-1)$ SIMO channel under transmit power constraint P/M:

$$C_{\mathrm{SIMO}} \triangleq \max_{p(x_1,\,\mathbf{x_r}|s)} \mathcal{I}_{\mathrm{SIMO}} \quad (7)$$

$$= \log\big(1 + (\|\sqrt{g}\,\mathbf{h_r}\|_F^2 + \|\mathbf{h_1}\|_F^2)P/M\big) \quad (8)$$

$$= \log\big(1 + (g(M-1) + M)P/M\big), \quad (9)$$

where $\|\cdot\|_F$ is the Frobenius norm, and the $\log$ is base 2. The next mutual information term is the instantaneous capacity of an $M \times M$ MIMO channel, for which we denote $C_{\mathrm{MIMO}} \triangleq \max_{p(\mathbf{x}|s)} \mathcal{I}_{\mathrm{MIMO}}$. Note that C_{MIMO} depends on the channel realization $\mathbf{H}$. In [23], [24] it was shown that the highest capacity from the best channel $\mathbf{H}$ subject to a fixed total channel power transfer $\|\mathbf{H}\|_F^2 = M^2$ is

$$\hat{C}_{\mathrm{MIMO}} \triangleq \max_{\mathbf{H}:\, \|\mathbf{H}\|_F^2 = M^2} C_{\mathrm{MIMO}} \quad (10)$$

$$= \max_{T:\, T=1,2,\ldots,M} T \log(1 + M^2 P/T^2). \quad (11)$$

The interpretation is that at low SNR, the best channel concentrates all power on one eigenmode, and at higher SNR, it distributes power evenly across additional eigenmodes. Since both C_{SIMO} and $\hat{C}_{\mathrm{MIMO}}$ are independent of the channel state, the upper bound becomes

$$R_{\mathrm{CS}} \le \min\{C_{\mathrm{SIMO}},\, \hat{C}_{\mathrm{MIMO}}\}. \quad (12)$$

B. Decode-and-forward achievable rate

In the decode-and-forward coding scheme [4], [7], [13], [19], transmission is done in blocks: the relay first fully decodes the source's message in one block, then in the ensuing block the relay and the source cooperatively send the message to the destination. The following rate can be achieved:

$$R_{\mathrm{DF}} = \mathrm{E}_S\Big[\max_{p(x_1,\,\mathbf{x_r}|s)} \min\{\mathcal{I}_{\mathrm{coop}}, \mathcal{I}_{\mathrm{MIMO}}\}\Big], \quad (13)$$

where $\mathcal{I}_{\mathrm{coop}} \triangleq I(x_1; \mathbf{y_r}\,|\,\mathbf{x_r}, S\!=\!s)$, and $\mathcal{I}_{\mathrm{MIMO}}$ is as defined above in the cut-set bound. Since finding the optimal input covariance matrix is computationally hard, we lower bound R_{DF} by considering a particular input covariance matrix $\mathbf{R_x} = \mathbf{I}_M$, i.e., by restricting the transmit cluster to sending only isotropic inputs:

$$R_{\mathrm{DF}} \ge \mathrm{E}_S\Big[\min\Big\{\mathcal{I}_{\mathrm{coop}}\big|_{\mathbf{R_x}=\mathbf{I}_M}, \mathcal{I}_{\mathrm{MIMO}}\big|_{\mathbf{R_x}=\mathbf{I}_M}\Big\}\Big]. \quad (14)$$

The first mutual information term can be interpreted as the rate at which cooperation information is conveyed from the source to the relay, and it is the capacity of a $1 \times (M-1)$ SIMO channel under transmit power constraint P/M:

$$C_{\mathrm{coop}} \triangleq \mathcal{I}_{\mathrm{coop}}\big|_{\mathbf{R_x}=\mathbf{I}_M} \quad (15)$$

$$= \log\big(1 + \|\sqrt{g}\,\mathbf{h_r}\|_F^2 P/M\big) \quad (16)$$

$$= \log\big(1 + g(M-1)P/M\big). \quad (17)$$

We note that when the relay is near the source ($g \gg 1$), $\|\sqrt{g}\,\mathbf{h_r}\|_F^2 \gg \|\mathbf{h_1}\|_F^2$, which implies $\mathcal{I}_{\mathrm{coop}} \approx \mathcal{I}_{\mathrm{SIMO}}$ and hence decode-and-forward is close to capacity-achieving.

