Joint Source and Channel Coding for MIMO Systems: Is It Better to be Robust or Quick?

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Abstract— **We optimize the diversity-multiplexing tradeoff inherent to MIMO systems to minimize total distortion in a joint source and channel code design. Our goal is to find the optimal balance between the increased data rate provided by multiplexing versus the robustness provided by diversity. We first consider concatenation of a vector quantizer and MIMO channel code. We show that in the high SIR regime we can obtain a closed form expression for the optimal multiplexing rate and the resulting end-to-end distortion can be expressed as a simple function of the optimal diversity-multiplexing tradeoff point. The optimization framework can be extended to a broad class of source and channel codes, which we demonstrate using an example of a progressive video source code combined with a space-time channel code. Similar ideas can be applied to MIMO networks and to delay distortion.**

I. Preliminaries

We assume a MIMO channel with M transmit antennas, N receive antennas, and AWGN. The channel gains between transmit and receive antennas are i.i.d. complex Gaussian with unit variance. These gains are assumed known at the receiver but not the transmitter. The channel is constant over a block of T symbols, with each block i.i.d.

Consider any family of codes for this channel. We define $P_e(SNR)$ as the average probability of error for the codebook and $R(SNR)$ as the number of bits per symbol for the codebook. We say that a channel coding scheme achieves multiplexing gain r and diversity gain d if $\lim_{\log SNR\to\infty} \frac{R(SNR)}{\log SNR} = r$, and $\lim_{\log SNR\to\infty} \frac{\log P_e(SNR)}{\log SNR} = -d$. The first limit implies that the rate of the channel code scales with SNR as $r \log SNR$. For each r we define the optimal diversity gain $d^*(r)$ as the supremum of the diversity gain achieved by any scheme. We make use of the following key result from [1].

Diversity-Multiplexing Tradeoff: Assume $T \geq M + N - 1$. Then $d^*(r) = (M - r)(N - R)$ for $0 \leq r \leq \min(M, N)$.

II. Joint Source and Channel Coding

We apply an s-bit quantizer to a data source $u \in \Re^k$ with pdf $f(u)$. We assume the source encoder/decoder pair achieves the high-resolution pth order noiseless distortion [2] of $D_s = 2^{-ps/k+O(1)}$. From [2], the end-to-end distortion of this quantized source transmitted over an error-prone channel with equally likely channel codeword error P_e can be bounded by $D_T \leq D_s + O(1)P_e$.

By matching the source and channel code rate $s = Tr \log SNR$, substituting for the P_e associated with the diversity/multiplexing MIMO tradeoff, matching exponents in both distortion terms of D_T, and taking the high SNR limit we obtain the following two theorems. These theorems yield a closed-form solution for the optimal multiplexing rate of the MIMO channel code and a bound on the corresponding end-to-end distortion.

Theorem 1: For $\frac{1}{\min(M,N)-1} \leq pT/k \leq (M-1)(N-1)$, choosing r^* such that $(N-r)(M-r) = pTr/k$ yields the multiplexing rate that minimizes end-to-end distortion D_T as $\log SNR \to \infty$.

Theorem 2: The asymptotic minimum distortion D_T is bounded by the diversity associated with the optimal multiplexing rate:

$$\lim_{\log SNR\to\infty} \frac{D_T}{\log SNR} \leq -d^*(r^*).$$

We can also find the optimal multiplexing rate r^* to minimize total distortion for large, but finite, SNR by solving the convex optimization $\min_r\ 2^{-\frac{pT}{k} r \log SNR} + 2^{-(M-r)(N-r)\log SNR}$. A similar convex optimization can be used to find the optimal multiplexing/diversity tradeoff for a broad range of source and channel codes. In Fig. 1 we show this optimization for the distortion vs. multiplexing rate of a progressive video source code and MIMO channel code. The same optimization can be applied to MIMO networks and delay distortion.

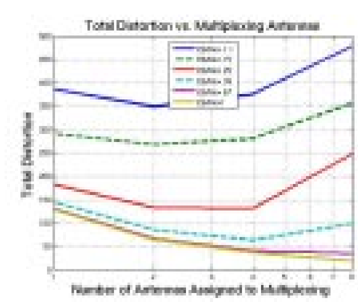

Fig. 1. Total distortion vs. antennas for multiplexing.

References

[1] L. Zheng, D. Tse, "Diversity and Multiplexing: the Optimal Tradeoff in Multiple Antenna Channels", *IEEE Trans. Inform. Theory*, pp. 1073–1096, May 2003.

[2] B. Hochwald, K. Zeger, "Tradeoff Between Source and Channel Coding", *IEEE Trans. Inform. Theory*, pp. 1412–1424, Sept. 1997.