The second mutual information term corresponds to the capacity of an $M \times M$ MIMO channel without CSI at the transmitter. In this case, the transmitter sends isotropic inputs and achieves the rate $C_{\mathbf{I}_M} \triangleq \mathcal{I}_{\mathrm{MIMO}}|_{\mathbf{R_x}=\mathbf{I}_M}$. Given a fixed total channel power transfer ($\|\mathbf{H}\|_F^2 = M^2$), an orthogonal channel (i.e., $\mathbf{H}\mathbf{H}^H = M\mathbf{I}_M$) maximizes capacity when the channel is unknown to the transmitter [25, Sec. 4.3]. Let $C_\perp$ be the $M \times M$ orthogonal channel capacity, then

$$C_{\mathbf{I}_M} < C_\perp = \log\big|\,\mathbf{I}_M + \mathbf{H}\mathbf{H}^H P/M\big|\,\Big|_{\mathbf{H}\mathbf{H}^H = M\mathbf{I}_M} \quad (18)$$

$$= M\log(1+P). \quad (19)$$

As $C_\perp$ is greater than $C_{\mathbf{I}_M}$, the orthogonal channel capacity does not serve to produce a lower bound on the decode-and-forward rate R_{DF}; rather, in Section III-C we employ $C_\perp$ to derive a lower bound to an SNR threshold, below which the cooperative capacity is lower bounded by the isotropic-input MIMO capacity $\mathrm{E}_S[\,C_{\mathbf{I}_M}]$.

C. Cooperative capacity in low-SNR and high-SNR regions

The decode-and-forward rate is lower bounded by restricting the transmit cluster to sending isotropic inputs, as described in (14). We want to determine the SNR region in which the cooperative capacity is at least as high as an isotropic-input MIMO channel. This corresponds to the SNR region when $C_{\mathrm{coop}} \ge C_{\mathbf{I}_M}$, i.e., when the cooperation link between the source and relay does not form a bottleneck. Since $C_{\mathbf{I}_M}$ depends on the channel state, we will consider its upper bound $C_\perp$ to obtain an SNR threshold lower bound that is independent of $\mathbf{H}$. We define the SNR threshold lower bound P_L as the positive transmit power level where C_{coop} and $C_\perp$ meet: $P_L \triangleq P > 0 : C_{\mathrm{coop}} = C_\perp$. In our scenario of interest ($g \gg M$), it can be observed that $C_\perp$ intersects with C_{coop} exactly once over positive P. Therefore, P_L is the solution to

$$(1+P_L)^M - g(M-1)P_L/M - 1 = 0, \quad (20)$$

which is an Mth degree polynomial in P_L, and its roots can be found numerically. Consequently, when P is lower than the SNR threshold lower bound, we have that

$$C_{\mathrm{coop}}\big|_{P<P_L} > C_\perp\big|_{P<P_L} > C_{\mathbf{I}_M}\big|_{P<P_L}, \quad (21)$$

which implies the decode-and-forward achievable rate in (14) is at least as high as the isotropic-input MIMO capacity:

$$R_{\text{DF}}\big|_{P<P_L} \geq \text{E}_S\Big[\min\Big\{C_{\text{coop}}\big|_{P<P_L}, C_{\mathbf{I}_M}\big|_{P<P_L}\Big\}\Big] \quad (22)$$

$$= \text{E}_S\big[C_{\mathbf{I}_M}\big|_{P<P_L}\big]. \quad (23)$$

Hence we refer to the range of SNR up to P_L as the MIMO-gain region. While imposing isotropic inputs restricts the MIMO channel capacity, in [26] it was shown that the capacity loss is upper bounded by approximately 0.53 bits per antenna.

Next, we investigate the SNR region under which the cooperative system is bounded away from achieving MIMO capacity gain. Recall that the capacity cut-set bound is upper bounded by (12). We define the SNR threshold upper bound P_U as the positive transmit power level where C_{SIMO} and $C_\perp$ meet: $P_U \triangleq P > 0 : C_{\text{SIMO}} = C_\perp$. Similarly, when $g \gg M$, P_U is the solution to

$$(1+P_U)^M - (1+g(M-1)/M)P_U - 1 = 0, \quad (24)$$

which can be solved numerically. For any P higher than the SNR threshold upper bound, we have

$$C_{\text{SIMO}}\big|_{P>P_U} < C_\perp\big|_{P>P_U} \leq \hat{C}_{\text{MIMO}}\big|_{P>P_U}. \quad (25)$$

When we apply (25) to the capacity upper bound in (12), it follows that the cooperative capacity is strictly less than the orthogonal MIMO capacity:

$$R_{\text{CS}}\big|_{P>P_U} \leq \min\Big\{C_{\text{SIMO}}\big|_{P>P_U}, \hat{C}_{\text{MIMO}}\big|_{P>P_U}\Big\} \quad (26)$$

$$= C_{\text{SIMO}}\big|_{P>P_U} < C_\perp\big|_{P>P_U}. \quad (27)$$

In particular, we observe that when $P > P_U$, the cooperative system capacity is limited by the SIMO channel capacity, which has a multiplexing gain of one. Hence we refer to the range of SNR above P_U as the coordination-limited region. We note that when the relay is near the source ($g \gg 1$), $\|\sqrt{g}\,\mathbf{h_r}\|_F^2 \gg \|\mathbf{h_1}\|_F^2$, which implies $C_{\text{coop}} \approx C_{\text{SIMO}}$, and hence P_L, P_U are approximately equal.

Numerical solutions of P_L, P_U for different values of M and g are plotted in Fig. 2. Note that since g is large, $P_L \approx P_U$ as expected, and their plots overlap. The SNR thresholds increase with g, thus when the cooperating nodes are closer together, the MIMO-gain region extends to a higher SNR. The number of cooperating antennas M, on the other hand, represents a coordination overhead. When M is large, the coordination-limited region sets in at a lower SNR.

We note that as the SNR increases, or the number of cooperating antennas becomes large, the cooperative system will eventually operate in the coordination-limited region, which is consistent with the capacity results in the asymptotic regime for high SNR [16], or large cooperative networks [17].

IV. Numerical Results

For computational tractability, we consider a small cooperating cluster network where $M = 2$. A two-dimensional ZMCSCG input vector $\mathbf{x} \in \mathbb{C}^2$, $\mathbf{x} \triangleq [x_1\ x_r]^T$ is completely specified by its Hermitian positive semi-definite covariance matrix $\mathbf{R_x} \in \mathbb{C}^{2\times 2}$. Since it is non-trivial to compute the

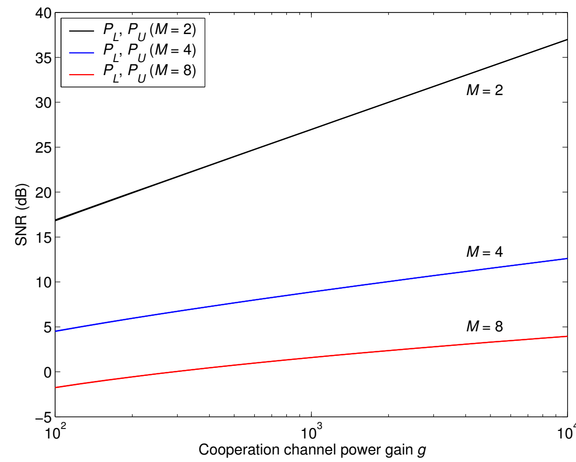

Fig. 2. Cooperative capacity gain regions: SNR threshold lower and upper bounds based on the network geometry and number of cooperating antennas.

optimal power allocation across the fading states, we tighten the power constraints to require they be satisfied in each fading block, i.e., $\text{E}[\,|x_1|^2] = \text{E}[\,|x_r|^2] = 1$, where the expectation is taken within a fading block. Applying the short-term power constraints, we have $\mathbf{R_x} = [\,[\,1\ \rho^*]^T\ [\,\rho\ 1\,]^T]$, where $\rho \in \mathbb{C}$ is the correlation coefficient between x_1 and x_r: $\rho \triangleq \text{E}[\,x_1 x_r^*]$, $|\rho|^2 \leq 1$, and ρ needs to be optimized for each channel state. The capacity upper bounds from Section III-A are valid since the tightened constraints cannot increase capacity, and the capacity lower bounds from Section III-B still apply because isotropic inputs satisfy the short-term power constraints.

Under the ZMCSCG input vector $\mathbf{x}$, the relay channel cut-set bound in (4) becomes

$$R_{\text{CS}} = \text{E}_S\Big[\max_{|\rho|^2\leq 1}\min\Big\{\log\big(1+(g+2)(1-|\rho|^2)P/2\big), \log\big|\,\mathbf{I}_2 + \mathbf{H}\mathbf{R_x}\mathbf{H}^H P/2\,\big|\Big\}\Big], \quad (28)$$

and the decode-and-forward rate (13) achieves

$$R_{\text{DF}} = \text{E}_S\Big[\max_{|\rho|^2\leq 1}\min\Big\{\log\big(1+g(1-|\rho|^2)P/2\big), \log\big|\,\mathbf{I}_2 + \mathbf{H}\mathbf{R_x}\mathbf{H}^H P/2\,\big|\Big\}\Big]. \quad (29)$$

The cut-set bound and the decode-and-forward rate are numerically evaluated via Monte Carlo simulation over 1000 instances of channel realizations. Under each channel realization, to ease computation complexity, the optimization is relaxed to find the maximum rate with the correlation coefficient ρ varying in discrete steps of $0.05e^{j0.1}$.

The relay channel cut-set bound R_{CS} and decode-and-forward rate R_{DF} are shown in Fig. 3, along with the SNR threshold lower and upper bounds P_L, P_U, and the multiple-antenna capacity bounds $C_\perp$, $\text{E}_S[C_{\mathbf{I}_M}]$, C_{SIMO}, and C_{coop}. For comparison, also plotted in Fig. 3 is the non-cooperative capacity $C_{\text{nc}} = \log(1+2P)$, which corresponds to the case where the relay is not used, and the source is under power constraint P. We assume the relay is near the source ($g = 100$); decode-and-forward is close to capacity-achieving as expected, and plots of R_{CS}, R_{DF} appear overlapped. We

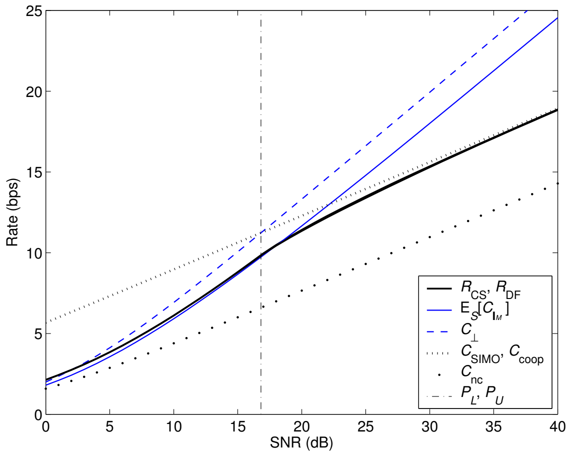

Fig. 3. MIMO-gain and coordination-limited SNR regions of a 2×2 cooperative system.

observe that in the MIMO-gain region when $P < P_L$, the relay rate R_{DF} outperforms the isotropic-input MIMO capacity $\mathrm{E}_S[C_{\mathbf{I}_M}]$. On the other hand, in the coordination-limited region, as $P > P_U$, the relay cut-set bound R_{CS} fails to parallel the orthogonal channel capacity $C_\perp$. Indeed, the cooperative capacity is bottlenecked by the SIMO channel capacity C_{SIMO}, and which scales with the SNR as $\Theta(\log P)$, instead of $\Theta(2 \log P)$.

V. Conclusion

We have shown that in a cooperative system, up to a lower bound to an SNR threshold, MIMO capacity gain is achievable. However, beyond an upper bound to the SNR threshold, the cooperative capacity is limited by that of a SIMO channel, which has a multiplexing gain of one. The SNR threshold depends on the network geometry (the power gain g between the source and relay) and the number of cooperating antennas M; when the relay is close to the source ($g \gg 1$), the SNR threshold lower and upper bounds are approximately equal.

As the cooperating nodes draw closer to each other, i.e., as g increases, the MIMO-gain region extends to a higher SNR. On the other hand, a populous cluster, i.e., when M is large, represents a more demanding cooperation overhead, and the coordination-limited region sets in at a lower SNR. Therefore, for a given cooperative system (i.e., fixed geometry and number of cooperating antennas), a MIMO-gain SNR region can be established within which cooperation is efficient.

In this paper we assume a phase fading environment where the channel gain has constant magnitude. To incorporate the effects of fading channel magnitude on cooperative performance, the analysis may be extended under Rayleigh fading. Further work includes considering cooperative systems for which the dominant coordination cost resides in the receive cluster (the source has multiple antennas, and the relay is near the single-antenna destination), or possibly in both the transmit and receive clusters (one relay is near the single-antenna source, and a second relay is near the single-antenna destination).

References

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[2] ——, "User cooperation diversity—Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, Nov. 2003.

[3] T. E. Hunter and A. Nosratinia, "Cooperation diversity through coding," in *Proc. IEEE Int. Symp. Inform. Theory*, 2002, p. 220.

[4] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[5] N. Jindal, U. Mitra, and A. Goldsmith, "Capacity of ad-hoc networks with node cooperation," in *Proc. IEEE Int. Symp. Inform. Theory*, 2004, p. 271, also in preparation for *IEEE Trans. Inform. Theory*.

[6] C. T. K. Ng and A. J. Goldsmith, "Transmitter cooperation in ad-hoc wireless networks: Does dirty-payer coding beat relaying?" in *Proc. IEEE Inform. Theory Workshop*, 2004, pp. 277–282.

[7] A. Host-Madsen, "Capacity bounds for cooperative diversity," *IEEE Trans. Inform. Theory*, submitted.

[8] M. A. Khojastepour, A. Sabharwal, and B. Aazhang, "Improved achievable rates for user cooperation and relay channels," in *Proc. IEEE Int. Symp. Inform. Theory*, 2004, p. 4.

[9] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channel," *IEEE Trans. Inform. Theory*, vol. 51, no. 6, pp. 2020–2040, June 2005.

[10] A. Host-Madsen, "On the achievable rate for receiver cooperation in ad-hoc networks," in *Proc. IEEE Int. Symp. Inform. Theory*, 2004, p. 272.

[11] ——, "A new achievable rate for cooperative diversity based on generalized writing on dirty paper," in *Proc. IEEE Int. Symp. Inform. Theory*, 2003, p. 317.

[12] I. Maric, R. D. Yates, and G. Kramer, "The discrete memoryless compound multiple access channel with conferencing encoders," in *Proc. IEEE Int. Symp. Inform. Theory*, 2005, pp. 407–410.

[13] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, no. 9, pp. 3037–3063, Sept. 2005.

[14] C. T. K. Ng and A. J. Goldsmith, "Capacity gain from transmitter and receiver cooperation," in *Proc. IEEE Int. Symp. Inform. Theory*, 2005, pp. 397–401.

[15] ——, "Capacity and power allocation for transmitter and receiver cooperation in fading channels," in *Proc. IEEE Internat. Conf. Commun.*, 2006, accepted.

[16] A. Host-Madsen, "The multiplexing gain of wireless networks," in *Proc. IEEE Int. Symp. Inform. Theory*, 2005, pp. 2065–2069.

[17] M. Gastpar and M. Vetterli, "On the capacity of large Gaussian relay networks," *IEEE Trans. Inform. Theory*, vol. 51, no. 3, pp. 765–779, Mar. 2005.

[18] E. C. van der Meulen, "Three-terminal communication channels," *Adv. Appl. Prob.*, vol. 3, pp. 120–154, 1971.

[19] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. 25, no. 5, pp. 572–584, Sept. 1979.

[20] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley-Interscience, 1991.

[21] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Europ. Trans. Telecommun.*, vol. 10, pp. 585–595, Nov. 1999.

[22] B. Wang, J. Zhang, and A. Host-Madsen, "On the capacity of MIMO relay channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 1, pp. 29–43, Jan. 2005.

[23] N. Chiurtu and B. Rimoldi, "Varying the antenna locations to optimize the capacity of multi-antenna Gaussian channels," in *Proc. IEEE Internat. Conf. Acous., Speech, Signal Proc.*, 2000, pp. 3121–3123.

[24] Z. Shen, J. G. Andrews, and B. L. Evans, "Upper bounds on MIMO channel capacity with channel Frobenius norm constraints," in *Proc. IEEE Globecom Conf.*, 2005.

[25] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge University Press, 2003.

[26] T. Philosof and R. Zamir, "The cost of uncorrelation and non-cooperation in MIMO channels," in *Proc. IEEE Int. Symp. Inform. Theory*, 2005, pp. 770–774.